Aircraft Engine Construction - turbofan engine

Elaborated by phd Robert JAKUBOWSKI, Rzeszow University of Technology

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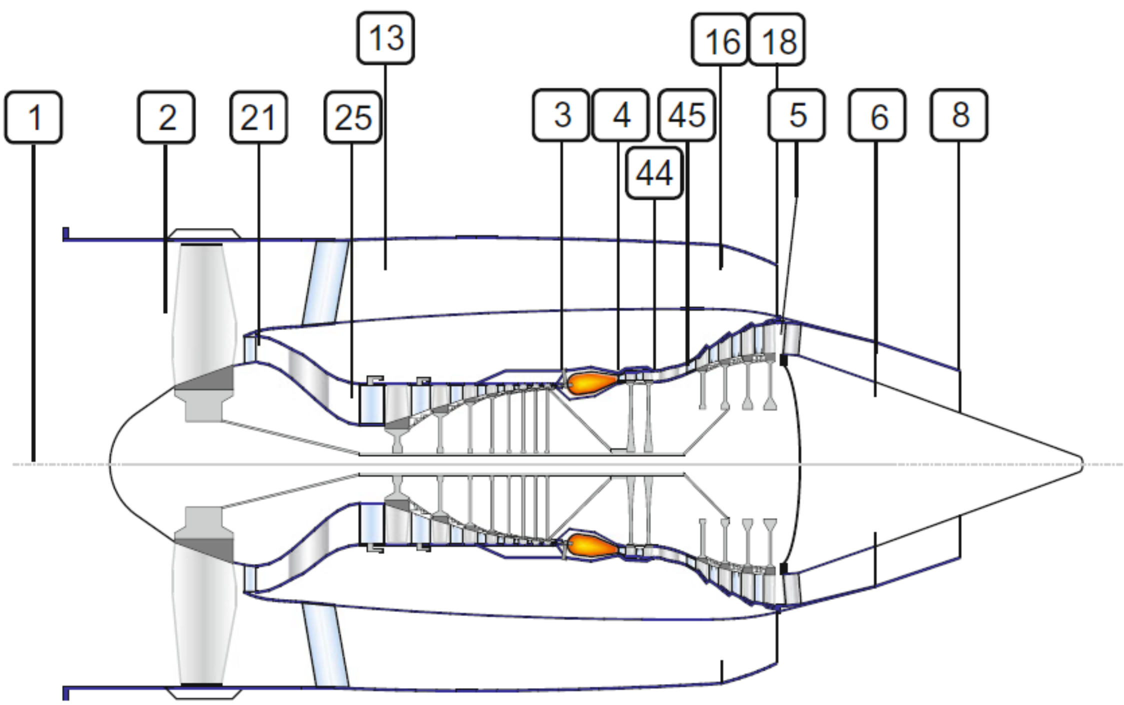
IDEAL TURBOFAN ENGINE

En example of turbofan engin calculation is presented. The assumption is that external and internal engine nozzles expanison is to ambient pressure



Given

Flight conditions: T0=217 K, P0=22 kPa, M0=0.82, bypass ratio 10, fan pressure ratio 1.49, compressor pressure ratio 20, Turbine inlet temperature Tt4=1650 K, mass flow m=60 kg/s.

Gas parameters:

Air: k=1.4; cp=1005 J/kg/K, R=287 J/kg/K,

Fumes in turbine and nozzle kt=1.33, cpt=1170 J/kg/K, Rt=290 J/kg/K,

For combustion in combustor cpB=1200 J/kg/K,

Fuel heat value: FHV=43 MJ/kg

Flight Mach No

```
M0 = 0.8200
```

Air Mass flow [kg/s]

```
m0 = 60
```

Bypass ratio

```
BPR = 10
```

Turbine inlet temperature [K]

```
Tt4 = 1650
```

Fan pressure ratio

```
FPR = 1.4900
```

Compressor pressure ratio

```
CPR = 20
```

Ambient conditions

Static temperature [K]

```
T0 = 217
```

Static pressure [Pa]

```
P0 = 22000
```

TURBOFAN ENGINE CALCULATION

Section 0

Total temperature [K]

$$T_{t0} = T_0\left(1 + \frac{k-1}{2} M_0^2\right)$$

```
Tt0_id = 246.1822
```

Total pressure [Pa]

$$P_{t0} = P_0\left(1 + \frac{k-1}{2} M_0^2\right)^{\frac{k}{k-1}}$$

```
Pt0_id = 3.4215e+04
```

Speed of sound [m/s]

$a_0 = \sqrt{k * R * T_0}$ - like for ideal engine

```
a0 = 295.2805
```

Flight speed [m/s]

$V_0 = M_0 * a_0$ - like for ideal engine

```
V0_id = 242.1300
```

Section 2 Inlet exit

Total temperature [K]

$T_{t2} = T_{t0}$ - like for ideal engine

```
Tt2_id = 246.1822
```

Total pressure [Pa]

$P_{t2} = P_{t0}$

```
Pt2_id = 3.4215e+04
```

Section 21/13 - Fan outlet

Total temperature [K]

$T_{t21} = T_{t13} = T_{t2} * \text{FPR}^{\frac{k-1}{k}}$

```
Tt21_id = 275.8915
Tt13_id = 275.8915
```

Total pressure [Pa]

$P_{t21} = P_{t13} = P_{t2} * \text{FPR}$

```
Pt21_id = 5.0980e+04
Pt13_id = 5.0980e+04
```

FAN

Fan work [J/kg]

$W_F = c_p * (T_{t21} - T_{t2})$

```
WF_id = 2.9858e+04
```

Fan power [W]

$P_F = m_0 * W_F$

```
PC_id = 1.7915e+06
```

STREAM SPLIT

Internal duct mass flow [kg/s]

$$m_{21} = m_0 * \frac{1}{1 + \mathrm{BPR}}$$

```
m21 = 5.4545
```

External duct mass flow [kg/s]

$$m_{13} = m_0 * \frac{\mathrm{BPR}}{1 + \mathrm{BPR}}$$

```
m13 = 54.5455
```

INTERNAL DUCT / CORE ENGINE

Section 3 - Compressor outlet / Burner inlet

Total temperature [K]

$$T_{\mathrm{t3}} = T_{\mathrm{t21}} * \mathrm{CPR}^{\frac{k-1}{k}}$$

```
Tt3_id = 649.3237
```

Total pressure [Pa]

$$P_{\mathrm{t3}} = P_{\mathrm{t21}} * \mathrm{CPR}$$

```
Pt3_id = 1.0196e+06
```

COMPRESSOR

Compressor work [J/kg]

$$W_C = c_p * (T_{\mathrm{t3}} - T_{\mathrm{t21}})$$

```
WC_id = 3.7530e+05
```

Compressor power [W]

$$P_C = m_{25} * W_C$$

```
PC_id = 2.0471e+06
```

Section 4 Burner outlet / Turbine inlet

Total temperature [K]

$$T_{\mathrm{t4}}$$

```
Tt4_id = 1650
```

Total pressure [Pa]

$$P_{\mathrm{t4}} = P_{\mathrm{t3}}$$

```
Pt4_id = 1.0196e+06
```

BURNER

Fuel-air ratio

$$f_B = \frac{m_f}{m_{25}} = c_{\mathrm{pB}} * \frac{T_{\mathrm{t4}} - T_{\mathrm{t3}}}{\mathrm{FHV} * \eta_B}$$

```
fB_id = 0.0279
```

Fuel mass flow [kg/s]

$$m_{\mathrm{fB}} = m_{21} * f_B$$

```
mfB_id = 0.1523
```

Section 45 High Pressure Turbine outlet

Total temperature [K]

$$T_{\mathrm{t45}} = T_{\mathrm{t4}} - \frac{W_C}{(1 + f_B) * c_{\mathrm{pt}}}$$

```
Tt45_id = 1.3379e+03
```

Total pressure [Pa]

$$P_{\mathrm{t45}} = P_{\mathrm{t4}} \left(\frac{T_{\mathrm{t45}}}{T_{\mathrm{t4}}} \right)^{\frac{\mathrm{kt}}{\mathrm{kt}-1}}$$

```
Pt45_id = 4.3801e+05
```

HPT Pressure ratio

$$H\mathrm{PT\ PR} = \frac{P_{\mathrm{t4}}}{P_{\mathrm{t45}}}$$

```
HPT_PR_ID = 2.3278
```

Section 5 Low Pressure Turbine outlet

Total temperature [K]

$$T_{\mathrm{t5}} = T_{\mathrm{t45}} - \frac{(1 + \mathrm{BPR}) W_F}{(1 + f_B) * c_{\mathrm{pt}}}$$

```
Tt5_id = 1.0649e+03
```

Total pressure [Pa]

$$P_{\mathrm{t5}} = P_{\mathrm{t45}} \left(\frac{T_{\mathrm{t5}}}{T_{\mathrm{t45}}} \right)^{\frac{\mathrm{kt}}{\mathrm{kt}-1}}$$

```
Pt5_id = 1.7454e+05
```

LPT Pressure ratio

$$\mathrm{LPT\ PR} = \frac{P_{\mathrm{t45}}}{P_{\mathrm{t5}}}$$

```
LPT_PR_ID = 2.5095
```

Section 8 Core Engine Nozzle outlet

Total temperature [K]

$$T_{t9} = T_{t5}$$

```
Tt9_id = 1.0649e+03
```

Total pressure [Pa]

$$P_{t9} = P_{t5}$$

```
Pt9_id = 1.7454e+05
```

Static pressure [Pa]

$$P_9 = P_0$$

```
P9_id = 22000
```

Static temperature [K]

$$T_9 = T_{t9} * \left(\frac{P_9}{P_{t9}}\right)^{\frac{\mathrm{kt}-1}{\mathrm{kt}}}$$

```
T9_id = 636.9630
```

Jet stream Mach No

$$M_9 = \sqrt{\left(\frac{T_{t9}}{T_9} - 1\right) * \frac{2}{\mathrm{kt} - 1}}$$

```
M9_id = 2.0178
```

Speed of sound [m/s]

$$a_9 = \sqrt{\mathrm{kt} * \mathrm{Rt} * T_9}$$

```
a9_id = 495.6578
```

Jet speed [m/s]

$$V_9 = M_9 * a_9$$

Section 19 External Engine Nozzle outlet

Total temperature [K]

$$T_{t19} = T_{t13}$$

Total pressure [Pa]

$$P_{t19} = P_{t13}$$

```
Pt19_id = 5.0980e+04
```

Static pressure [Pa]

$P_{19} = P_0$

```
P19_id = 22000
```

Static temperature [K]

$$T_{19} = T_{t19} * \left(\frac{P_{19}}{P_{t19}}\right)^{\frac{k-1}{k}}$$

```
T19_id = 217.0000
```

