REAL TURBOJET

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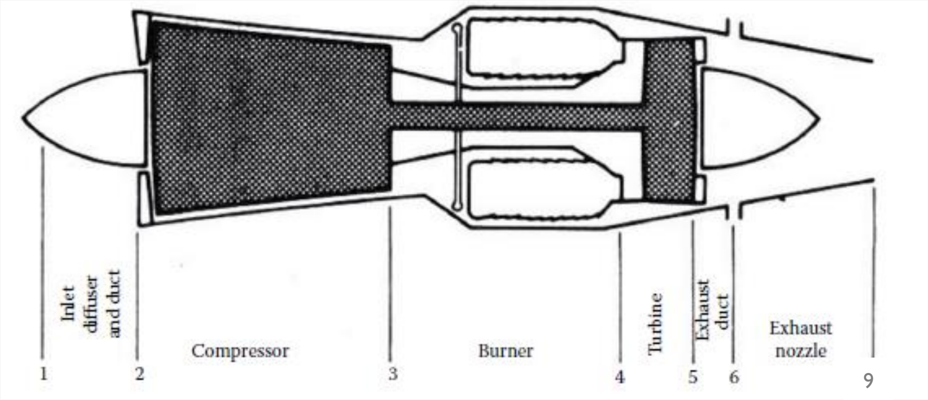
LITERATURE:

- **Jack D. Mattingly, Elements of Propulsion: Gas Turbines and Rockets, AIAA Education Series 2006 (Chapter 7)**
- **Jack D. Mattingly, Elements of Gas Turbine Propulsion, Tata McGraw Hill Education Private Limited, 2013 (Chapter 7)**
- **Gordon C. Oates, Aerothermodynamics of Gas Turbine and Rocket Propulsion, AIAA Education Series, 1997 (Chapter 7)**

AMBIENT CONDITIONS, INLET

Engine work in static conditions $V_0=0$ → $P_{t0} = P_0, \ T_{t0} = T_0$



Ram pressure recovery for flight condition $(M_0 > 0)$

$$P_{t0} = P_0\left(1 + \frac{k-1}{2}{M_0}^2\right)^{k/(k-1)} \qquad T_{t0} = T_0\left(1 + \frac{k-1}{2}{M_0}^2\right)$$

INLET pressure losses → $P_{t2} = \pi_D P_{t0}$

No thermal losses $T_{t2} = T_{t0}$

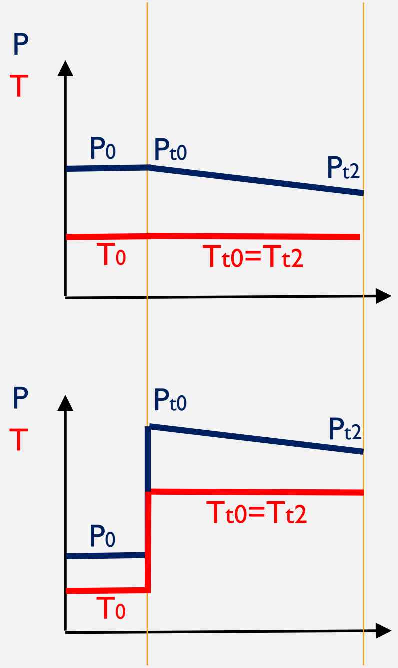
Typical range:

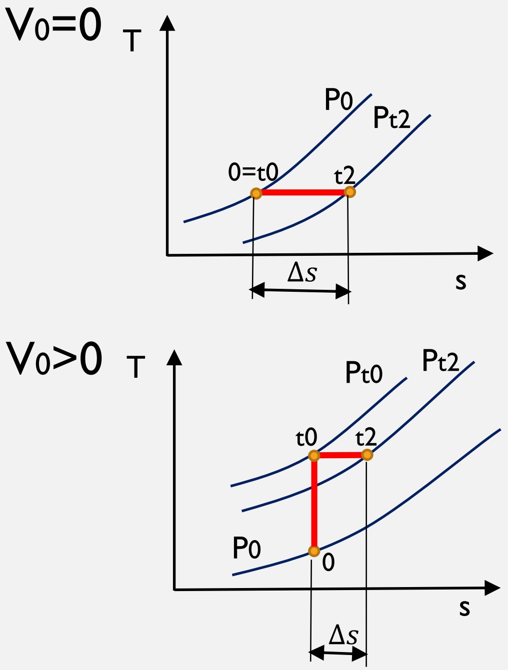
$\pi_D = 0{,}95 - 0{,}99, \quad \pi_D = P_{t2} \,/\, P_{t0}$

π_D is lower for high supoersonic speed

Pressure losses are visible in entropy grow:

$$\Delta s = -R ln(P_{t2}/P_{t0}) = R ln(1/\pi_D)$$





COMPRESSOR

COMPRESSOR (2 – 3)

Compressor work is politropic

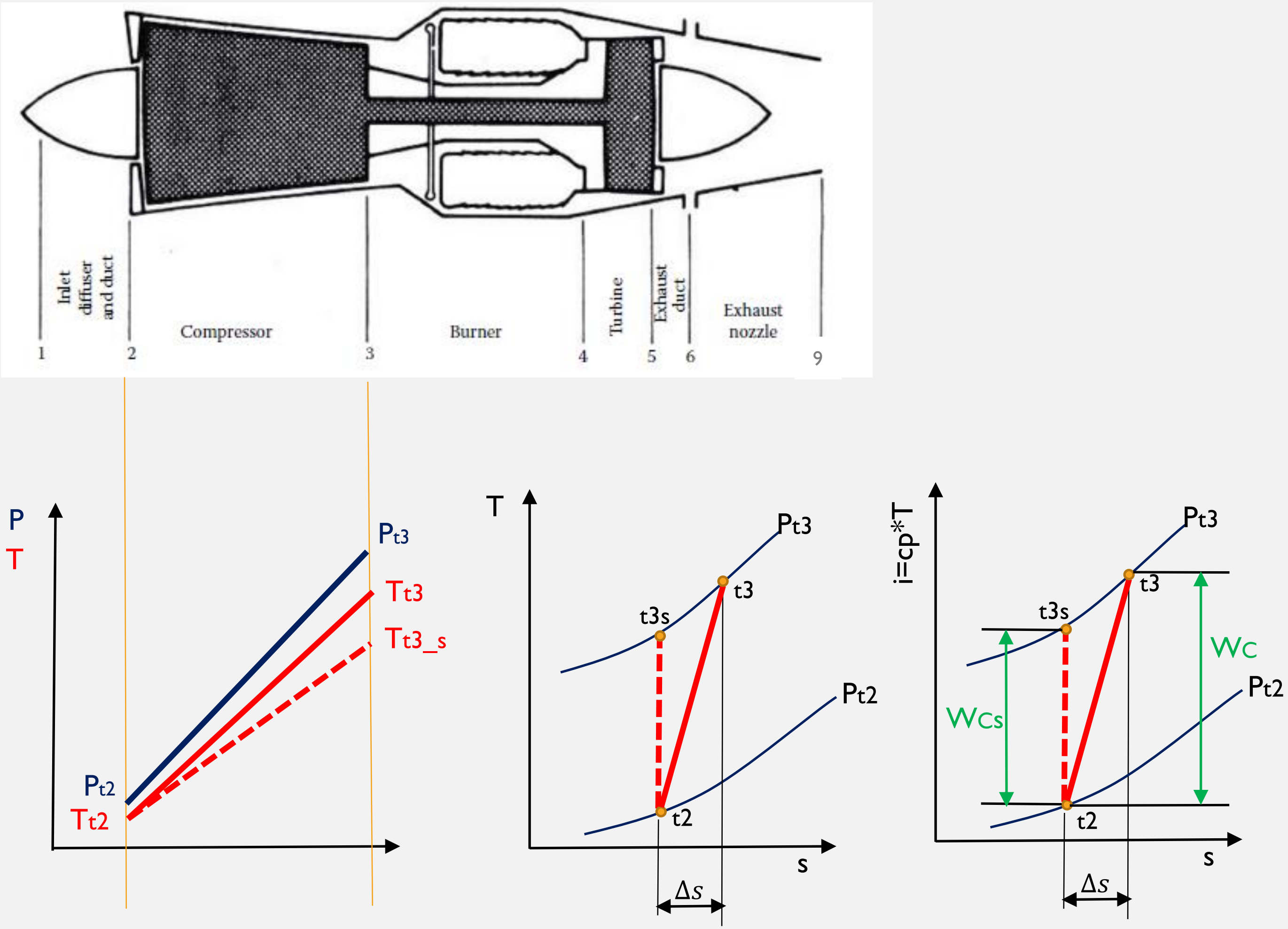
$$\pi_C = \frac{P_{t3}}{P_{t2}} = \text{CPR}$$

Compressor loss is visible in entropy grow:
$\Delta s = Cp * ln(T_{t3}/T_{t2}) - Rln(P_{t3}/P_{t2}))$

Isentropic efficiency

$$\eta_C = \frac{W_{Cs}}{W_C} = \frac{Cp(T_{t3s} - T_{t2})}{Cp(T_{t3} - T_{t2})} = \frac{T_{t3s}/T_{t2} - 1}{T_{t3}/T_{t2} - 1}$$

$$= \frac{(P_{t3}/P_{t2})^{\frac{k-1}{k}} - 1}{T_{t3}/T_{t2} - 1} = \frac{(\pi_C)^{\frac{k-1}{k}} - 1}{T_{t3}/T_{t2} - 1}$$



Compressor outlet temeparture is higher and copressor entropy grows dueto losses.

