# Turbine – 2 D analysis

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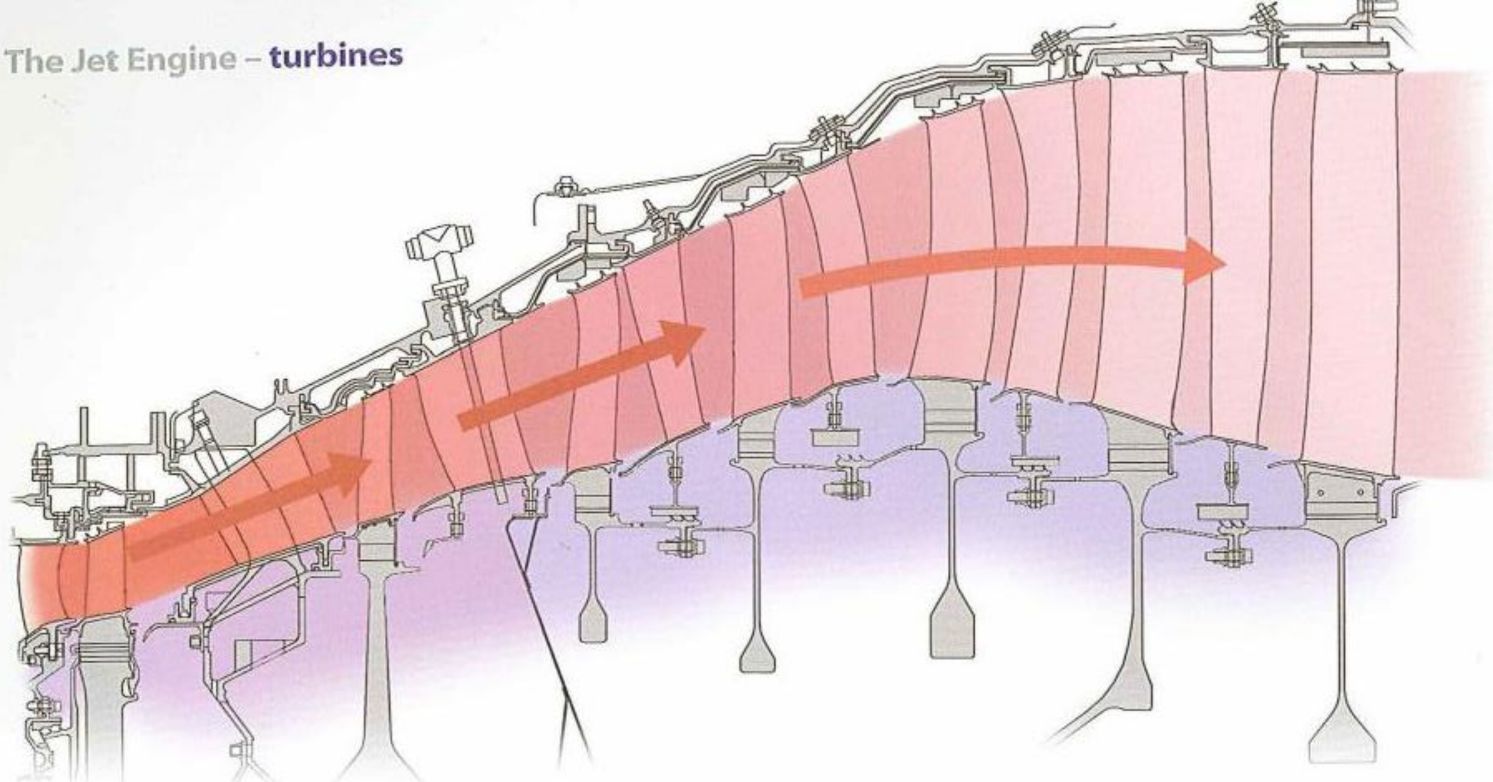
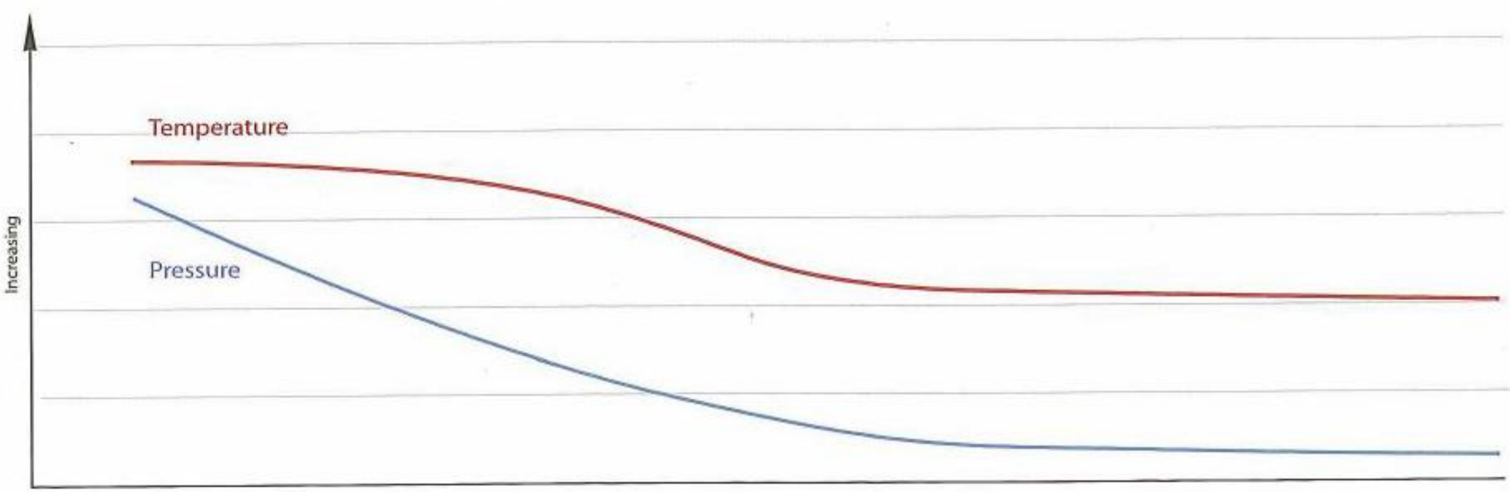