Jet stream Mach No

$$M_{19} = \sqrt{\left(\frac{T_{t19}}{T_{19}} - 1\right) * \frac{2}{k-1}}$$

```
M19_id = 1.1649
```

Speed of sound [m/s]

$$a_{19} = \sqrt{k * R * T_{19}}$$

```
a19_id = 295.2805
```

Jet speed [m/s]

$$V_{19} = M_{19} * a_{19}$$

```
V19_id = 343.9667
```

TURBOFAN ENGINE PERFORMANCE CALCULATION

Thrust [N]

$$T = m_{21} * (1 + f_B) * V_9 + m_{21} * \mathrm{BPR} * V_{19} - m_{21} * (1 + \mathrm{BPR}) * V_0$$

```
T_id = 9.8415e+03
```

Specific thrust [Ns/kg]

$$\mathrm{ST} = \frac{T}{m_0} = \frac{(1 + f_B) * V_9 + \mathrm{BPR} * V_{19} - (1 + \mathrm{BPR}) * V_0}{1 + \mathrm{BPR}}$$

```
ST_id = 164.0257
```

Specific fuel consumption [kg/N/s]

$$\mathrm{SFC} = \frac{m_{\mathrm{fB}}}{T} = \frac{f_B}{(1 + \mathrm{BPR}) * \mathrm{ST}}$$

```
SFC_id = 1.5478e-05
```

Specific fuel consumption [kg/N/h]

$$\mathrm{SFC} = \mathrm{SFC} * 3600$$

```
SFC_id = 0.0557
```

Thermal efficiency

$$\eta_{\mathrm{th}} = \frac{(1+f_B)*V_9{}^2 + \mathrm{BPR}*V_{19}{}^2 - (1+\mathrm{BPR})V_0{}^2}{2*f_B*\mathrm{FHV}}$$

```
etha_th_id = 0.6522
```

Propulsive efficiency

$$\eta_p = \frac{2*V_0*\mathrm{ST}*(1+\mathrm{BPR})}{(1+f_B)*V_9{}^2 + \mathrm{BPR}*V_{19}{}^2 - (1+\mathrm{BPR})V_0{}^2}$$

```
etha_p_id = 0.5578
```

Overall efficiency

$$\eta_o = \frac{V_0*\mathrm{ST}*(1+\mathrm{BPR})}{f_B*\mathrm{FHV}} = \eta_{\mathrm{th}}*\eta_p$$

```
etha_o_id = 0.3638
```

Temperature, pressure vs engine sections plot

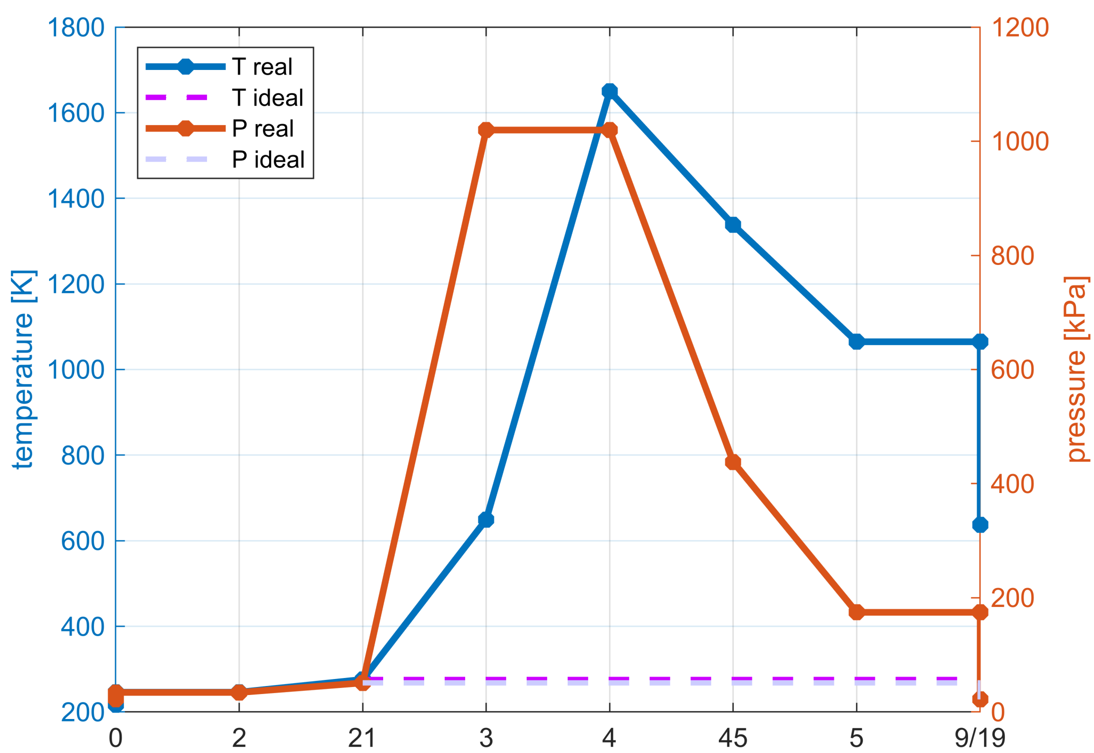


Tabela = 12×3 table

	Section	T ideal [K]	P ideal [kPa]
1	'0'	217	22
2	't0'	246	34.2146
3	't2'	246	34.2146

	Section	T ideal [K]	P ideal [kPa]
4	't21'	276	50.9798
5	't3'	649	1.0196e+03
6	't4'	1650	1.0196e+03
7	't45'	1338	438.0132
8	't5'	1065	174.5393
9	't9'	1065	174.5393
10	'9'	637	22
11	't19'	276	50.9798
12	'19'	217	22

Performance of ideal turbofan engine

```
Tabela = 9×3 table
```

	Parameter	Unit	Ideal turbofan
1	'Thrust'	'kN'	9.8415
2	'Specific Thrust'	'N*s/kg'	164.0257
3	'Fuel consumption'	'kg/s'	0.1523
4	'Specific fuel consump'	'kg/N/h'	0.0557
5	'therm. efficiency'	'-'	0.6522
6	'prop. efficiency'	'-'	0.5578
7	'overall efficiency'	'-'	0.3638
8	'V9'	'm/s'	1.0001e+03
9	'V19_id'	'm/s'	343.9667

CONCLUSIONS

In the ideal turbofan

- Temperature and pressure rise in the fan and compressor and drop in turbines is isentropic
- Specific thrust is lower than in the turbojet engine (see turbojet engine calculation)
- Higher temperature after compressor causes lower fuel consumption of the real engine - TIT (turbine inlet temperature) is the same in both cases
- Lower total temperature in the nozzle inlet and higher static temperature in the nozzle outlet causes lower outlet flow velocity and by this way lower thrust and specific thrust of real jet engine
- Specific fuel consumption is higher due to lower thrust and thermal and overall efficiences are lower in real engine

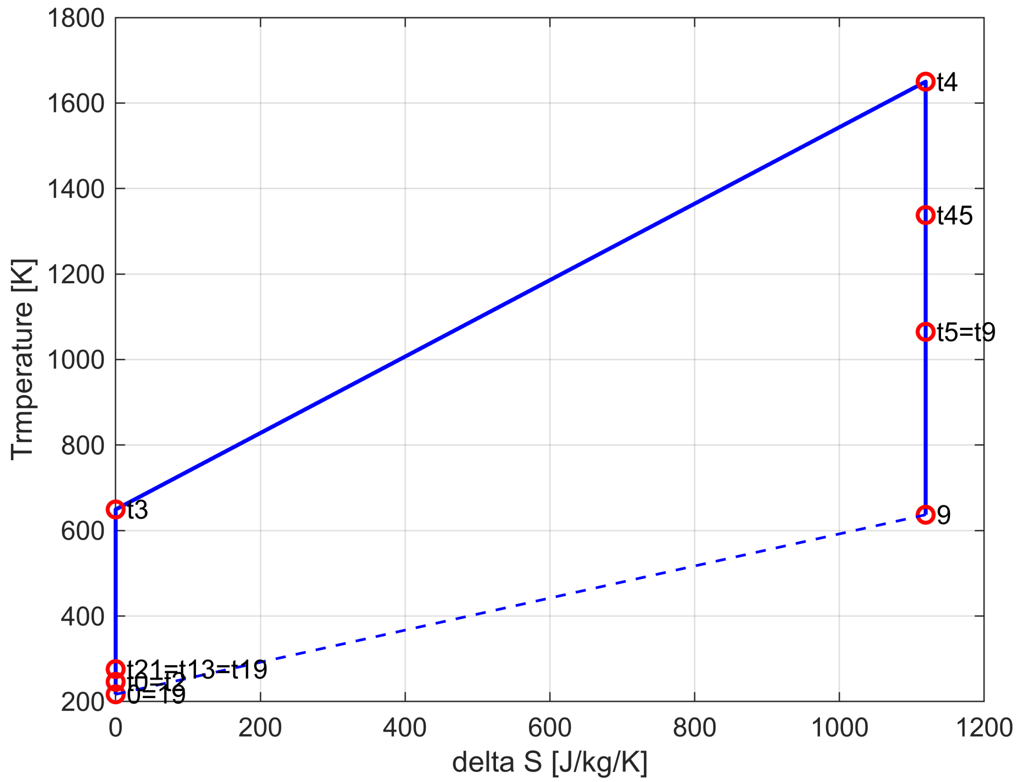
Temperature - entropy plot

Entropy growth is calculated from equations:

Combustor entropy grow   [J/kg/K] :

$$\Delta s_B = c_{\mathrm{pB}} * \ln \frac{T_{\mathrm{t4}}}{T_{\mathrm{t3}}}$$

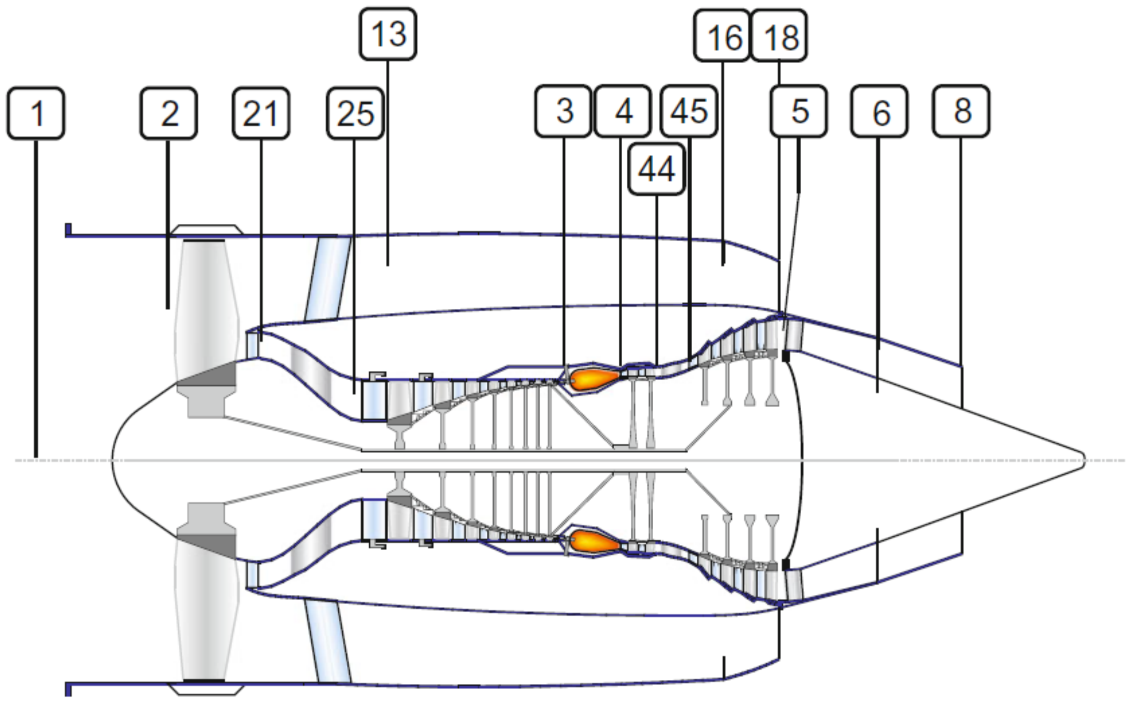
```
dS_B = 1.1191e+03
```



CONCLUSIONS

TURBOFAN ENGINE WITH LOSSES

Calculation of a turbofan engin with losses of the same work parameters like an ideal turbofan. The assumption is that an external and internal engine nozzles expanison is to ambient pressur. The losses coefcients and efficiences are specified below.



Given

Flight conditions: T0=217 K, P0=22 kPa, M0=0.88, bypass ratio 9, fan pressure ratio 1.55, compressor pressure ratio 20, Turbine inlet temperature Tt4=1650 K, mass flow m=60 kg/s.

Gas parameters:

Air: k=1.4; cp=1005 J/kg/K, R=287 J/kg/K,

Fumes in turbine and nozzle kt=1.33, cpt=1170 J/kg/K, Rt=290 J/kg/K,

For combustion in combustor cpB=1200 J/kg/K,

Fuel heat value: FHV=43 MJ/kg

Engine losses coefficients:

inlet pressure losses coefficient σ_{IN} 0.98, burner pessure losses coefficient σ_B 0.98, internal nozzle pressure losses coefficient $\sigma_{N\,\text{int}}$ 0.97, external nozzle pressure losses coefficient $\sigma_{N\,\text{ext}}$ 0.96, fun efficiency η_F 0,91, compressor efficeincy η_C 0.83, high pressure turbine (HPT) efficiency η_{HPT} 0.88, low pressure turbine (LPT) efficiency η_{LPT} 0.9, burner efficiency η_B 0.98, mechanical efficiency low pressure spool $\eta_{M\,\text{LP}}$= 0.995 ,mechanical efficiency high pressure spool $\eta_{M\,\text{HP}}$= 0.99.

Flight Mach No

```
M0 = 0.8200
```

Air Mass flow [kg/s]

```
m0 = 60
```

Bypass ratio

```
BPR = 10
```

Turbine inlet temperature [K]

```
Tt4 = 1650
```

Fan pressure ratio

```
FPR = 1.4900
```

Compressor pressure ratio

```
CPR = 20
```

Ambient conditions

Static temperature [K]

```
T0 = 217
```

Static pressure   [Pa]

```
P0 = 22000
```

TURBOFAN ENGINE WITH LOSSES

Section 0

Total temperature   [K]

$T_{t0} = T_0\left(1 + \frac{k-1}{2} M_0^2\right)$ - like for ideal engine

```
Tt0 = 246.1822
```

Total pressure   [Pa]

$P_{t0} = P_0\left(1 + \frac{k-1}{2} {M_0}^2\right)^{\frac{k}{k-1}}$ - like for ideal engine

```
Pt0 = 3.4215e+04
```

Speed of sound   [m/s]

$a_0 = \sqrt{k * R * T_0}$ - like for ideal engine

```
a0 = 295.2805
```

Flight speed   [m/s]

$V_0 = M_0 * a_0$ - like for ideal engine

```
V0 = 242.1300
```

Section 2 Compressor inlet

Total temperature [K]

$T_{t2} = T_{t0}$ - like for ideal engine

```
Tt2 = 246.1822
```

Total pressure [Pa]

$P_{t2} = \sigma_{IN} * P_{t0}$

```
Pt2 = 3.3530e+04
```

Section 21/13 - Fan outlet

Total temperature [K]

$$T_{t21} = T_{t13} = T_{t2} * \left(1 + \frac{FPR^{\frac{k-1}{k}} - 1}{\eta_F}\right)$$

```
Tt21 = 278.8298
Tt13 = 278.8298
```

Total pressure [Pa]

$P_{t21} = P_{t13} = P_{t2} * FPR$

```
Pt21 = 4.9960e+04
```

FAN

Fan work [J/kg]

$W_F = c_p * (T_{t21} - T_{t2})$

```
WF = 3.2811e+04
```

Fan power [W]

$P_F = m_0 * W_F$

```
PF = 1.9687e+06
```

STREAM SPLIT

Internal duct mass flow [kg/s]

$$m_{21} = m_0 * \frac{1}{1 + BPR}$$

```
m21 = 5.4545
```

External duct mass flow [kg/s]

$$m_{13} = m_0 * \frac{\text{BPR}}{1 + \text{BPR}}$$

```
m13 = 54.5455
```

INTERNAL DUCT / CORE ENGINE

Section 3 - Compressor outlet / Burner inlet

Total temperature [K]

$$T_{t3} = T_{t21} * \left(1 + \frac{\text{CPR}^{\frac{k-1}{k}} - 1}{\eta_C}\right)$$

```
Tt3 = 733.5397
```

Total pressure [Pa]

$$P_{t3} = P_{t21} * \text{CPR}$$

```
Pt3 = 9.9920e+05
```

COMPRESSOR

Compressor work [J/kg]

$$W_C = c_p * (T_{t3} - T_{t21})$$

```
WC = 4.5698e+05
```

Compressor power [W]

$$P_C = m_{21} * W_C$$

```
PC = 2.4926e+06
```

Section 4 Burner outlet / Turbine inlet

Total temperature [K]

$$T_{t4}$$

```
Tt4 = 1650
```

Total pressure [Pa]

$$P_{t4} = \sigma_B * P_{t3}$$

```
Pt4 = 9.7922e+05
```

BURNER

Fuel-air ratio

$$f_B = c_{pB} * \frac{T_{t4} - T_{t3}}{\text{FHV} * \eta_B}$$

```
fB = 0.0261
```

Fuel mass flow [kg/s]

$$m_{\text{fB}} = m_{25} * f_B$$

```
mfB = 0.1424
```

Section 45 High Pressure Turbine (HPT) outlet / Low Pressure Turbine (LPT) inlet

Total temperature [K]

$$T_{\text{t45}} = T_{\text{t4}} - \frac{W_C}{\eta_{M\ \text{HP}} * (1 + f_B) * c_{\text{pt}}}$$

```
Tt45 = 1.2655e+03
```

Total pressure [Pa]

$$P_{\text{t45}} = P_{\text{t4}}\left(\frac{\eta_{T\ \text{HP}} + \frac{T_{\text{t45}}}{T_{\text{t4}}} - 1}{\eta_T}\right)^{\frac{\text{kt}}{\text{kt}-1}}$$

```
Pt45 = 2.8343e+05
```

High pressure turbine pressure ratio

$$H\text{PT PR} = \frac{P_{\text{t4}}}{P_{\text{t45}}}$$

```
HPT_PR = 3.4549
```

Section 5 LPT outlet / Nozzle inlet

Total temperature [K]

$$T_{\text{t5}} = T_{\text{t45}} - \frac{(1 + \text{BPR})W_F}{\eta_{M\ \text{LP}} * (1 + f_B) * c_{\text{pt}}}$$

```
Tt5 = 963.3615
```

Total pressure [Pa]

$$P_{\text{t5}} = P_{\text{t45}}\left(\frac{\eta_{\text{LPT}} + \frac{T_{\text{t5}}}{T_{\text{t45}}} - 1}{\eta_{\text{LPT}}}\right)^{\frac{\text{kt}}{\text{kt}-1}}$$

```
Pt5 = 8.1822e+04
```

Low pressure turbine pressure ratio

$$\text{LPT PR} = \frac{P_{\text{t45}}}{P_{\text{t5}}}$$

```
LPT_PR = 3.4640
```

Section 9 Core engine nozzle outlet / Internal duct nozzle outlet

Total temperature [K]

$$T_{t9} = T_{t5}$$

```
Tt9 = 963.3615
```

Total pressure [Pa]

$$P_{t9} = P_{t5} * \sigma_{N\,\text{int}}$$

```
Pt9 = 7.9368e+04
```

Static pressure [Pa]

$$P_9 = P_0$$

```
P9 = 22000
```

Static temperature [K]

$$T_9 = T_{t9} * \left(\frac{P_9}{P_{t9}}\right)^{\frac{\text{kt}-1}{\text{kt}}}$$

```
T9 = 700.6989
```

Jet stream Mach No

$$M_9 = \sqrt{\left(\frac{T_{t9}}{T_9} - 1\right) * \frac{2}{\text{kt} - 1}}$$

```
M9 = 1.5073
```

Speed of sound [m/s]

$$a_9 = \sqrt{\text{kt} * \text{Rt} * T_9}$$

```
a9 = 519.8650
```

Jet speed [m/s]

$$V_9 = M_9 * a_9$$

```
V9 = 783.5775
```

Section 19 External Engine Nozzle outlet

Total temperature [K]

$$T_{t19} = T_{t13}$$

Total pressure [Pa]

$$P_{t19} = \sigma_{N\,\text{ext}} * P_{t13}$$

```
Pt19 = 4.7962e+04
```

Static pressure [Pa]

$P_{19} = P_0$

```
P19 = 22000
```

Static temperature [K]

$$T_{19} = T_{t19} * \left(\frac{P_{19}}{P_{t19}}\right)^{\frac{k-1}{k}}$$

```
T19 = 223.1684
```

Jet stream Mach No

$$M_{19} = \sqrt{\left(\frac{T_{t19}}{T_{19}} - 1\right) * \frac{2}{k-1}}$$

```
M19 = 1.1167
```