COMPRESSOR POLYTROPIC EFFICIENCY

$$e_C = \frac{\text{ideal work of compression for a differential pressure change}}{\text{actual work of compression for a differential pressure change}}$$

$$e_C = \frac{di_{ts}}{di_t} = \frac{dT_{ts}}{dT_t} = \frac{dT_{ts}/T_t}{dT_t/T_t}$$

$$dT_{ts}/T_t = \frac{k-1}{k} dP_t/P_t$$

rearangment

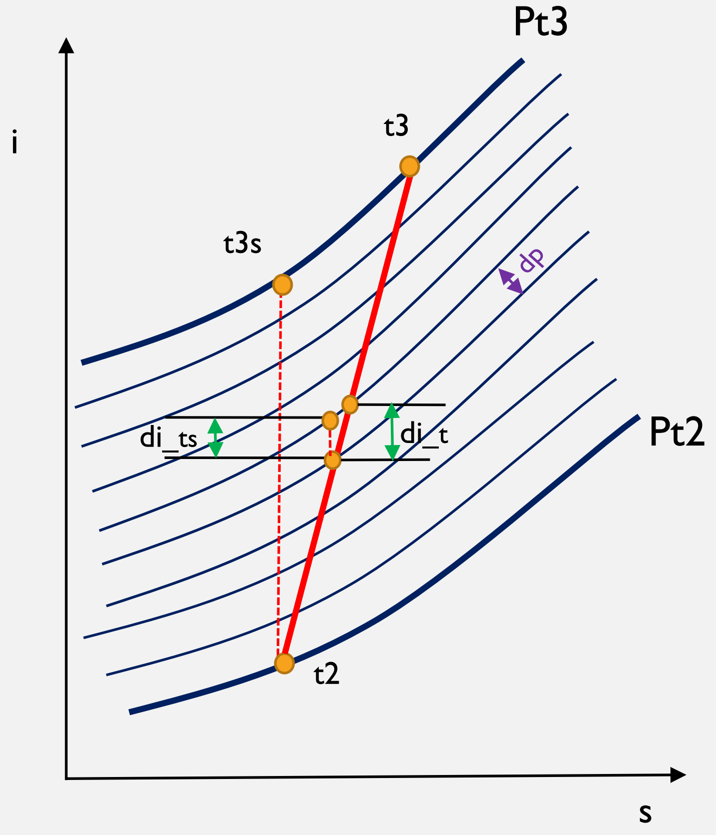
$$dT_t/T_t = \frac{k-1}{e_C k} dP_t/P_t$$

after differentiation

$$\ln(T_{t3}/T_{t2}) = \frac{k-1}{e_C k} \ln(P_{t3}/P_{t2})$$

$$e_C = \frac{k-1}{k} \ln(P_{t3}/P_{t2}) / \ln(T_{t3}/T_{t2})$$

$$T_{t3}/T_{t2} = (P_{t3}/P_{t2})^{\frac{k-1}{e_C k}}$$



Polytropic efficiency is treated as a constant for specific compressor. It is value independent of number of stages in the compressor (CPR). Higher polytropic efficiency is for modern compressors

Typical range:

e_C = 0,88 – 0,92

POLYTROPIC VS. ISENTROPIC EFFICIENCY OF COMPRESSOR

Isentropic efficiency

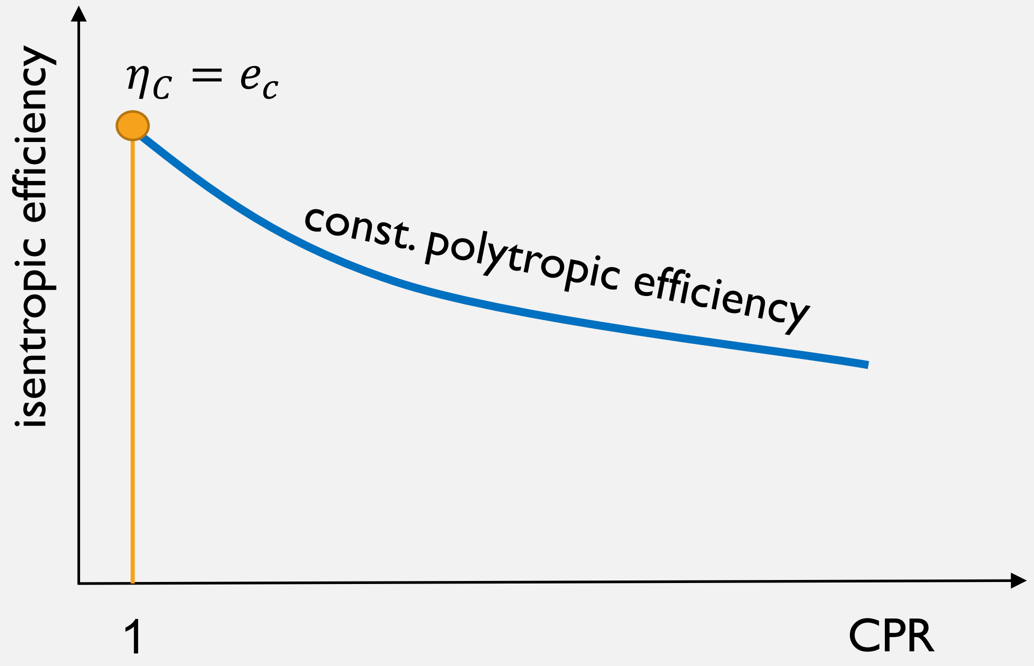
$$\eta_C = \frac{(P_{t3}/P_{t2})^{\frac{k-1}{k}} - 1}{T_{t3}/T_{t2} - 1}$$

Polytropic efficiency relation

$$T_{t3}/T_{t2} = (P_{t3}/P_{t2})^{\frac{k-1}{e_C\,k}}$$

Isentropic vs. polytropic compressor efficiency

$$\eta_C = \frac{(P_{t3}/P_{t2})^{\frac{k-1}{k}} - 1}{(P_{t3}/P_{t2})^{\frac{k-1}{e_C\,k}} - 1}$$



Isentropic efficiency decreases with compressor pressure ratio (CPR) growing for defined politropic efficiency.

Compressor isentropic efficiency for CPR>1 is lower than politropic efficiency`

COMPRESSOR WORK AND POWER

Compressor work:

$$W_C = Cp(T_{t3} - T_{t2})$$

for isentropic efficiency →

$$W_C = CpT_{t2}\left(\frac{(\pi_C)^{\frac{k-1}{k}} - 1}{\eta_C}\right)$$

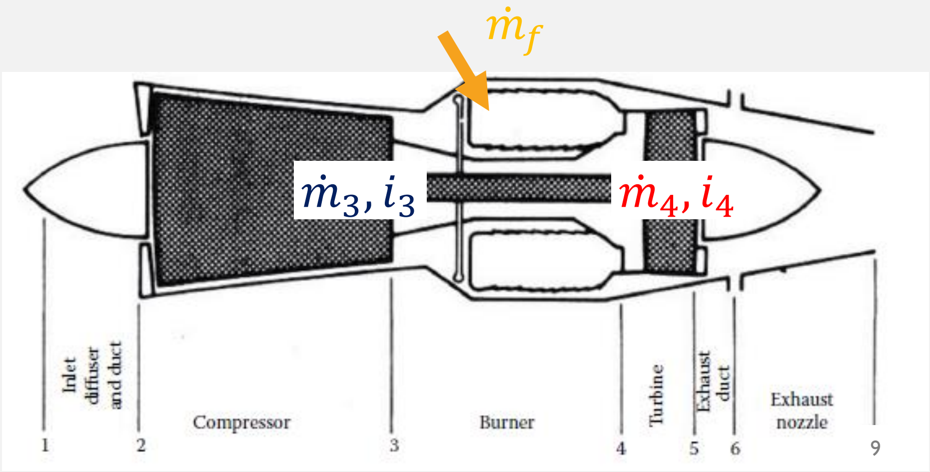
for polytropic efficiency →

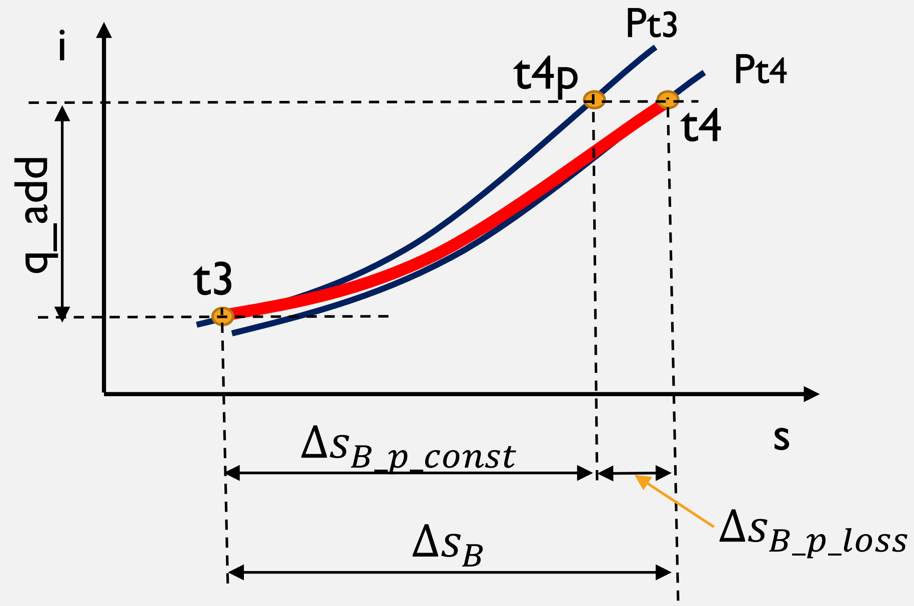
$$W_C = CpT_{t2}\left((\pi_C)^{\frac{k-1}{e_C\,k}} - 1\right)$$