Reducing pressure and temperature through turbines

# Task example

Given to turbine stage design:

Work: 179800 J/kg

Inlet total temperature: 1300 K

Inlet total pressure: 10^6 Pa

Inlet flow speed : 180 m/s

Inlet flow angle: 0

Axial velocity is constant

Outlet flow angle: 12

Rotation speed U: 300 m/s

Pressure losses in stator: 0.985

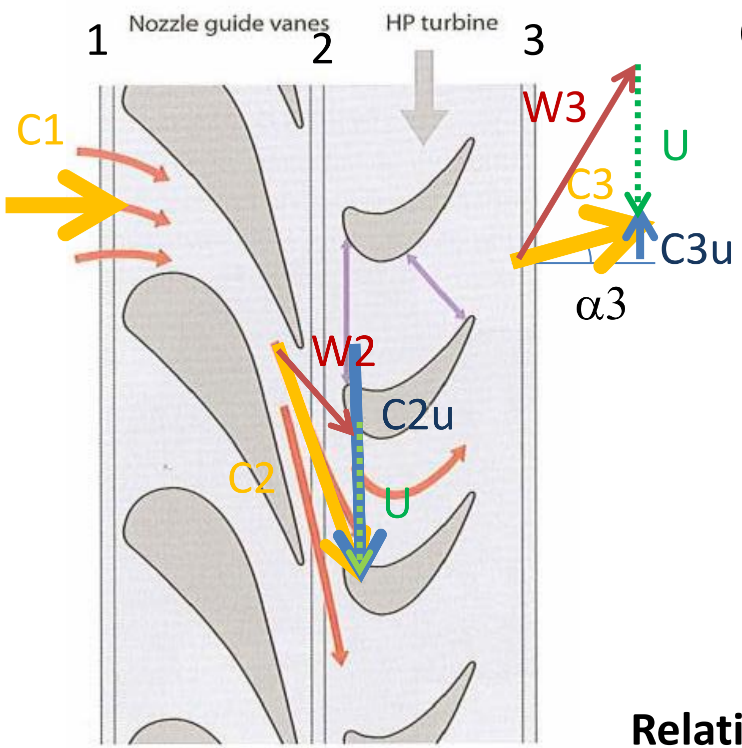
Pressure losses in rotor: 0.98

Cp = 1200 J/kg/K
k = 1.33
R = 289 J/kg/K

To do:
Velocity triangles in all section
Temperature and pressure in turbine sections
Turbine parameters:

- Reaction degree
- Stage loading
- Isentropic and polytrophic efficiency
- Turbine pressure ratio
- Turbine stage geometry angles

# Single turbine stage velocity triangles




Turbine driven by the impulse of the gas flow and its subsequent reaction as it accelerates through the converging blade passage