Speed of sound [m/s]

$$a_{19} = \sqrt{k * R * T_{19}}$$

```
a19 = 299.4480
```

Jet speed [m/s]

$$V_{19} = M_{19} * a_{19}$$

```
V19 = 334.4005
```

TURBOFAN ENGINE PERFORMANCE CALCULATION

Thrust [N]

$$T = m_{21} * (1 + f_B) * V_9 + m_{21} * \text{BPR} * V_{19} - m_{21} * (1 + \text{BPR}) * V_0$$

```
T = 8.0978e+03
```

Specific thrust [Ns/kg]

$$\text{ST} = \frac{T}{m_0} = \frac{(1 + f_B) * V_9 + \text{BPR} * V_{19} - (1 + \text{BPR}) * V_0}{1 + \text{BPR}}$$

```
ST = 134.9638
```

Specific fuel consumption [kg/N/s]

$$\text{SFC} = \frac{m_{\text{fB}}}{T} = \frac{f_B}{(1 + \text{BPR}) * \text{ST}}$$

```
SFC = 1.7579e-05
```

Specific fuel consumption [kg/N/h]

$$\text{SFC} = \text{SFC} * 3600$$

```
SFC = 0.0633
```

Thermal efficiency

$$\eta_{\mathrm{th}} = \frac{(1+f_B)*{V_9}^2 + \mathrm{BPR}*{V_{19}}^2 - (1+\mathrm{BPR})*{V_0}^2}{2*f_B*\mathrm{FHV}}$$

```
etha_th = 0.4916
```

Propulsive efficiency

$$\eta_p = \frac{2*V_0*\mathrm{ST}*(1+\mathrm{BPR})}{(1+f_B)*{V_9}^2 + \mathrm{BPR}*{V_{19}}^2 - (1+\mathrm{BPR})*{V_0}^2}$$

```
etha_p = 0.6516
```

Overall efficiency

$$\eta_o = \frac{V_0*\mathrm{ST}*(1+\mathrm{BPR})}{f_B*\mathrm{FHV}} = \eta_{\mathrm{th}}*\eta_p$$

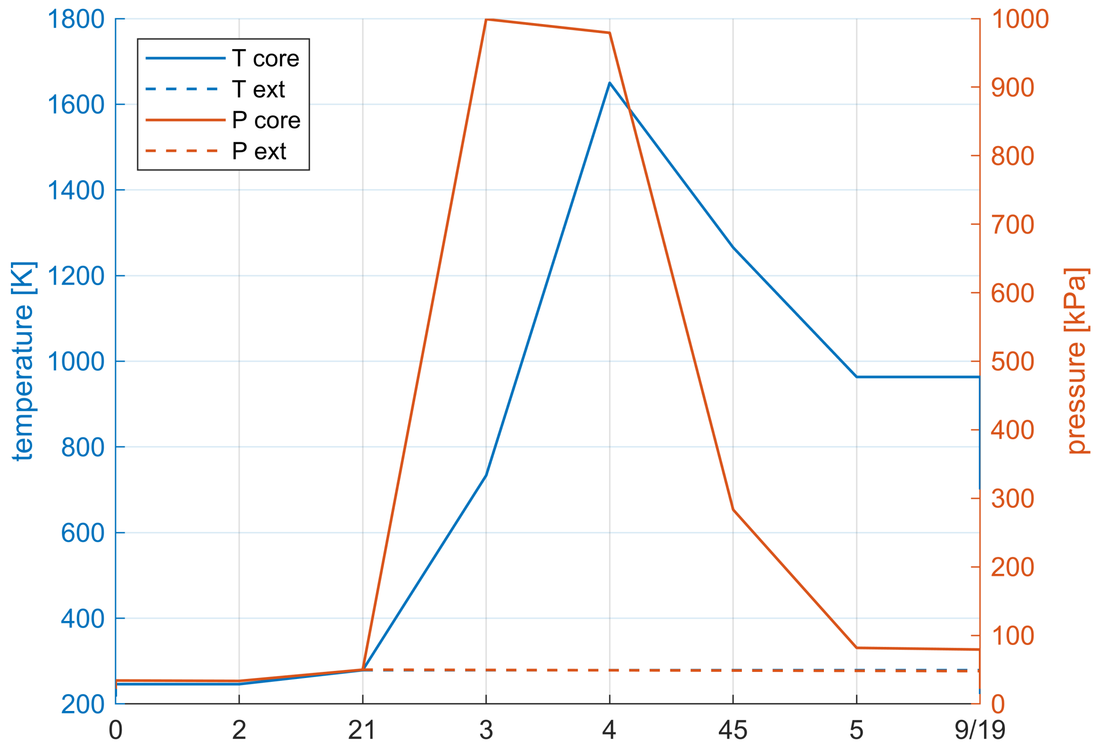
```
etha_o = 0.3203
```

Results sumarization

Temperature, presure vs engine sections

```
Tabela2 = 12×3 table
```

	Section	T real [K]	P real [kPa]
1	'0'	217	22
2	't0'	246	34
3	't2'	246	34
4	't21'	279	50
5	't3'	734	999
6	't4'	1650	979
7	't45'	1266	283
8	't5'	963	82
9	't9'	963	79
10	'9'	701	22
11	't19'	279	48
12	'19'	223	22



Performance summarization

Tabela = 11×3 table

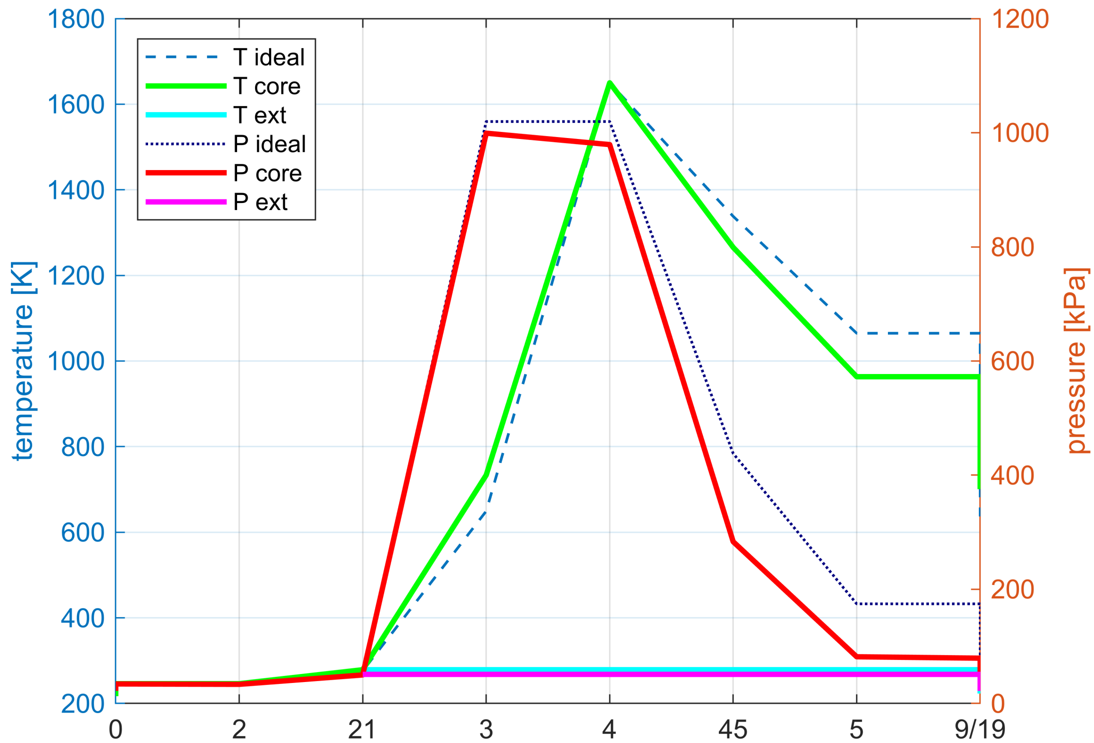
	Parameter	Unit	Value
1	'Thrust'	'kN'	8.0978
2	'Specific Thrust'	'N*s/kg'	135
3	'Fuel consumption'	'kg/s'	0.1424
4	'Specific fuel consump'	'kg/N/h'	0.0633
5	'therm. efficiency'	'-'	0.4916
6	'prop. efficiency'	'-'	0.6516
7	'overall efficiency'	'-'	0.3203
8	'V9'	'm/s'	784
9	'V19'	'm/s'	334
10	'HPT_PR'	'-'	3.4549
11	'LPT_PR'	'-'	3.4640

REAL vs IDEAL TURBOFAN

Tabela = 12×5 table

	Section	T ideal [K]	T real [K]	P ideal [kPa]	P real [kPa]
1	'0'	217	217	22	22

	Section	T ideal [K]	T real [K]	P ideal [kPa]	P real [kPa]
2	't0'	246	246	34.2146	34.2146
3	't2'	246	246	34.2146	33.5303
4	't21'	276	279	50.9798	49.9602
5	't3'	649	734	1.0196e+03	999.2036
6	't4'	1650	1650	1.0196e+03	979.2195
7	't45'	1338	1266	438.0132	283.4293
8	't5'	1065	963	174.5393	81.8223
9	't9'	1065	963	174.5393	79.3677
10	'9'	637	701	22	22
11	't19'	276	279	50.9798	47.9618
12	'19'	217	223	22	22



Performance comparison of real vs ideal turbofan engine

`Tabela = 11×4 table`

	Parameter	Unit	Ideal	real
1	'Thrust'	'kN'	9.8415	8.0978
2	'Specific Thrust'	'N*s/kg'	164.0257	134.9638
3	'Fuel consumption'	'kg/s'	0.1523	0.1424

	Parameter	Unit	Ideal	real
4	'Specific fuel consump'	'kg/N/h'	0.0557	0.0633
5	'therm. efficiency'	'-'	0.6522	0.4916
6	'prop. efficiency'	'-'	0.5578	0.6516
7	'overall efficiency'	'-'	0.3638	0.3203
8	'V9'	'm/s'	1.0001e+03	783.5775
9	'V19'	'm/s'	343.9667	334.4005
10	'HPT_PR'	'-'	2.3278	3.4549
11	'LPT_PR'	'-'	2.5095	3.4640

CONCLUSIONS

Real to ideal jet engine comparison shows

- Total pressure in engine sections is lower in real engine
- Total temperature after compressor is higher, but after turbine is lower in real engine
- Higher temperature after compressor causes lower fuel consumption of the real engine - TIT (turbine inlet temperature) is the same in both cases
- Lower total temperature in the nozzle inlet and higher static temperature in the nozzle outlet causes lower outlet flow velocity and by this way lower thrust and specific thrust of real jet engine
- Specific fuel consumption is higher due to lwer thrust and thermal and overall efficiences are lower in real engine

Temperature - entropy plot

Entropy growth is calculated from equations:

Inlet entropy grow [J/kg/K] :

$$\Delta s_{\mathrm{IN}} = -R * \ln(\sigma_{\mathrm{IN}})$$

```
dS_IN = 5.7982
```

Fan entropy grow [J/kg/K] :

$$\Delta s_F = c_p * \ln\frac{T_{\mathrm{t21}}}{T_{\mathrm{t2}}} - R * \ln(\mathrm{FPR})$$

```
dS_F = 10.7038
```

Compressor entropy grow [J/kg/K] :

$$\Delta s_C = c_p * \ln\frac{T_{\mathrm{t3}}}{T_{\mathrm{t21}}} - R * \ln(\mathrm{CPR})$$

```
dS_C = 112.3414
```

Combustor entropy grow [J/kg/K] :

$$\Delta s_B = c_{\text{pB}} * \ln\frac{T_{\text{t4}}}{T_{\text{t3}}} - R_t * \ln(\sigma_B)$$

```
dS_B = 978.6373
```

HPT entropy grow [J/kg/K] :

$$\Delta s_{\text{HPT}} = c_{\text{pt}} * \ln\frac{T_{\text{t45}}}{T_{\text{t4}}} - R_t * \ln\left(\frac{P_{\text{t45}}}{P_{\text{t4}}}\right)$$

```
dS_HPT = 49.1342
```

LPT entropy grow [J/kg/K] :

$$\Delta s_{\text{LPT}} = c_{\text{pt}} * \ln\frac{T_{\text{t5}}}{T_{\text{t4}}5} - R_t * \ln\left(\frac{P_{\text{t5}}}{P_{\text{t45}}}\right)$$

```
dS_LPT = 41.1262
```

Core nozzle entropy grow [J/kg/K] :

$$\Delta s_{N\text{ int}} = -\text{Rt} * \ln(\sigma_{N\text{ int}})$$

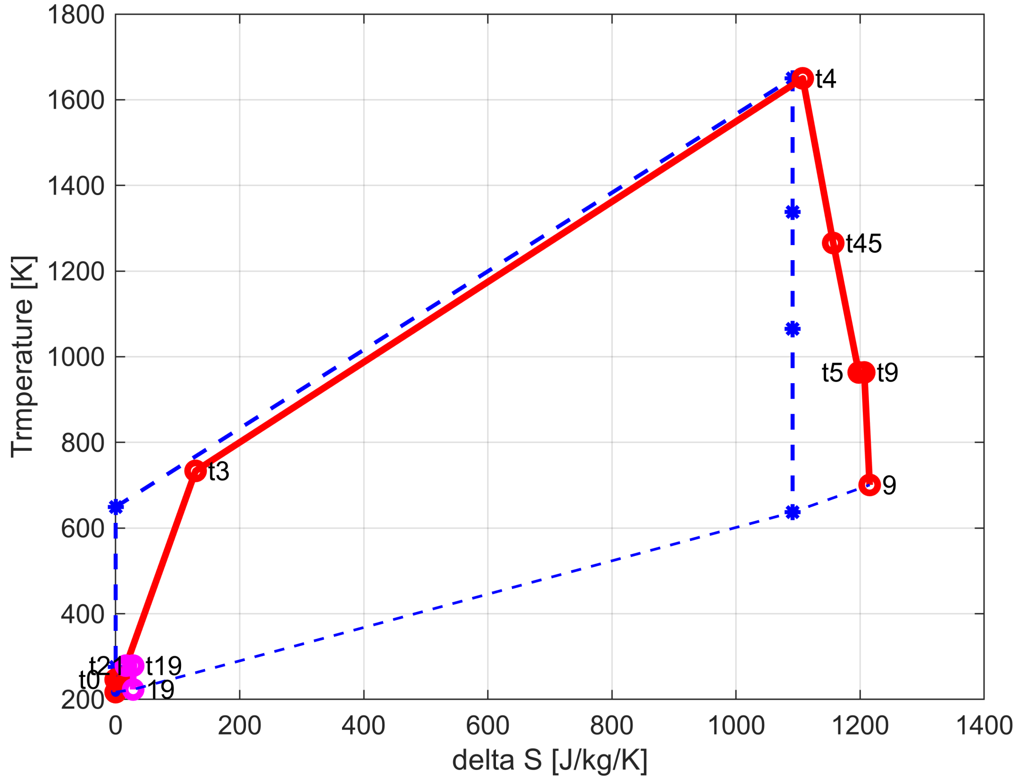
```
dS_N_int = 8.8332
```

External nozzle entropy grow [J/kg/K] :

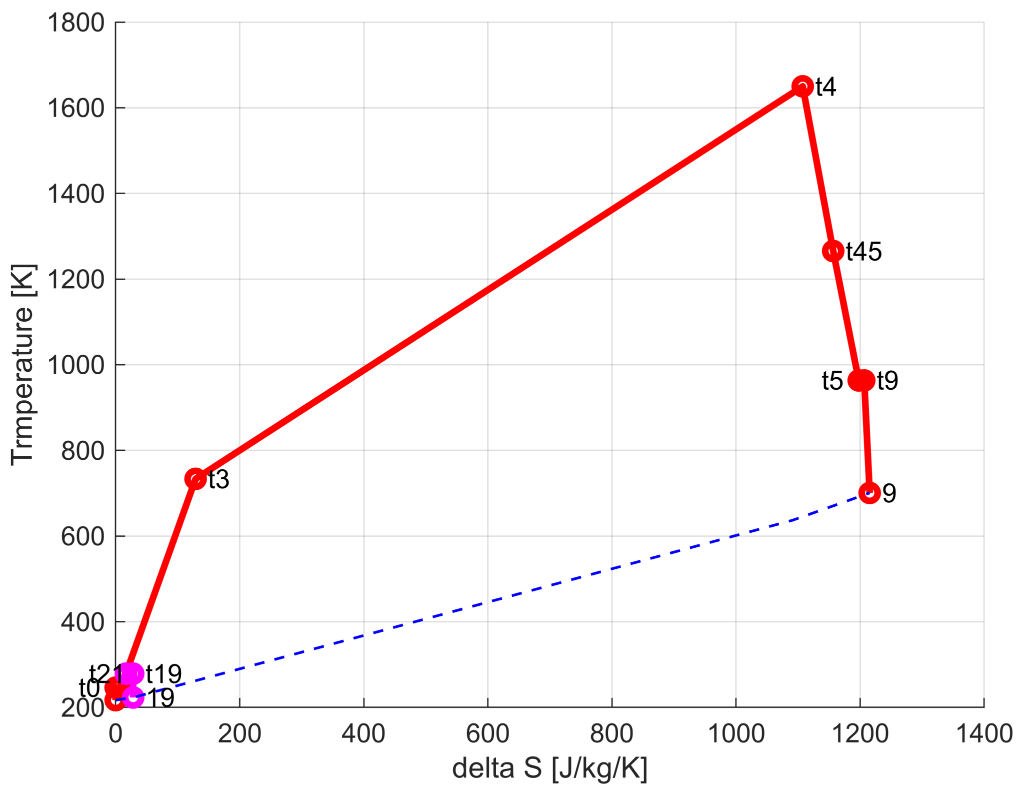
$$\Delta s_{N\text{ ext}} = -R * \ln(\sigma_{N\text{ ext}})$$

```
dS_N_ext = 11.7159
```

Temperature-entropy plot for real and ideal turbofan



Temperature-entropy plot for real turbofan



Real engine entropy rise in engine component

```
Tabela3 = 8×2 table
```

	Parameter	entropy [J/kg/K]
1	'ds_IN'	6
2	'ds_F'	11
3	'ds_C'	112
4	'ds_B'	979
5	'ds_HPT'	49
6	'ds_LPT'	41
7	'ds_N_int'	9
8	'ds_N_ext'	12

TURBOFAN ENGINE WITH CONVERGENT NOZZLES

Maximum possible gas expansion in convergent nozzle is to critical pressure - speed of jet stream is equal speed of sound (M=1). Engine calculation for other components than nozzle looks like for the engine with full expansion in the nozzle. Difference starts from nozzle parameters calculation therefore the example presented below refers to the real engine calculation presented above.

Example of turbofan engine calculation with convergent nozzles of internal and external duct for flight condition and engine work parameters like in example above.

All parameters calculated to total section t19 and t9 are like above. Difference starts from static parameters calculation in both sections.

Internal nozzle calculatios:

Total temperature in section 9

```
Tt9 = 963.3615
```

Total pressure in section 9

```
Pt9 = 7.9368e+04
```

Total to critical presure paramether calculation:

$$\beta = \left(\frac{k_t + 1}{2}\right)^{\frac{k_t}{k_t - 1}}$$

```
beta = 1.8506
```

Total pressure in internal nozzle to static ambient pressure calculation:

$$\frac{P_{t9}}{P_0} =$$

```
ans = 3.6076
```

Pt9/P0 is higher than β then full expansion in the convergent nozzle isn't possible. The critical pressure is available in the nozzle only (IE indext - incomplete expansion):

$$P_{9\,\mathrm{IE}} = \frac{P_{\mathrm{t}9}}{\beta}$$ [Pa]

```
P9_IE = 4.2887e+04
```

Static pressure in the nozzle exit

$$T_{9\,\mathrm{IE}} = \frac{2 * T_{\mathrm{t}9}}{k_t + 1}$$ [K]

```
T9_IE = 826.9197
```

Secrion 9 density calculation:

$$\rho_{9\,\mathrm{IE}} = \frac{P_{9\,\mathrm{IE}}}{R_t * T_{9\,\mathrm{IE}}}$$

```
rho_9_IE = 0.1788
```

Jet speed in the propelling nozzle exit

$$V_{9\,\mathrm{IE}} = a_9 = \sqrt{k_t * R_t * T_{9\,\mathrm{IE}}}$$ [m/s]

```
V9_IE = 564.7503
```

Speed of gas streem after expansion to ambient pressure outside the nozzle:

$$V_{9e} = V_{9\,\mathrm{IE}} + \frac{(P_{9\,\mathrm{IE}} - P_0)}{\rho_{9\,\mathrm{IE}} * V_{9\,\mathrm{IE}}}$$

```
V9e = 771.5548
```

Static temperaure in 9e

$$P_{9e} = P_0$$

```
P9e = 22000
```

Static temperature in section 9e

$$T_{9e} = T_{\mathrm{t}9} - \frac{V_{9e}^2}{2 * \mathrm{cp}_t}$$

```
T9e = 708.9611
```

External nozzle calculatios:

Total temperature in section 19

```
Tt19 = 278.8298
```

Total pressure in section 19

```
Pt19 = 4.7962e+04
```

Total to critical presure paramether calculation in external nozzle (air):

$$\beta = \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}$$

```
beta = 1.8929
```