Compressor power:

$$P_C = \dot{m}_C W_C = \dot{m}_0 Cp(T_{t3} - T_{t2})$$

BURNER/COMBUSTER





Energy balance:

$$\eta_B \dot{m}_f FHV = \dot{m}_4(i_{t4} - i_{t3}) = \dot{m}_4 Cp_t T_{t4} - \dot{m}_3 Cp_c T_{t3}$$

Burner efficiency

$$\eta_B = \frac{heat\ added\ to\ the\ gas\ flow\ through\ the\ combastor}{heat\ contained\ in\ fuel}$$

$$\eta_B \dot{m}_f FHV = \dot{m}_3 Cp_B(T_{t4} - T_{t3})$$

Fuel mass flow

$$\dot{m}_f = \frac{\dot{m}_3 Cp_B(T_{t4} - T_{t3})}{\eta_B FHV}$$

Fuel-air ratio

$$f = \frac{\dot{m}_f}{\dot{m}_0} = \frac{Cp_B(T_{t4} - T_{t3})}{\eta_B FHV}$$

Pressure losses:

$$\pi_B = \frac{P_{t4}}{P_{t3}}$$

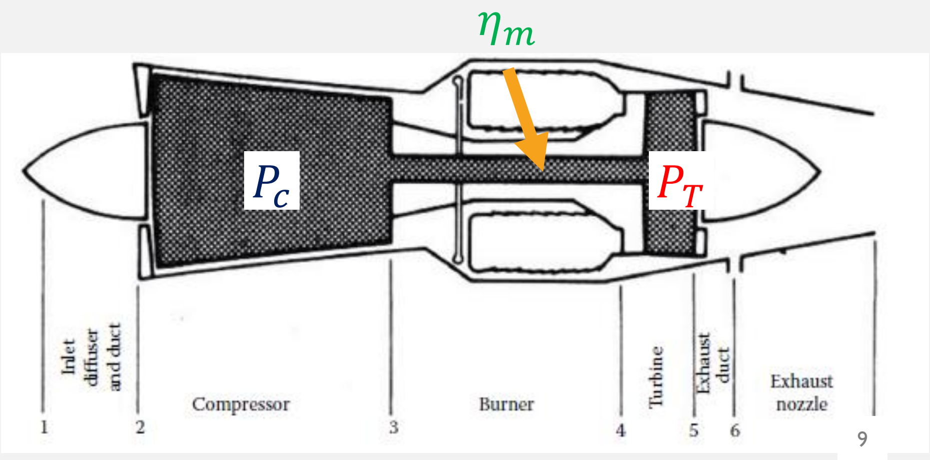
Entropy increase in a burner:

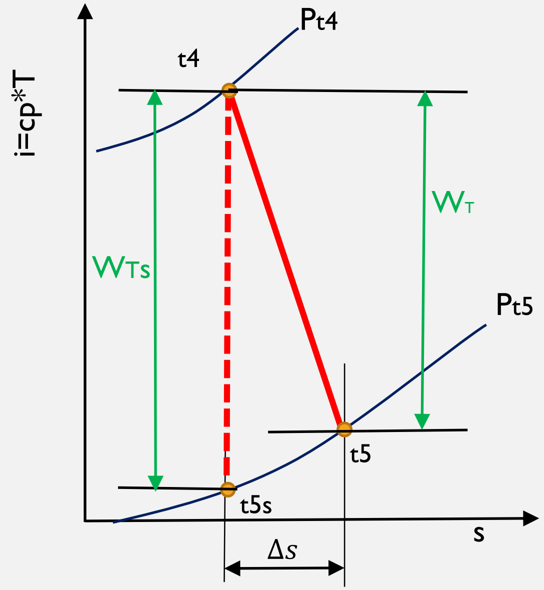
for izobaric process $\Delta s_{B\_p\_const} = cp_B ln\frac{T_{t4}}{T_{t3}}$

for pressure losses $\Delta s_{B\_p\_loss} = -R_t ln\frac{P_{t4}}{P_{t3}}$

for all $\Delta s_B = cp_B ln\frac{T_{t4}}{T_{t3}} - R_t ln\frac{P_{t4}}{P_{t3}}$

TURBINE





Compressor-turbine energy balance:

$$P_T = \dot{m}_4 Cp_T(T_{t4} - T_{t5}) = \frac{1}{\eta_m} P_C$$

Turbine outlet temperature

$$T_{t5} = T_{t4} - \frac{P_C}{\eta_m \dot{m}_4 Cp_T(T_{t4} - T_{t5})}$$

for: $\dot{m}_2 = \dot{m}_0$ and $\dot{m}_4 = \dot{m}_0 + \dot{m}_f$

$$T_{t5} = T_{t4} - \frac{W_C}{\eta_m(1+f)Cp_T(T_{t4} - T_{t5})}$$

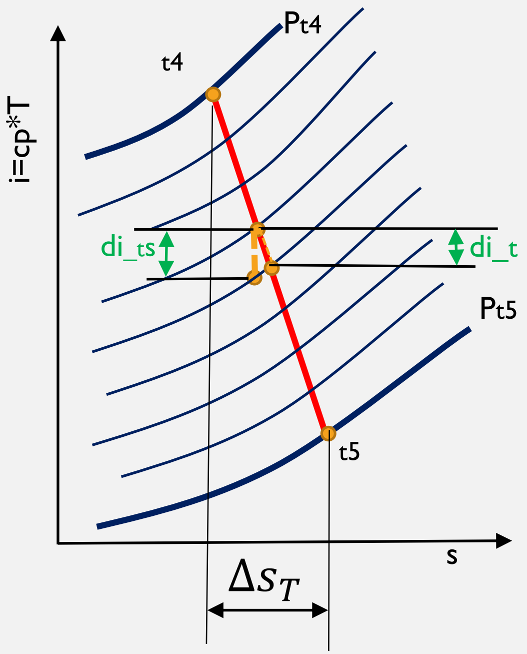
Turbine pressure drop

$$\pi_T = P_{t4}/P_{t5} > 1 \qquad \pi_T = \left(1 - \frac{1 - T_{t5}/T_{t4}}{\eta_T}\right)^{\frac{k_t}{k_t - 1}}$$

Turbine isentropic efficiency

$$\eta_T = \frac{W_T}{W_{Ts}} = \frac{Cp_t(T_{t4} - T_{t5})}{Cp_t(T_{t4} - T_{t5s})} = \frac{1 - T_{t5}/T_{t4}}{1 - T_{t5s}/T_{t4}} = \frac{1 - T_{t5}/T_{t4}}{1 - (P_{t5}/P_{t4})^{\frac{k_t - 1}{k_t}}}$$

TURBINE POLYTROPIC EFFICIENCY



$$e_T = \frac{\text{actual turbine work for a differential pressure change}}{\text{ideal work of turbine for a differential pressure change}}$$

$$e_T = \frac{di_t}{di_{ts}} = \frac{dT_t}{dT_{ts}} = \frac{dT_t/T_t}{dT_{ts}/T_t}$$

$$dT_{ts}/T_t = \frac{k_t - 1}{k_t} dP_t/P_t$$

rearangment

$$dT_t/T_t = e_T \frac{k_t - 1}{k_t} dP_t/P_t$$

after differentiation

$$\ln(T_{t5}/T_{t4}) = e_T \frac{k_t - 1}{k_t} ln(P_{t5}/P_{t4})$$

$$e_T = \frac{k_t}{k_t - 1} ln\,(T_{t3}/T_{t2})/ln(P_{t3}/P_{t2})$$

$$T_{t5}/T_{t4} = (P_{t5}/P_{t4})^{\frac{e_T(k_t-1)}{k_t}}$$

Turbine entropy grow:

$$\Delta s_T = Cp_t * ln(T_{t5}/T_{t4}) - R_t ln(P_{t5}/P_{t4}))$$

Polytropic efficiency is assumed as a constant for the turbine. It's value is independent of number of stages in the turbine (TPR). Higher polytropic efficiency is for modern turbine.

Typical range:

e_C = 0,87 – 0,9

POLYTROPIC VS. ISENTROPIC EFFICIENCY OF TURBINE

Isentropic efficiency

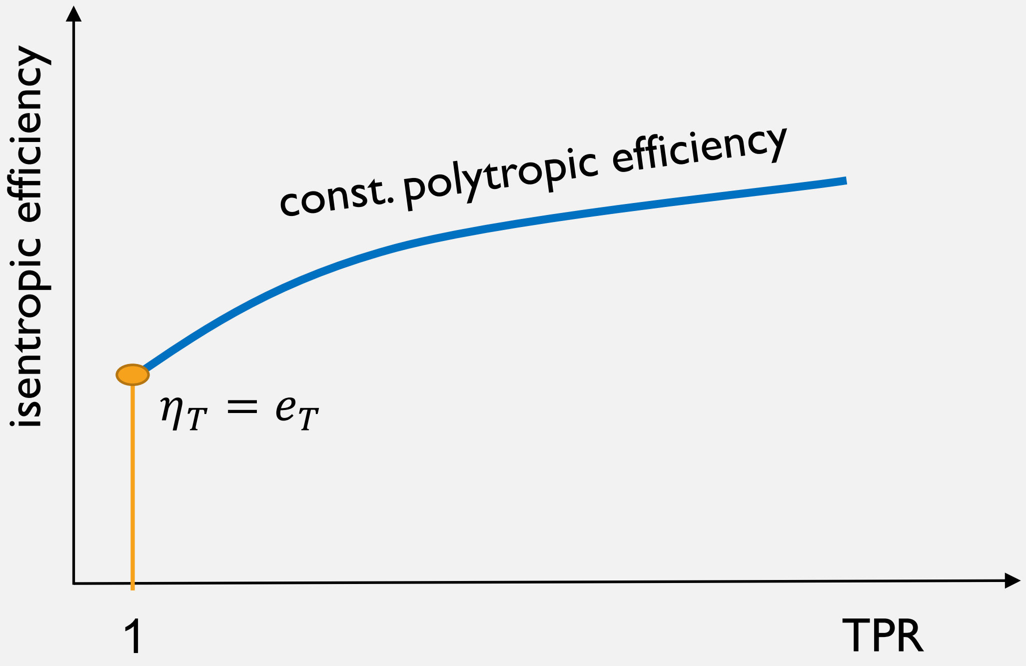
$$\eta_T = \frac{1 - T_{t5}/T_{t4}}{1 - (P_{t5}/P_{t4})^{\frac{k_t-1}{k_t}}}$$

Polytropic efficiency relation

$$T_{t5}/T_{t4} = (P_{t5}/P_{t4})^{\frac{e_T(k_t-1)}{k_t}}$$

Isentropic vs. polytropic compressor efficiency

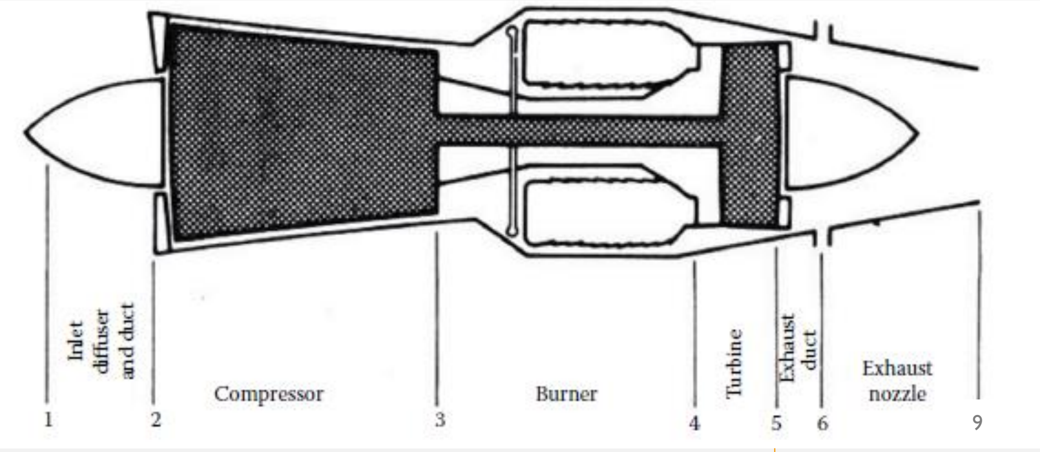
$$\eta_T = \frac{1 - (P_{t5}/P_{t4})^{\frac{e_T(k_t-1)}{k_t}}}{1 - (P_{t5}/P_{t4})^{\frac{k_t-1}{k_t}}}$$



Isentropic efficiency increases with growing of turbine pressure ratio (TPR) for defined politropic efficiency.

Turbine isentropic efficiency for TPR>1 is higher than politropic efficiency`

FULL EXPANSION IN THE NOZZLE WITH LOSESS



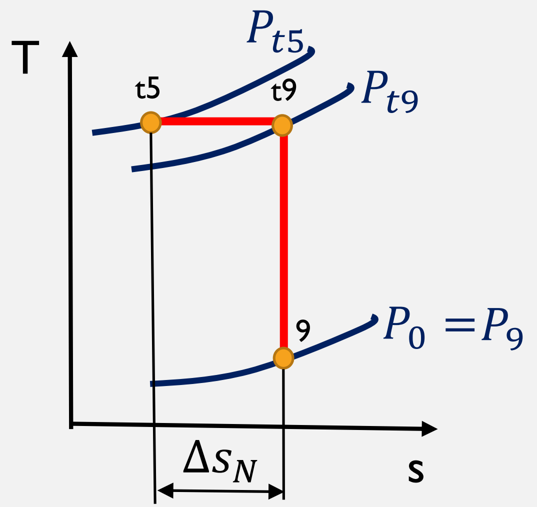
NOZZLE (5-9)

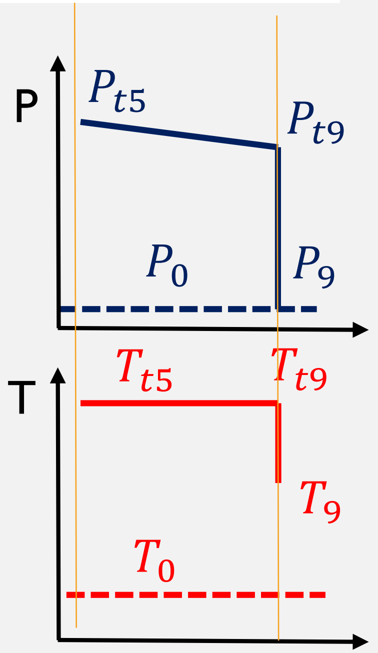
Pressure losses: $\pi_N = \frac{P_{t9}}{P_{t5}} < 1 \quad P_{t9} = \pi_N P_{t5}$

No heat losess $\quad T_{t9} = T_{t5}$

Full expansion: $\quad P_9 = P_0$

Entropy growth: $\Delta s_N = R_t ln \frac{1}{\pi_N}$



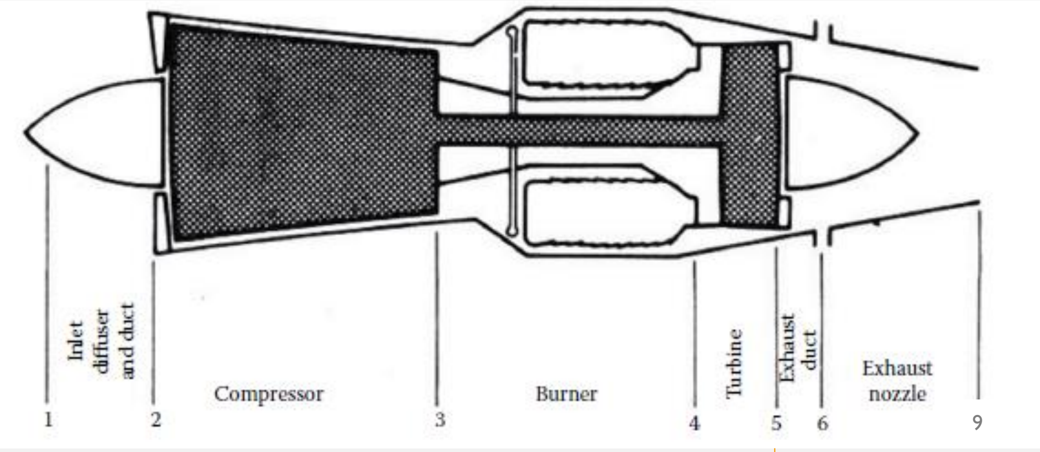


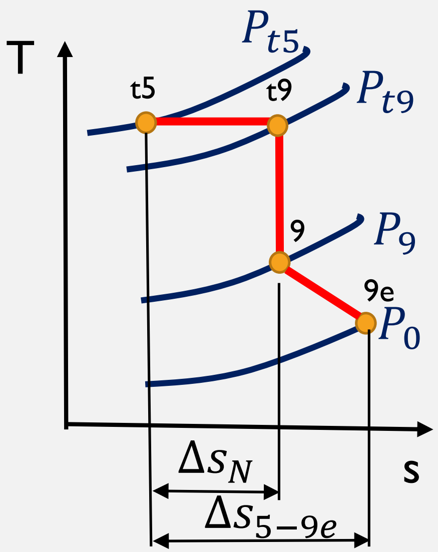
$$c_9 = \sqrt{2Cp_t(T_{9t} - T_9)}$$ - for incompressible flow