Total pressure in internal nozzle to static ambient pressure calculation:

$$\frac{P_{t19}}{P_0} =$$

```
ans = 2.1801
```

Pt19/P0 is higher than β then full expansion in the convergent nozzle isn't possible. The critical pressure is available in the external nozzle only (IE indext - incomplete expansion):

$$P_{19\,\mathrm{IE}} = \frac{P_{t19}}{\beta} \quad \text{[Pa]}$$

```
P19_IE = 2.5337e+04
```

Static pressure in the nozzle exit

$$T_{19\,\mathrm{IE}} = \frac{2 * T_{t19}}{k+1} \quad \text{[K]}$$

```
T19_IE = 232.3582
```

Secrion 9 density calculation:

$$\rho_{19\,\mathrm{IE}} = \frac{P_{19\,\mathrm{IE}}}{R * T_{19\,\mathrm{IE}}}$$

```
rho_19_IE = 0.3799
```

Jet speed in the propelling nozzle exit

$$V_{19\,\mathrm{IE}} = a_{19} = \sqrt{k * R * T_{19\,\mathrm{IE}}} \quad \text{[m/s]}$$

```
V19_IE = 305.5512
```

Speed of gas streem after expansion to ambient pressure outside the nozzle:

$$V_{19e} = V_{19\,\mathrm{IE}} + \frac{(P_{19\,\mathrm{IE}} - P_0)}{\rho_{19\,\mathrm{IE}} * V_{19\,\mathrm{IE}}}$$

```
V19e = 334.2983
```

Static temperaure in 9e

$$P_{19e} = P_0$$

```
P19e = 22000
```

Static temperature in section 9e

$$T_{19e} = T_{t19} - \frac{V_{19}^2}{2 * \mathrm{cp}}$$

```
T19e = 223.2301
```

Performance parameters calculation for incomplete expansion in propelling nozzles

Thrust [N]

$$T = m_{21} * (1 + f_B) * V_{9e} + m_{21} * \mathrm{BPR} * V_{19e} - m_{21} * (1 + \mathrm{BPR}) * V_0$$

```
T_IE = 8.0250e+03
```

Specific thrust [Ns/kg]

$$\mathrm{ST} = \frac{T}{m_0} = \frac{(1 + f_B) * V_{9e} + \mathrm{BPR} * V_{19e} - (1 + \mathrm{BPR}) * V_0}{1 + \mathrm{BPR}}$$

```
ST_IE = 133.7493
```

Specific fuel consumption [kg/N/s]

$$\mathrm{SFC} = \frac{m_{\mathrm{fB}}}{T} = \frac{f_B}{(1 + \mathrm{BPR}) * \mathrm{ST}}$$

```
SFC_IE = 1.7738e-05
```

Specific fuel consumption [kg/N/h]

$$\mathrm{SFC} = \mathrm{SFC} * 3600$$

```
SFC_IE = 0.0639
```

Thermal efficiency

$$\eta_{\mathrm{th}} = \frac{(1 + f_B) * {V_{9e}}^2 + \mathrm{BPR} * {V_{19e}}^2 - (1 + \mathrm{BPR}) * {V_0}^2}{2 * f_B * \mathrm{FHV}}$$

```
etha_th_IE = 0.4828
```

Propulsive efficiency

$$\eta_p = \frac{2 * V_0 * \mathrm{ST} * (1 + \mathrm{BPR})}{(1 + f_B) * {V_{9e}}^2 + \mathrm{BPR} * {V_{19e}}^2 - (1 + \mathrm{BPR}) * {V_0}^2}$$

```
etha_p_IE = 0.6576
```

Overall efficiency

$$\eta_o = \frac{V_0 * \mathrm{ST} * (1 + \mathrm{BPR})}{f_B * \mathrm{FHV}} = \eta_{\mathrm{th}} * \eta_p$$

```
etha_o_IE = 0.3174
```

Performance comparisson of full expansion turbofan and turbofan with convergent nozzle

```
Tabela = 11×4 table
```

	Parameter	Unit	CONV Nozzles	FULL EXPANS
1	'Thrust'	'kN'	8.0250	8.0978
2	'Specific Thrust'	'N*s/kg'	133.7493	134.9638
3	'Fuel consumption'	'kg/s'	0.1424	0.1424
4	'Specific fuel consump'	'kg/N/h'	0.0639	0.0633
5	'therm. efficiency'	'-'	0.4828	0.4916
6	'prop. efficiency'	'-'	0.6576	0.6516
7	'overall efficiency'	'-'	0.3174	0.3203
8	'V9'	'm/s'	564.7503	783.5775
9	'V9e'	'm/s'	771.5548	783.5775
10	'V19'	'm/s'	305.5512	334.4005
11	'V19e'	'm/s'	334.2983	334.4005

CONCLUSIONS:

Incomplete expansion in the engine propelling nozzle causes:

- Lower thrust and specific thrust than it is in full decompression mode
- Higher specific fuel consumption
- Additional entropy increas caused by jet decompression outside the nozzle

ENTROPY GROW FOR INCOIMPLET EXPANSION

Core nozzle entropy grow   [J/kg/K] :

$$\Delta s_{N\ \mathrm{int}} = -\mathrm{Rt} * \ln(\sigma_{N\ \mathrm{int}})$$

```
dS_N_int = 8.8332
```

Core nozzle entropy grow outside the nozzle [J/kg/K] :

$$\Delta s_{9-9e} = \mathrm{cp}_t \ln\left(\frac{T_{9e}}{T_9}\right) - \mathrm{Rt} * \ln\left(\frac{P_{9e}}{P_9}\right)$$

```
ds_9_9e = 13.5145
```

External nozzle entropy grow   [J/kg/K] :

$$\Delta s_{N\ \mathrm{ext}} = -R * \ln(\sigma_{N\ \mathrm{ext}})$$

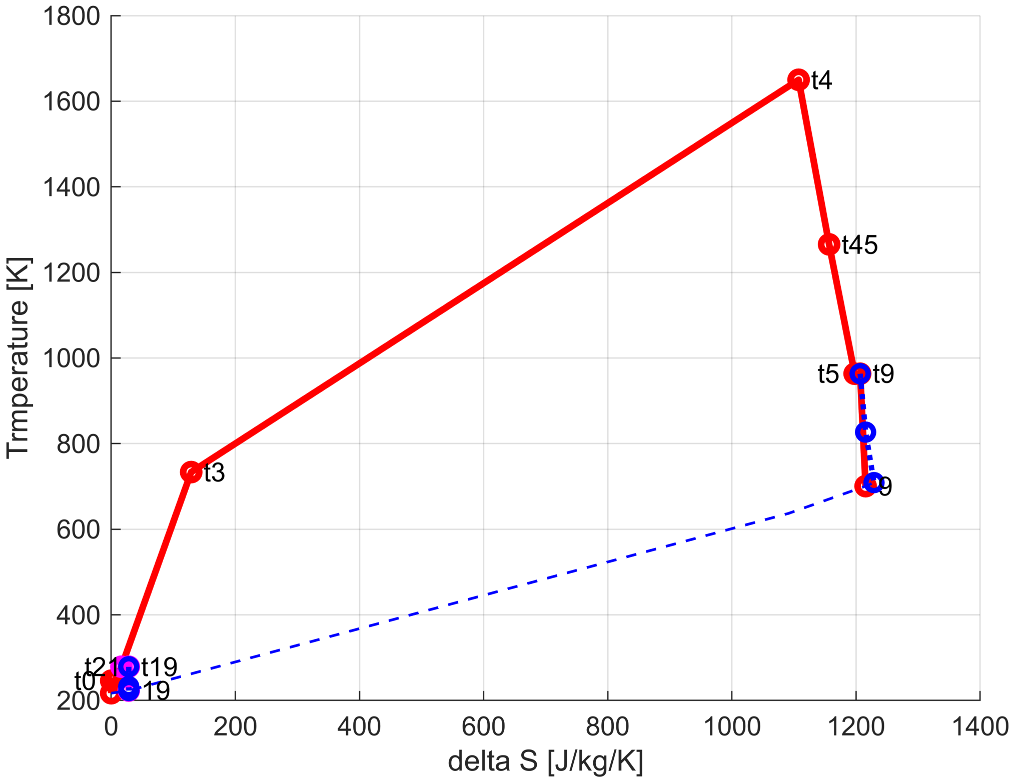
```
dS_N_ext = 11.7159
```

External nozzle entropy grow outside the nozzle [J/kg/K] :

$$\Delta s_{19-19e} = \mathrm{cp} * \ln\left(\frac{T_{19e}}{T_{19}}\right) - R * \ln\left(\frac{P_{19e}}{P_{19}}\right)$$

```
ds_19_19e = 0.2577
```

Temperature entropy plot for turbofan engine with full expansion nozzles and convergent nozzles



Core nozzle entropy grow [J/kg/K] :

$$\Delta s_{N\ \mathrm{int}} = -\mathrm{Rt} * \ln(\sigma_{N\ \mathrm{int}})$$

```
dS_N_int = 8.8332
```

Core nozzle entropy grow outside the nozzle [J/kg/K] :

$$\Delta s_{9-9e} = \mathrm{cp}_t \ln\left(\frac{T_{9e}}{T_9}\right) - \mathrm{Rt} * \ln\left(\frac{P_{9e}}{P_9}\right)$$

```
ds_9_9e = 13.5145
```

External nozzle entropy grow [J/kg/K] :

$$\Delta s_{N\ \mathrm{ext}} = -R * \ln(\sigma_{N\ \mathrm{ext}})$$

```
dS_N_ext = 11.7159
```

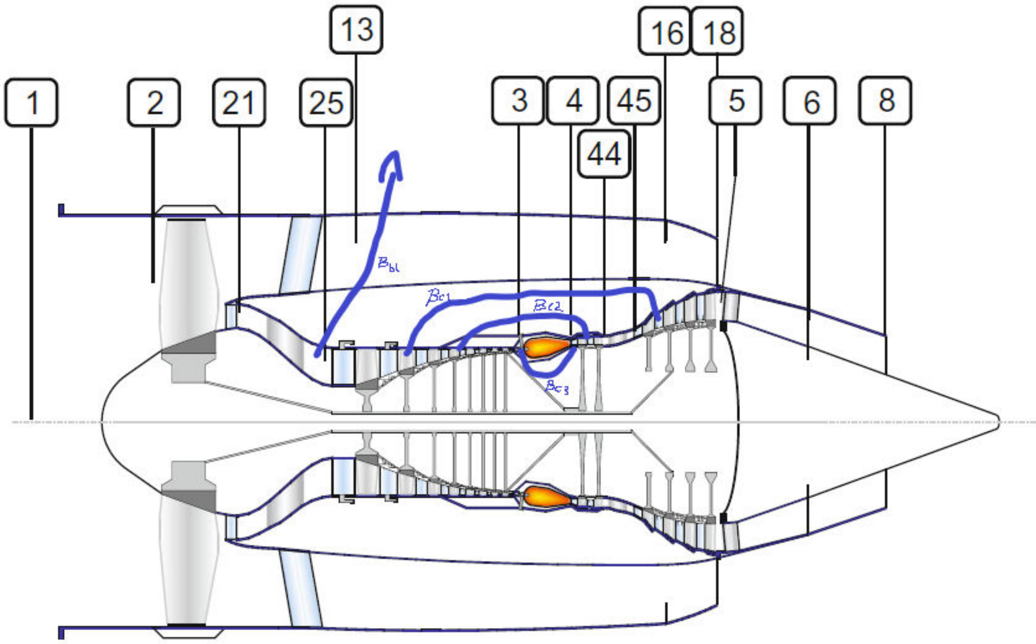
External nozzle entropy grow outside the nozzle [J/kg/K] :

$$\Delta s_{19-19e} = \mathrm{cp} * \ln\left(\frac{T_{19e}}{T_{19}}\right) - R * \ln\left(\frac{P_{19e}}{P_{19}}\right)$$

```
ds_19_19e = 0.2577
```

TURBOFAN ENGINE WITH BLEEDS AND TURBINE COOLING

Calculation of a turbofan engin with losses of the same work parameters like the turbofan above whith the compressor air bleeding and turbine cooling. The assumption is that in the external and internal engine is full expansion. The losses coefcients and efficiences are specified below.



Given

Flight conditions: T0=217 K, P0=22 kPa, M0=0.88, bypass ratio 9, fan pressure ratio 1.55, compressor pressure ratio 20, Turbine inlet temperature Tt4=1650 K, mass flow m=60 kg/s.

Customer bleed relative flow β_{BL}=0,02 is located at CPR_BL=1.6, LPT turbine cooling bleed relative folw β_{C1}=0.025 is located at CPR =3.2, HPT cooling bleed relative flow β_{C2}= 0.03 is located at CPR=9.3, and HPT nozzle cooling relative flow β_{C3}=0.08 is located after compressor.

Gas parameters:

Air: k=1.4; cp=1005 J/kg/K, R=287 J/kg/K,

Fumes in turbine and nozzle kt=1.33, cpt=1170 J/kg/K, Rt=290 J/kg/K,

For combustion in combustor cpB=1200 J/kg/K,

Air in turbine cooling process cp_c=1100 J/kg/K,

Fuel heat value: FHV=43 MJ/kg

Engine losses coefficients:

inlet pressure losses coefficient σ_{IN} 0.98, burner pressure losses coefficient σ_B 0.98, internal nozzle pressure losses coefficient $\sigma_{N\,\text{int}}$ 0.97, external nozzle pressure losses coefficient $\sigma_{N\,\text{ext}}$ 0.96, fun efficiency η_F 0,91, compressor efficeincy η_C 0.83, high pressure turbine (HPT) efficiency η_{HPT} 0.88, low pressure turbine (LPT) efficiency η_{LPT} 0.9, burner efficiency η_B 0.98, mechanical efficiency low pressure spool $\eta_{M\,\mathrm{LP}}$= 0.995 ,mechanical efficiency high pressure spool $\eta_{M\,\mathrm{HP}}$= 0.99.

Flight Mach No

```
M0 = 0.8200
```

Air Mass flow [kg/s]

```
m0 = 60
```

Bypass ratio

```
BPR = 10
```

Turbine inlet temperature [K]

```
Tt4 = 1650
```

Fan pressure ratio

```
FPR = 1.4900
```

Compressor pressure ratio

```
CPR = 20
```

Ambient conditions

Static temperature [K]

```
T0 = 217
```

Static pressure [Pa]

```
P0 = 22000
```

TURBOFAN ENGINE WITH LOSSES and BLEEDING

Section 0

Total temperature [K]

$T_{t0} = T_0\left(1 + \frac{k-1}{2} M_0^2\right)$ - like for ideal engine

```
Tt0 = 246.1822
```

Total pressure [Pa]

$P_{t0} = P_0\left(1 + \frac{k-1}{2} M_0{}^2\right)^{\frac{k}{k-1}}$ - like for ideal engine

```
Pt0 = 3.4215e+04
```

Speed of sound [m/s]

$a_0 = \sqrt{k * R * T_0}$ - like for ideal engine

```
a0 = 295.2805
```

Flight speed [m/s]

$V_0 = M_0 * a_0$ - like for ideal engine

```
V0 = 242.1300
```

Section 2 Compressor inlet

Total temperature [K]

$T_{t2} = T_{t0}$

```
Tt2 = 246.1822
```

Total pressure [Pa]

$P_{t2} = \sigma_{IN} * P_{t0}$

```
Pt2_1 = 3.3530e+04
```

Section 21/13 - Fan outlet

Total temperature [K]

$$T_{t21} = T_{t13} = T_{t2} * \left(1 + \frac{\mathrm{FPR}^{\frac{k-1}{k}} - 1}{\eta_F}\right)$$

```
Tt21 = 278.8298
Tt13 = 278.8298
```

Total pressure [Pa]

$P_{t21} = P_{t13} = P_{t2} * \mathrm{FPR}$

```
Pt21 = 4.9960e+04
```

FAN

Fan work [J/kg]

$W_F = c_p * (T_{t21} - T_{t2})$

```
WF = 3.2811e+04
```

Fan power [W]

$P_F = m_0 * W_F$

```
PF = 1.9687e+06
```

STREAM SPLIT

Internal duct mass flow [kg/s]

$$m_{21} = m_0 * \frac{1}{1 + \mathrm{BPR}}$$

```
m21 = 5.4545
```

External duct mass flow [kg/s]

$$m_{13} = m_0 * \frac{\mathrm{BPR}}{1 + \mathrm{BPR}}$$

```
m13 = 54.5455
```

INTERNAL DUCT / CORE ENGINE

Section 3 - Compressor outlet / Burner inlet

Total temperature [K]

$$T_{t3} = T_{t21} * \left(1 + \frac{\mathrm{CPR}^{\frac{k-1}{k}} - 1}{\eta_C}\right)$$

```
Tt3 = 733.5397
```

Total pressure [Pa]

$$P_{t3} = P_{t21} * \mathrm{CPR}$$

```
Pt3 = 9.9920e+05
```

BLEED CALCULATION

Politropic efficiency calculation

$$e_C = \frac{k-1}{k} \frac{\ln \frac{P_{t3}}{P_{t21}}}{\ln \frac{T_{t3}}{T_{t21}}}$$

```
ec = 0.8849
```

Customer bleed

Total temperature

$$T_{tBL} = T_{t21} * \mathrm{CPR_{BL}}^{\frac{k-1}{k\,\mathrm{ec}}}$$

```
TtBL = 324.5239
```

Total pressure

$$P_{tBL} = P_{t21} * \mathrm{CPR_{BL}}$$

```
PtBL = 7.9936e+04
```

Air bleed mass flow [kg/s]

$m_{BL} = \beta_{BL}\, m_{21}$

```
m_BL = 0.1091
```

LPT coolant bleed

Total temperature

$$T_{tC1} = T_{t21} * CPR_{C1}^{\frac{k-1}{k\, ec}}$$

```
TtC1 = 405.9243
```

Total pressure

$$P_{tC1} = P_{t21} * CPR_{C1}$$

```
PtC1 = 1.5987e+05
```

Air bleed mass flow [kg/s]

$$m_{C1} = \beta_{C1}\, m_{21}$$

```
m_C1 = 0.1364
```

HPT coolant bleed

Total temperature

$$T_{tC2} = T_{t21} * CPR_{C2}^{\frac{k-1}{k\, ec}}$$

```
TtC2 = 572.8605
```

Total pressure

$$P_{tC2} = P_{t21} * CPR_{C2}$$

```
PtC2 = 4.6463e+05
```

Air bleed mass flow [kg/s]

$$m_{C2} = \beta_{C2}\, m_{21}$$

```
m_C2 = 0.1636
```

LPT nozzle coolant bleed

Total temperature

$$T_{tC3} = T_{t3}$$

```
TtC3 = 733.5397
```

Air bleed mass flow [kg/s]

$$m_{C3} = \beta_{C3}\, m_{21}$$

```
m_C3 = 0.4364
```

COMPRESSOR

Bleeding parameters sumarization

```
TabelaC = 4×6 table
```

	Param.	Units	custom bl	LPT cool	HPT cool	HPT_Noz Cool
1	'rel_flow'	'-'	0.0200	0.0250	0.0300	0.0800
2	'mass'	'kg/s'	0.1091	0.1364	0.1636	0.4364
3	'pressure'	'kPa'	79.9363	159.8726	464.6297	999.2036
4	'temp.'	'K'	324.5239	405.9243	572.8605	733.5397

Compressor work [J/kg]

$$W_C = c_p[(T_{\mathrm{tBL}} - T_{\mathrm{t21}}) + (1 - \beta_{\mathrm{BL}})(T_{\mathrm{tC1}} - T_{\mathrm{tBL}}) + (1 - \beta_{\mathrm{BL}} - \beta_{\mathrm{C1}})(T_{\mathrm{tC2}} - T_{\mathrm{tC1}}) + (1 - \beta_{\mathrm{BL}} - \beta_{\mathrm{C1}} - \beta_{\mathrm{C2}})(T_{\mathrm{t3}} - T_{\mathrm{tC2}})]$$

```
WC = 4.3569e+05
```

Compressor power [W]

$$P_C = m_{21} * W_C$$

```
PC = 2.3765e+06
```

Section 4 Burner outlet / Turbine inlet

Total temperature [K]

$$T_{\mathrm{t4}}$$

```
Tt4 = 1650
```

Total pressure [Pa]

$$P_{\mathrm{t4}} = \sigma_B * P_{\mathrm{t3}}$$

```
Pt4 = 9.7922e+05
```

BURNER

Fuel-air ratio

$$f_B = c_{\mathrm{pB}}(1 - \beta_{\mathrm{BL}} - \beta_{\mathrm{C1}} - \beta_{\mathrm{C2}} - \beta_{\mathrm{C3}})\frac{T_{\mathrm{t4}} - T_{\mathrm{t3}}}{\mathrm{FHV} * \eta_B}$$

```
fB = 0.0221
```