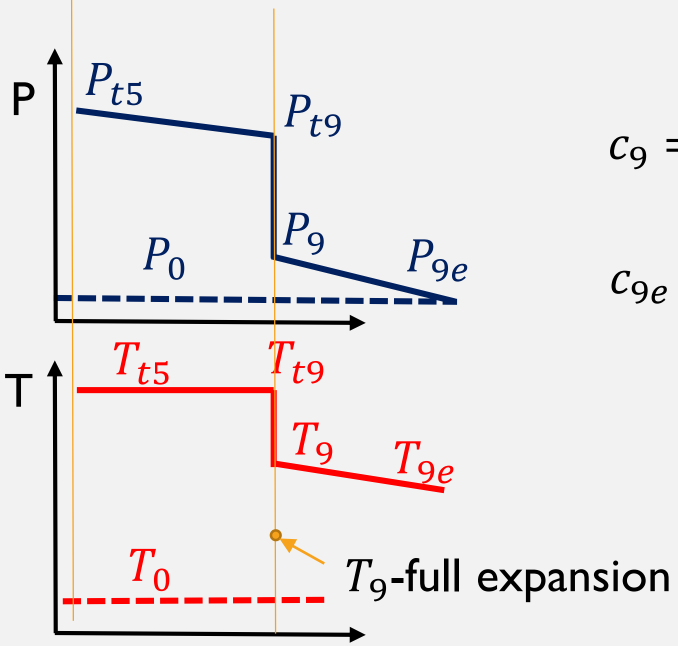
$$c_9 = a_9 M_9 = \sqrt{k_t R_t T_9} * \sqrt{\frac{2}{k_t - 1}\left(\frac{T_{t9}}{T_9} - 1\right)}$$ - for compressible flow

$$\frac{T_{t9}}{T_9} = \frac{P_{t9}}{P_9}^{(k_t-1)/k_t}$$ - isentropic relation betwen total and static paramethers

INCOMPLETE EXPANSION IN THE NOZZLE WITH LOSESS







NOZZLE (5-9)

Pressure losses: $\pi_N = \frac{P_{t9}}{P_{t5}} < 1 \quad P_{t9} = \pi_N P_{t5}$

No heat losess $\quad T_{t9} = T_{t5}$

Incomploete expansion: $\quad P_9 > P_0$

Entropy growth: $\Delta s_{5-9e} = Cp_t ln\frac{T_{9e}}{T_{t5}} - R_t ln\frac{P_0}{P_{t5}}$

$$c_9 = a_9 M_9 = \sqrt{k_t R_t T_9} * \sqrt{\frac{2}{k_t - 1}\left(\frac{T_{t9}}{T_9} - 1\right)}$$

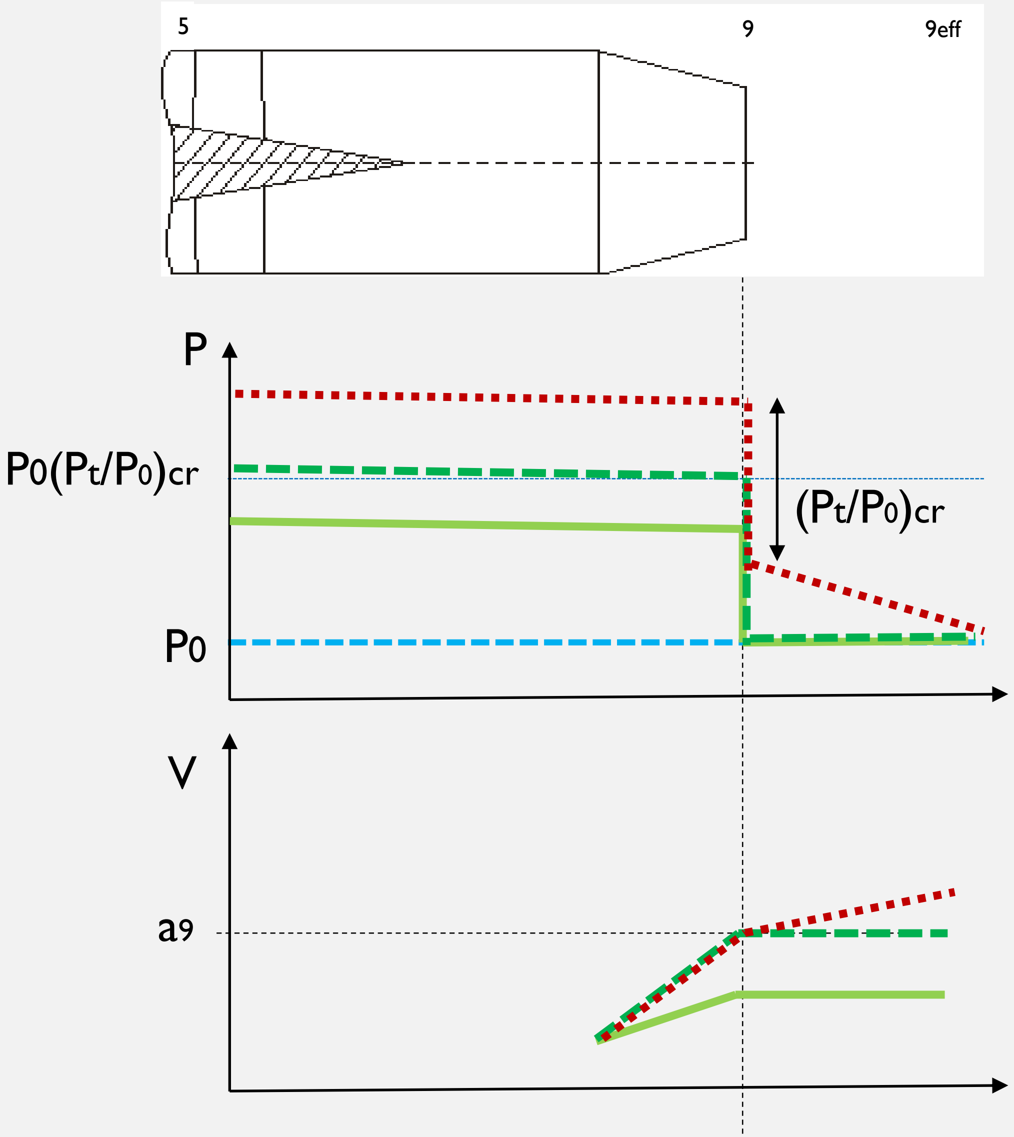
$$c_{9e} = c_9 + \frac{A_9(P_9 - P_o)}{\dot{m}_9} = c_9 + \frac{(P_9 - P_o)}{\rho_9 c_9}$$

$$= c_9 + \frac{R_9 T_9 (P_9 - P_o)}{P_9 c_9}$$

continuity equation:
$\dot{m}_9 = \rho_9 \rho_9 c_9$

$$\frac{T_{t9}}{T_9} = \left(\frac{P_{t9}}{P_9}\right)^{(k_t-1)/k_t}$$

TURBOJET ENGINE WITH SUBSONIC (CONVERGENT) NOZZLE



- Full gas expansion in the nozzle is available for subsonic flow speed ($C_9<a_9$)
- When total to static pressure ratio is critical the flow velocity in nozzle outlet is equal speed of sound
- When total to static pressure ratio is higher than critical the nozzle outlet velocity is equal speed of sound – chocked nozzle. Gas expansion process is continued outside the nozzle

CONVERGENT NOZZLE WITH LOSESS

NOZZLE (5-9)

Pressure losses: $\pi_N = \frac{P_{t9}}{P_{t5}} < 1 \quad P_{t9} = \pi_N P_{t5}$

No heat losess $\quad T_{t9} = T_{t5}$

Critical pressure ratio coefficient $\beta_{cr} = \left(\frac{1+k_t}{2}\right)^{\frac{k_t}{k_t-1}}$

If $\frac{P_{t9}}{P_{t5}} < \beta_{cr}$

Full expansion in the propelling nozzle

$$P_9 = P_0$$

Nozzle outlet parameters calculation like for full expansion

If $\frac{P_{t9}}{P_{t5}} = \beta_{cr}$

Full expansion in the propelling nozzle

$$P_9 = P_0$$

Outlet gas velocity is equal speed of sound

$$C_9 = a_9 = \sqrt{\frac{2k_t}{k_t + 1} R_t T_{t9}}$$

Nozzle outlet parameters calculation like for full expansion

If $\frac{P_{t9}}{P_{t5}} = \beta_{cr}$

Expansion to critical pressure in the propelling nozzle

$$P_9 = P_{t9}/\beta_{cr}$$

Nozzle outlet gas velocity is equal speed of sound

$$C_9 = a_9 = \sqrt{\frac{2k_t}{k_t + 1} R_t T_{t9}}$$

Gas expansion outside the nozzle. Parameters calculation like for full expansion

TURBOJET ENGINE WITH LOSSES

- Proesses in the inlet (diffuser) burner and nozzle are with losses –
  $$\pi_D < 1, \pi_B < 1, \ \pi_N < 1$$
- Compressor and turbine process isn't isentropic, $\eta_C(e_C) < 1, \eta_T(e_T) < 1$
- Combustor efficiency $\eta_B < 1$
- Mechanical efficiency $\eta_m < 1$