Fuel mass flow [kg/s]

$$m_{\mathrm{fB}} = m_{21} * f_B$$

```
mfB = 0.1203
```

Section 45 High Pressure Turbine (HPT) outlet / Low Pressure Turbine (LPT) inlet

Total temperature [K]

$$T_{t45} = -\frac{c_{pt}\,(1-\beta_{BL}-\beta_{C1}-\beta_{C2}-\beta_{C3})T_{t4} + c_{pc}\,\beta_{C3}\,T_{tC3} + c_{pc}\,\beta_{C2}\,T_{tC2} - \dfrac{W_C}{\eta_{M\ HP}}}{c_{pt}\,(1-\beta_{BL}-\beta_{C1}-\beta_{C2}-\beta_{C3}) + c_{pc}\,\beta_{C3} + c_{pc}\,\beta_{C2}}$$

```
Tt45 = 1.1100e+03
```

Total pressure [Pa]

$$P_{t45} = P_{t4}\left(\frac{\eta_{T\ HP} + \dfrac{T_{t45}}{T_{t4}} - 1}{\eta_T}\right)^{\frac{kt}{kt-1}}$$

```
Pt45 = 1.5030e+05
```

High pressure turbine pressure ratio

$$H\text{PT PR} = \frac{P_{t4}}{P_{t45}}$$

```
HPT_PR = 6.5153
```

Section 5 LPT outlet / Nozzle inlet

Total temperature [K]

$$T_{t5} = -\frac{c_{pt}\,(1-\beta_{BL}-\beta_{C1})T_{t45} + c_{pc}\,\beta_{C1}\,T_{tC1} - \dfrac{(1+\text{BPR})W_F}{\eta_{M\ LP}}}{c_{pt}\,(1-\beta_{BL}-\beta_{C1}) + c_{pc}\,\beta_{C1}}$$

```
Tt5 = 776.2853
```

Total pressure [Pa]

$$P_{t5} = P_{t45}\left(\frac{\eta_{LPT} + \dfrac{T_{t5}}{T_{t45}} - 1}{\eta_{LPT}}\right)^{\frac{kt}{kt-1}}$$

```
Pt5 = 2.9194e+04
```

Low pressure turbine pressure ratio

$$\text{LPT PR} = \frac{P_{t45}}{P_{t5}}$$

```
LPT_PR = 5.1481
```

Section 9 Core engine nozzle outlet / Internal duct nozzle outlet

Total temperature [K]

$$T_{t9} = T_{t5}$$

```
Tt9 = 776.2853
```

Total pressure [Pa]

$P_{t9} = P_{t5} * \sigma_{N\,\mathrm{int}}$

```
Pt9 = 2.8318e+04
```

Static pressure [Pa]

$P_9 = P_0$

```
P9 = 22000
```

Static temperature [K]

$$T_9 = T_{t9} * \left(\frac{P_9}{P_{t9}}\right)^{\frac{\mathrm{kt}-1}{\mathrm{kt}}}$$

```
T9 = 729.1492
```

Jet stream Mach No

$$M_9 = \sqrt{\left(\frac{T_{t9}}{T_9} - 1\right) * \frac{2}{\mathrm{kt} - 1}}$$

```
M9 = 0.6259
```

Speed of sound [m/s]

$a_9 = \sqrt{\mathrm{kt} * \mathrm{Rt} * T_9}$

```
a9 = 530.3139
```

Jet speed [m/s]

$V_9 = M_9 * a_9$

```
V9 = 331.9402
```

Section 19 External Engine Nozzle outlet

Total temperature [K]

$T_{t19} = T_{t13}$

Total pressure [Pa]

$P_{t19} = \sigma_{N\,\mathrm{ext}} * P_{t13}$

```
Pt19 = 4.7962e+04
```

Static pressure [Pa]

$P_{19} = P_0$

```
P19 = 22000
```

Static temperature [K]

$$T_{19} = T_{t19} * \left(\frac{P_{19}}{P_{t19}}\right)^{\frac{k-1}{k}}$$

```
T19 = 223.1684
```

Jet stream Mach No

$$M_{19} = \sqrt{\left(\frac{T_{t19}}{T_{19}} - 1\right) * \frac{2}{k-1}}$$

```
M19 = 1.1167
```

Speed of sound [m/s]

$$a_{19} = \sqrt{k * R * T_{19}}$$

```
a19 = 299.4480
```

Jet speed [m/s]

$$V_{19} = M_{19} * a_{19}$$

```
V19 = 334.4005
```

TURBOFAN ENGINE PERFORMANCE CALCULATION

Thrust [N]

$$T = m_{21} * (1 + f_B - \beta_{\mathrm{BL}}) * V_9 + m_{21} * \mathrm{BPR} * V_{19} - m_{21} * (1 + \mathrm{BPR}) * V_0$$

```
T = 5.5265e+03
```

Specific thrust [Ns/kg]

$$\mathrm{ST} = \frac{T}{m_0} = \frac{(1 + f_B - \beta_{\mathrm{BL}}) * V_9 + \mathrm{BPR} * V_{19} - (1 + \mathrm{BPR}) * V_0}{1 + \mathrm{BPR}}$$

```
ST = 92.1087
```

Specific fuel consumption [kg/N/s]

$$\mathrm{SFC} = \frac{m_{\mathrm{fB}}}{T} = \frac{f_B}{(1 + \mathrm{BPR}) * \mathrm{ST}}$$

```
SFC = 2.1765e-05
```

Specific fuel consumption [kg/N/h]

$$\mathrm{SFC} = \mathrm{SFC} * 3600$$

```
SFC = 0.0784
```

Thermal efficiency

$$\eta_{\mathrm{th}} = \frac{(1 + f_B) * {V_9}^2 + \mathrm{BPR} * {V_{19}}^2 - (1 + \mathrm{BPR}) * {V_0}^2}{2 * f_B * \mathrm{FHV}}$$

```
etha_th = 0.3090
```

Propulsive efficiency

$$\eta_p = \frac{2 * V_0 * \mathrm{ST} * (1 + \mathrm{BPR})}{(1 + f_B) * {V_9}^2 + \mathrm{BPR} * {V_{19}}^2 - (1 + \mathrm{BPR}) * {V_0}^2}$$

```
etha_p = 0.8374
```

Overall efficiency

$$\eta_o = \frac{V_0 * \mathrm{ST} * (1 + \mathrm{BPR})}{f_B * \mathrm{FHV}} = \eta_{\mathrm{th}} * \eta_p$$

```
etha_o = 0.2587
```

Temperature, pressure vs engine sections plot for turbofan with bleeding and without bleeding

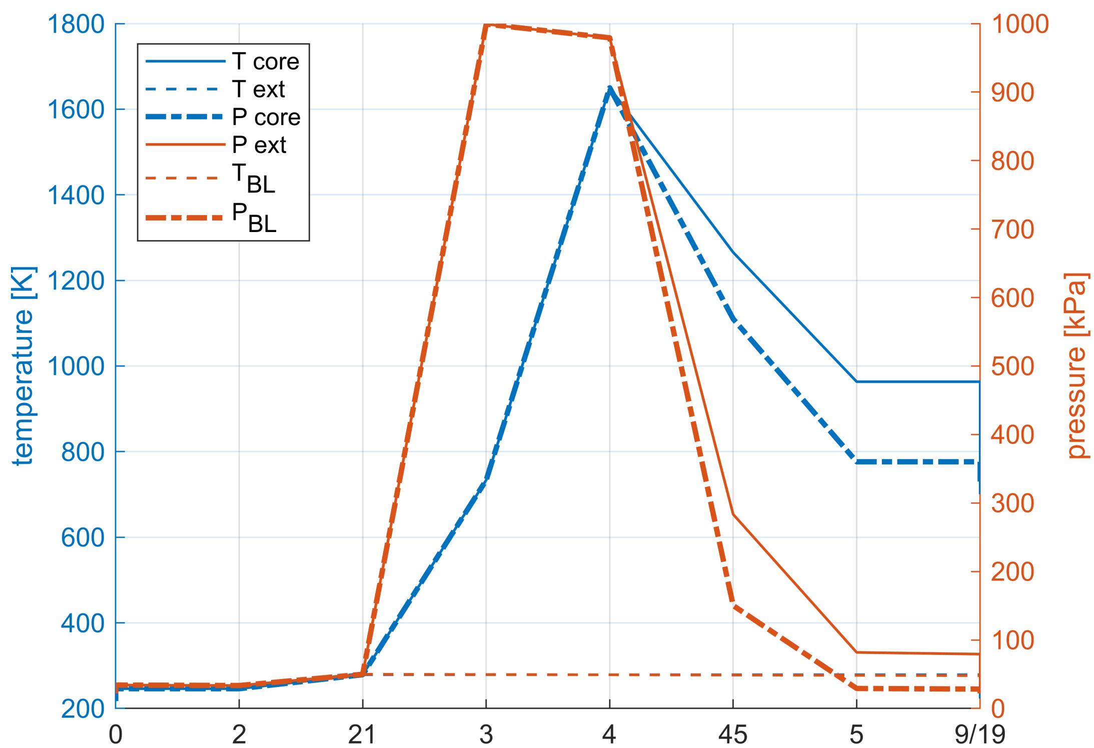


Tabela = 12×5 table

	SE	T no bl [K]	T bleed [K]	P no bl [kPa]	P bleed [kPa]
1	'0'	217	217	22	22
2	't0'	246	246	34.2146	34.2146
3	't2'	246	246	33.5303	33.5303
4	't21'	279	279	49.9602	49.9602
5	't3'	734	734	999.2036	999.2036
6	't4'	1650	1650	979.2195	979.2195

	SE	T no bl [K]	T bleed [K]	P no bl [kPa]	P bleed [kPa]
7	't45'	1266	1110	283.4293	150.2953
8	't5'	963	776	81.8223	29.1941
9	't9'	963	776	79.3677	28.3183
10	'9'	701	729	22	22
11	't19'	279	279	47.9618	47.9618
12	'19'	223	223	22	22

Performance comparison of real vs ideal turbofan engine

```
Tabela = 11×4 table
```

	Parameter	Unit	no bleed	bleed
1	'Thrust'	'kN'	8.0978	5.5265
2	'Specific Thrust'	'N*s/kg'	134.9638	92.1087
3	'Fuel consumption'	'kg/s'	0.1424	0.1203
4	'Specific fuel consump'	'kg/N/h'	0.0633	0.0784
5	'therm. efficiency'	'-'	0.4916	0.3090
6	'prop. efficiency'	'-'	0.6516	0.8374
7	'overall efficiency'	'-'	0.3203	0.2587
8	'V9'	'm/s'	783.5775	331.9402
9	'V19'	'm/s'	334.4005	334.4005
10	'HPT_PR'	'-'	3.4549	6.5153
11	'LPT_PR'	'-'	3.4640	5.1481

CONCLUSIONS

Bleeding turbofan vs no bleeding turbofan comparison summarization

- Total pressure and temperature are lower starting from section 45
- Lower fuel consumption caused lower air mass flow through burner
- Higher turbines pressure ratio TPR
- Lower thrust, specific thrust, and thermal and overall efficiences
- Higher specific fuel consumption, but lower fuel consumption