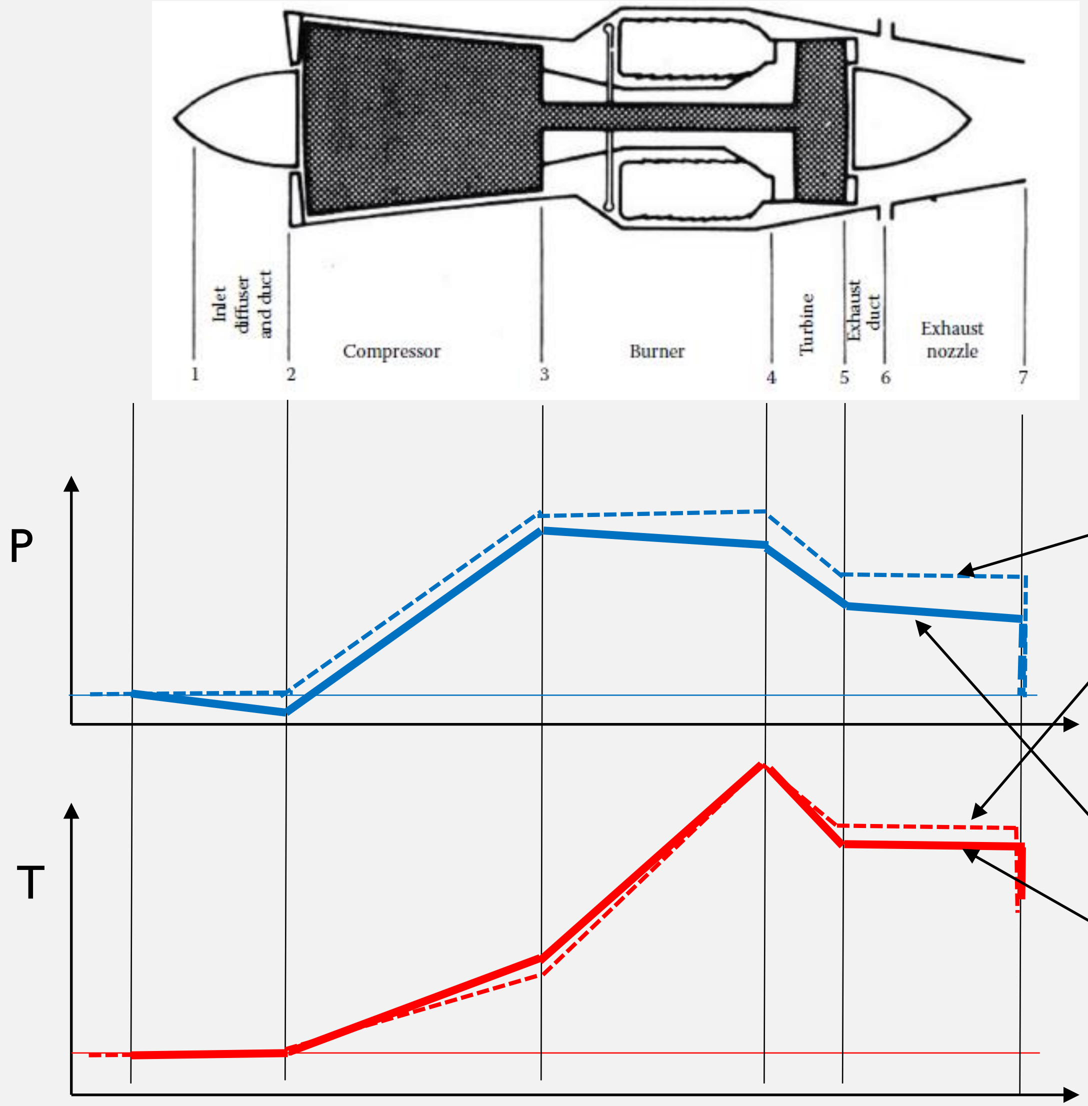
REAL TURBOJET ENGINE – FULL EXPANSION IN NOZZLE

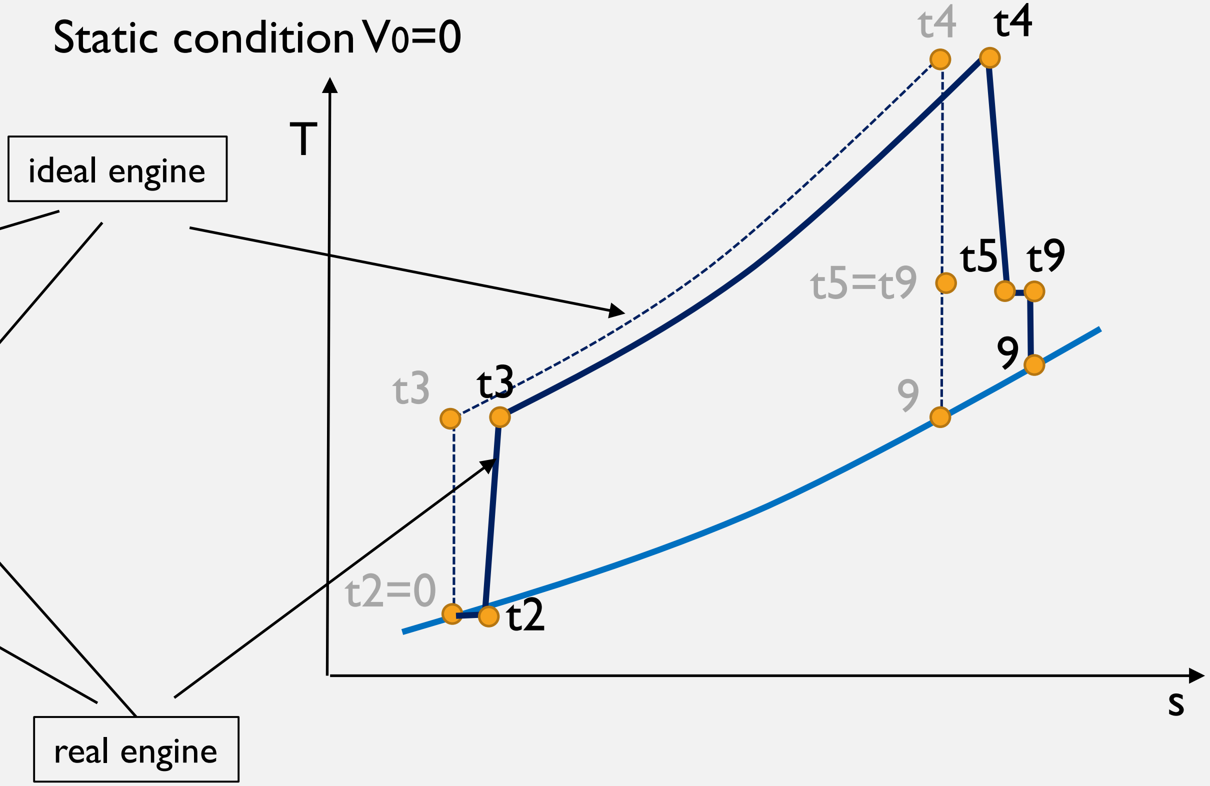
Proesses in the inlet (diffuser) burner and nozzle are with losses - $\pi_D < 1, \pi_B < 1, \ \pi_N < 1$

Compressor and turbine process isn't isentropic, $\eta_C < 1, \eta_T < 1$

Combustor efficiency $\eta_B < 1$

Mechanical efficiency $\eta_m < 1$





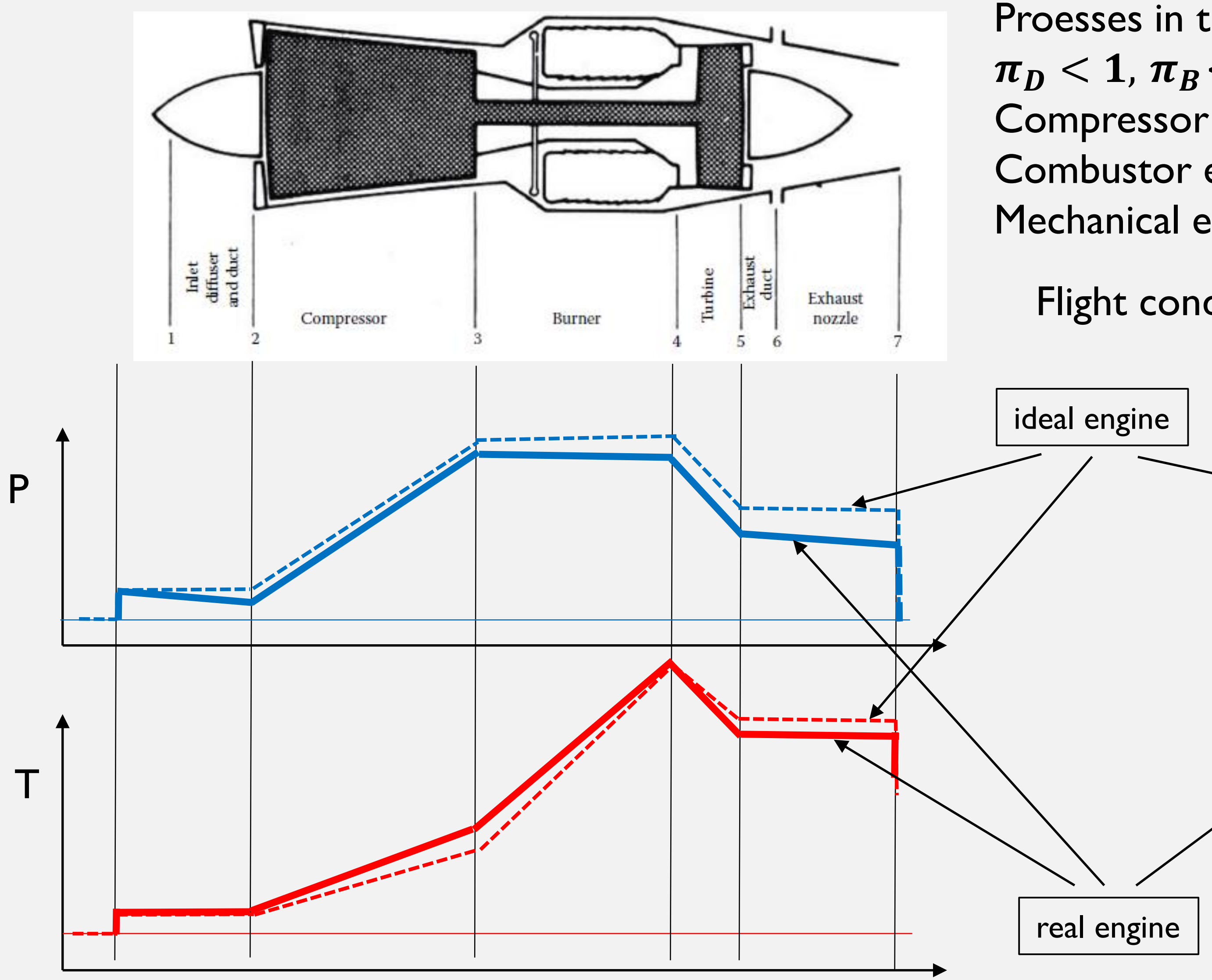
REAL TURBOJET ENGINE – FULL EXPANSION IN NOZZLE

Proesses in the inlet (diffuser) burner and nozzle are with losses - $\boldsymbol{\pi_D < 1,\ \pi_B < 1,\ \pi_N < 1}$

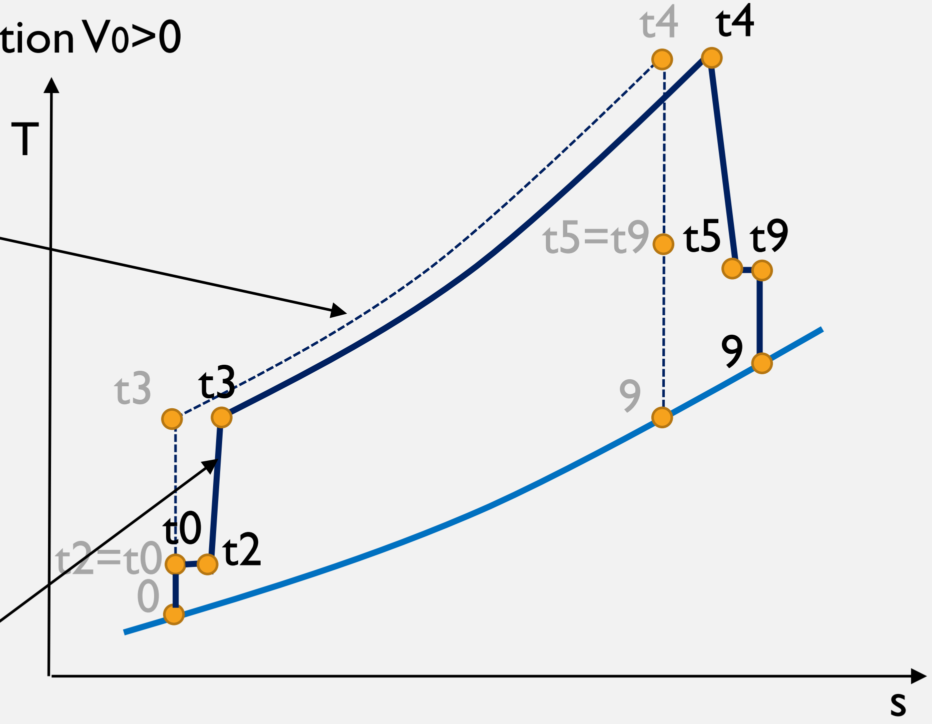
Compressor and turbine process isn't isentropic, $\boldsymbol{\eta_C < 1, \eta_T < 1}$

Combustor efficiency $\boldsymbol{\eta_B < 1}$

Mechanical efficiency $\boldsymbol{\eta_m < 1}$



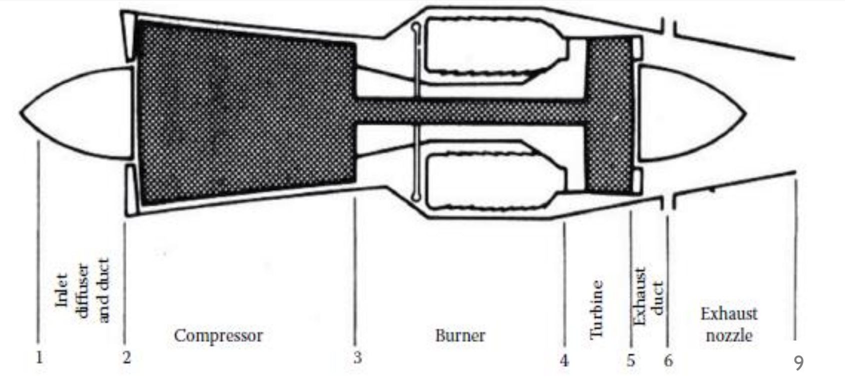
Flight condition V0>0



EXAMPLE OF TURBOJET ENGINE CALCULATION

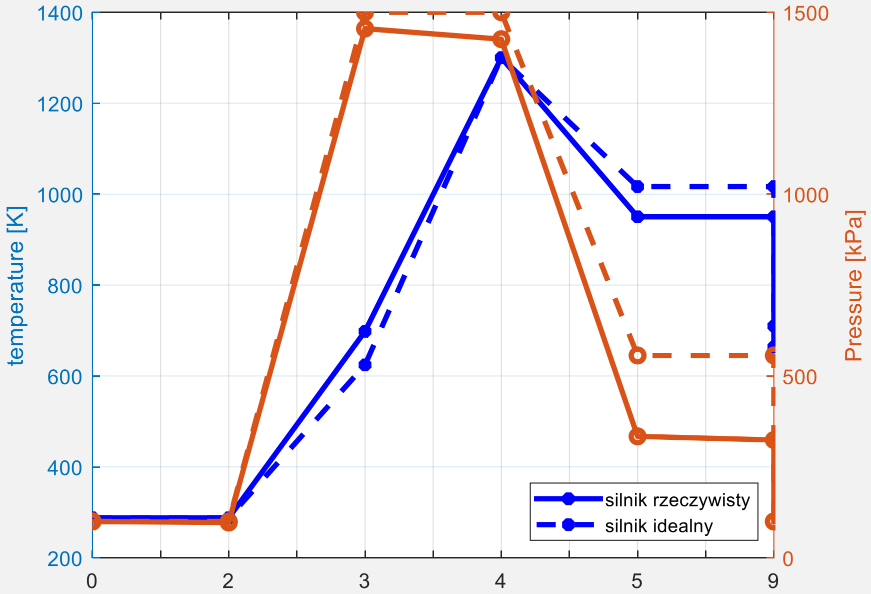
Given:

	Parameter	Value
1	'Altitude [m]'	0
2	'Flight speed Ma'	0
3	'air mass flow [kg/s]'	25
4	'CPR'	15
5	'T_t_4 [K]'	1300
6	'Inlet pressure losses \pi_D'	0.9700
7	'Burner pressure losses \pi_B'	0.9800
8	'Nozzle pressure losses \pi_N'	0.9700
9	'compressor efficiency \eta_C'	0.8200
10	'turbine efficiency \eta_T'	0.8900
11	'Burner efficiency \eta_B'	0.9800
12	'Mechanical efficiency \eta_M'	0.9900



Temperature and pressure comparison in engine cutsections:

	Section	Temp. [K] real	Temp. [K] ideal	Pressure [kPa] real	Pressure [kPa] ideal
1	'0'	288	288	100	100
2	't0'	288	288	100	100
3	't2'	288	288	97	100
4	't3'	698	624	1455	1500
5	't4'	1300	1300	1426	1500
6	't5'	950	1016	334	556
7	't9'	950	1016	324	556
8	'9'	710	664	100	100



TEMPERATURE PRESSURE IN ENGINE SECTIONS

Engine performance:

	Parameter	Unit	Real eng.	Ideal eng.
1	'Thrust'	'kN'	19.0608	23.1214
2	'Specific Thrust'	'N*s/kg'	762.4330	924.8565
3	'fuel consump'	'kg/s'	0.4285	0.4714
4	'Specific fuel consump'	'kg/N/h'	0.0809	0.0734
5	'prędkość V9'	'm/s'	749.5865	907.7404
6	'therm. efficiency'	'-'	0.3878	0.5177
7	'prop. efficiency'	'-'	0	0
8	'overall efficiency'	'-'	0	0

Real to ideal jet engine comparison shows:

- Total pressure in engine sections is lower in real engine
- Total temperature after compressor is higher, but after turbine is lower in real engine
- Higher temperature after compressor causes lower fuel consumption of the real engine - TIT (turbine inlet temperature) is the same in both cases
- Lower total temperature in the nozzle inlet and higher static temperature in the nozzle outlet causes lower outlet flow velocity and by this way lower thrust and specific thrust of real jet engine
- Specific fuel consumption is higher due to lower thrust and thermal and overall efficiencies are lower in real engine

Example of turbojet engine thermodynamic model: real turbojet engine model

EXAMPLE OF TURBOJET ENGINE CALCULATION WITH CONVERGENT NOZZLE

Given:

	Parameter	Value
1	'Altitude [m]'	0
2	'Flight speed Ma'	0
3	'air mass flow [kg/s]'	25
4	'CPR'	15
5	'T_t_4 [K]'	1300
6	'Inlet pressure losses \pi_D'	0.9700
7	'Burner pressure losses \pi_B'	0.9800
8	'Nozzle pressure losses \pi_N'	0.9700
9	'compressor efficiency \eta_C'	0.8200
10	'turbine efficiency \eta_T'	0.8900
11	'Burner efficiency \eta_B'	0.9800
12	'Mechanical efficiency \eta_M'	0.9900

Parameters and engine performance comparison for full expansion nozzle and convergent nozzle

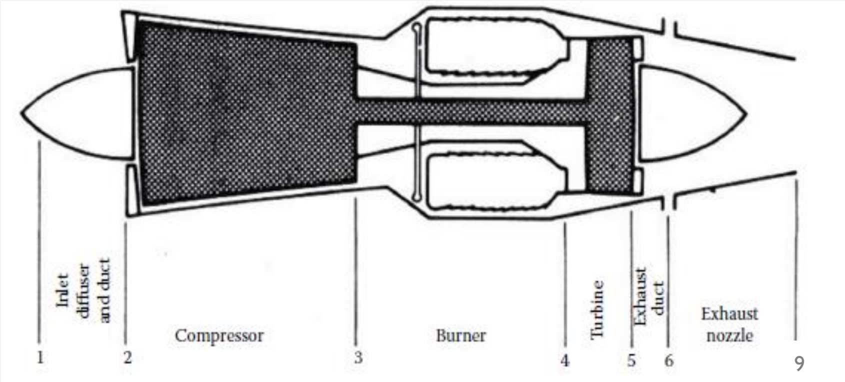
	Parameter	Unit	FULL EXP.	CONVERG. NOZZLE
1	'Tt9'	'K'	1.0330e+03	1.0330e+03
2	'Pt9'	'kPa'	143.4731	143.4731
3	'T9'	'K'	648.7254	886.8015
4	'V9'	'm/s'	947.8210	584.6740
5	'P9'	'kPa'	22	77.5530
6	'T9e'	'K'	648.7254	687.0761
7	'V9e'	'm/s'	947.8210	899.7531
8	'Thrust'	'kN'	14.0374	13.0560
9	'Specific Thrust'	'N*s/kg'	701.8725	652.8003
10	'Specific fuel consump'	'kg/N/h'	0.1072	0.1152
11	'therm. efficiency'	'-'	0.4711	0.4206
12	'prop. efficiency'	'-'	0.4407	0.4590
13	'overall efficiency'	'-'	0.2076	0.1931

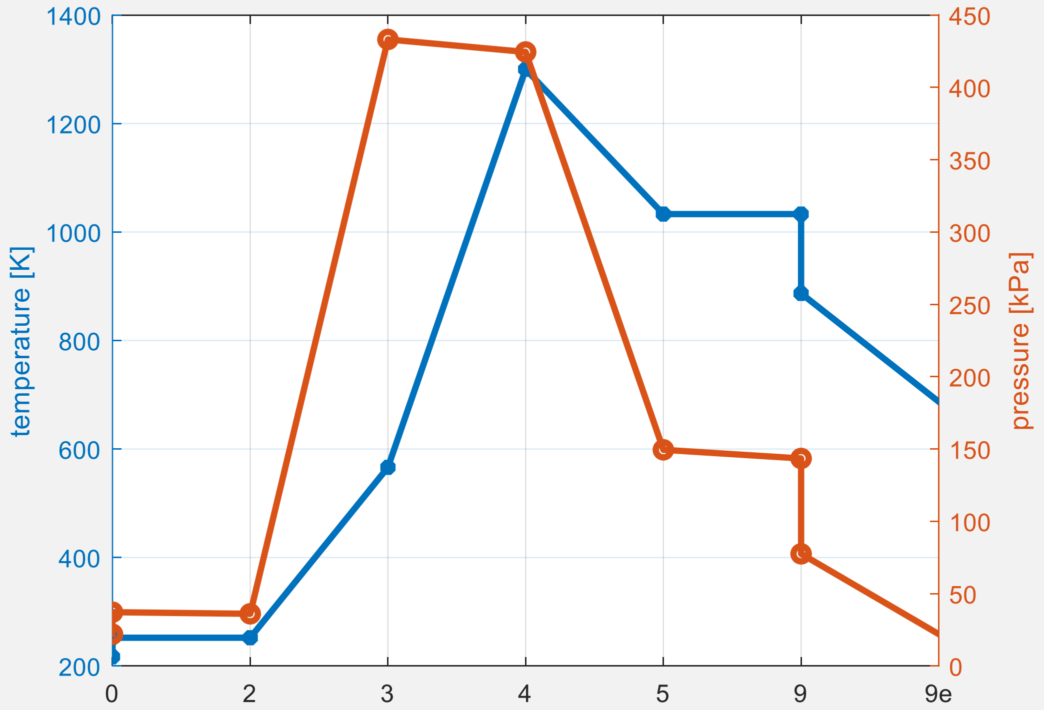
CONCLUSIONS:

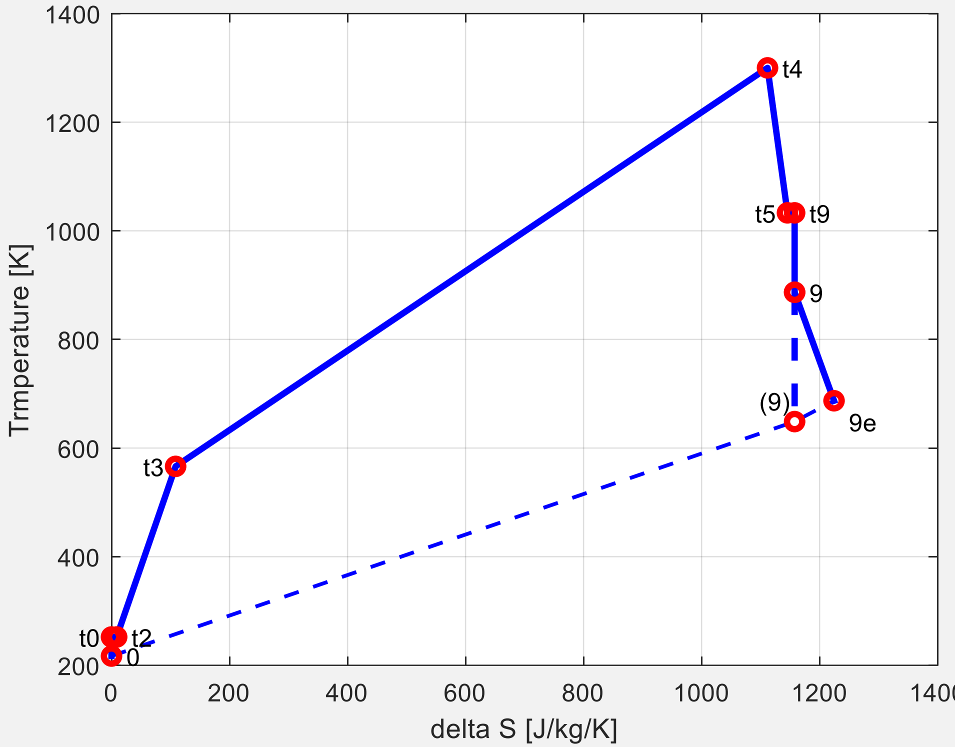
Incomplete expansion in the engine propelling nozzle causes:

- Lower thrust and specific thrust than it is in full decompression mode
- Higher specific fuel consumption
- Additional entropy increase caused by jet decompression outside the nozzle

EXAMPLE OF TURBOJET ENGINE CALCULATION WITH CONVERGENT NOZZLE







Example of turbojet engine with convergent nozzle calculation in: [real turbojet engine model]

THANKS FOR YOUR ATENTION

Questions and Comments ?

1. …….
2. …….
3. …….