Outlet temperature:

$$T_{03} = T_{01} - W/cp = 1300-179800/1200=1150 \text{ K}$$

Delta Cu:

$$\Delta c_u = W/U = 179800/300=600 \text{ m/s}$$

C3u: $c_{3u} = c_x * tg(\alpha_3) = 180 * tg(12°)=39 \text{ m/s}$

C3: $c_3 = \sqrt{c_{3u}^2 + c_x^2} = \sqrt{39^2 + 180^2} = 184 m/s$

C2u: $c_{2u} = \Delta c_u - c_{3u} = 600 - 39=561 \text{ m/s}$

C2: $c_2 = \sqrt{c_{2u}^2 + c_x^2} = \sqrt{561^2 + 180^2} = 589 m/s$

**Relative flow in rotor calculation**

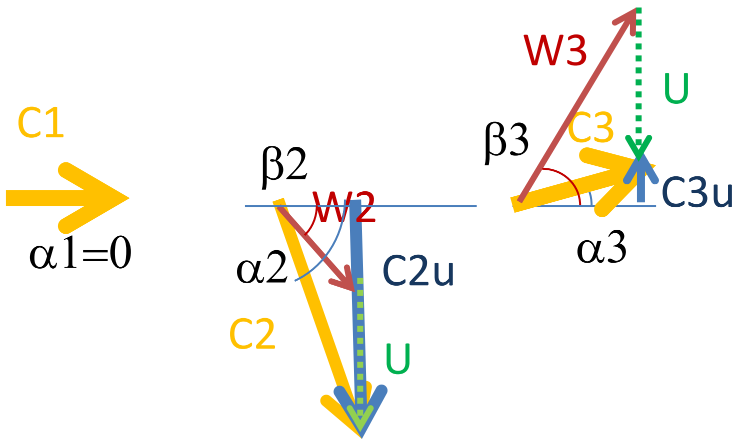
W2u: $W_{2u} = c_{2u} - U = 561 - 300 = 261\ m/s$

W2: $W_2 = \sqrt{W_{2u}^2 + c_x^2} = \sqrt{261^2 + 180^2} = 317 m/s$

W3u: $W_{3u} = c_{3u} + U = 39 + 300 = 339\ m/s$

W3: $W_3 = \sqrt{W_{3u}^2 + c_x^2} = \sqrt{339^2 + 180^2} = 383 m/s$

# Velocity angles




**Absolute** flow angles:

$\alpha 1 = 0$

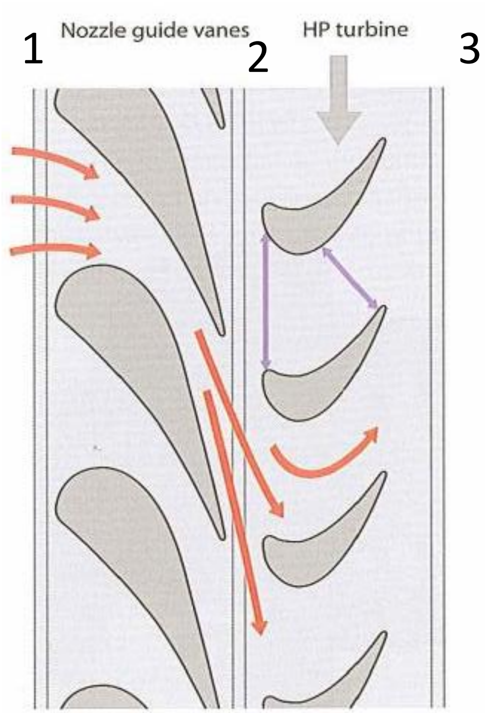
$\alpha 2 = \text{acos}(c_x/c_2)$=acos(180/589)=72

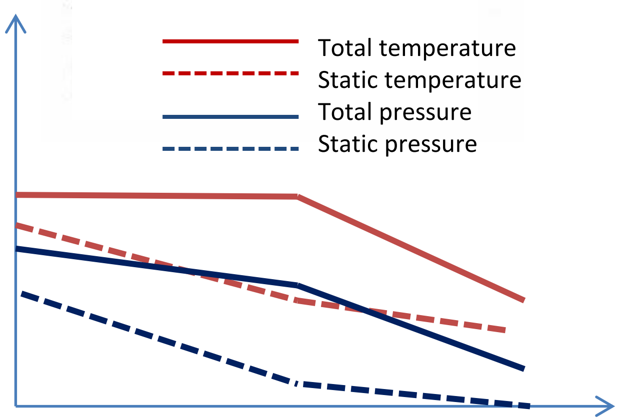
**Relative** flow angles:

$\beta 2 = \text{acos}(c_x/W_2)$=acos(180/317)=46

$\beta 3 = \text{acos}(c_x/W_3)$=acos(180/383)=62

# Temperature and pressure




**Total temperature in section 1 and 2**

$T_{01} = T_{02} = 1300\ K$

Outlet total temperature:

$T_{03} = T_{01} - W/cp$=1300-179800/1200=1150 K

Static temperature

$$T_1 = T_{01} - \frac{c_1^2}{2*c_p} = 1300 - \frac{180^2}{2*1200} = 1286.5\ K$$

$$T_2 = T_{02} - \frac{c_2^2}{2*c_p} = 1300 - \frac{589^2}{2*1200} = 1155\ K$$

$$T_3 = T_{03} - \frac{c_3^2}{2*c_p} = 1150 - \frac{184^2}{2*1200} = 1136\ K$$

**Total pressure in section 1 and 2**

$$p_{01} = 10^6\ \ Pa$$

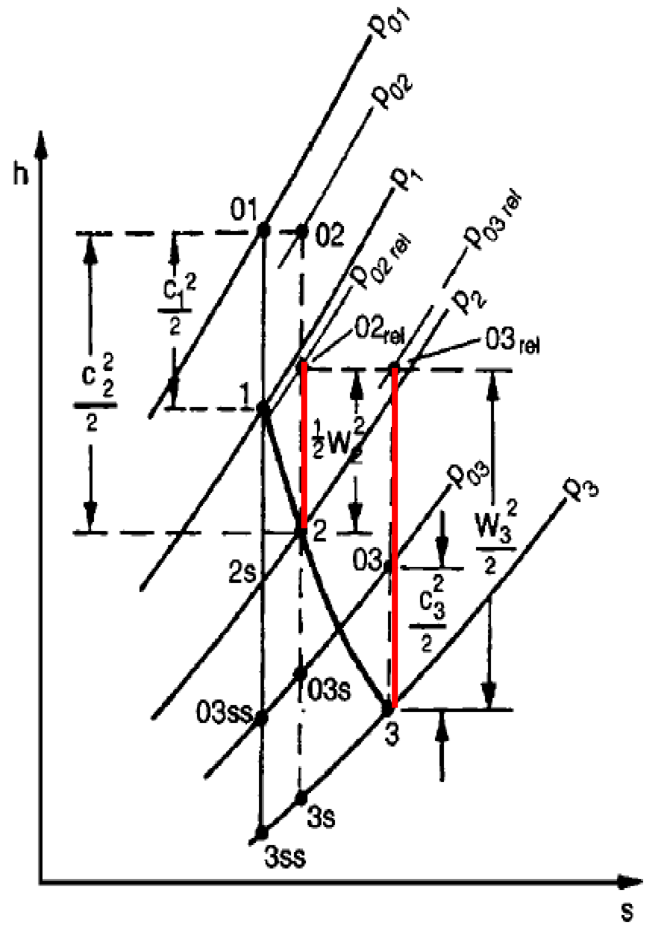
$$p_{02} = \sigma_{ST} * p_{01} = 0.985 * 10^6 = 985000\ Pa$$

**Static pressure in section 1 and 2**

$$p_1 = p_{01} * \left(\frac{T_1}{T_{01}}\right)^{\frac{k}{k-1}} = 10^6 * \left(\frac{1286.5}{1300}\right)^{\frac{1.33}{0.33}} = 955770 Pa$$

$$p_2 = p_{02} * \left(\frac{T_2}{T_{02}}\right)^{\frac{k}{k-1}} = 985000 * \left(\frac{1155}{1300}\right)^{\frac{1.33}{0.33}} = 590782 Pa$$

# Mellor diagram and pressure in section 3




Rotalpy equation to evaluate rotor relative parameters

$$T_{02rel} = T_2 + \frac{W_2^2}{2 * c_p} = 1155 + \frac{317^2}{2 * 1200} = 1197\ K$$

$$T_{03rel} = T_{02rel} = 1197\ K$$

$$T_{03rel} = T_3 + \frac{W_3^2}{2 * c_p}$$

Pressure in relative and absolute value in rotor

$$p_{02rel} = p_2 * \left(\frac{T_{02rel}}{T_2}\right)^{\frac{k}{k-1}} = 590783 * \left(\frac{1197}{1155}\right)^{\frac{1.33}{0.33}} = =689386\ Pa$$

$$p_{03rel} = p_{02rel} * \sigma_R = 689386 * 0.98 = 675598\ Pa$$

$$p_3 = p_{03rel} * \left(\frac{T_3}{T_{03rel}}\right)^{\frac{k}{k-1}} = 675598 * \left(\frac{1139}{1197}\right)^{\frac{1.33}{0.33}} = = 538250 Pa$$

$$p_{03} = p_3 * \left(\frac{T_{03}}{T_3}\right)^{\frac{k}{k-1}} = 538250 * \left(\frac{1150}{1139}\right)^{\frac{1.33}{0.33}} = = 567524\ Pa$$

# Other turbine parameters calculation

Turbine pressure ratio:

$$\pi_{T_{t-t}} = \frac{p_{01}}{p_{02}} = \frac{10^6}{567524} = 1,76$$

Degree of Reaction:

$$R_p = \frac{p_2 - p_3}{p_1 - p_3} = \frac{590782 - 538250}{955770 - 538250} = 0.12$$

$$R_i = \frac{T_2 - T_3}{T_1 - T_3} = \frac{1155 - 1136}{1286.5 - 1136} = 0.128$$

Stage loading:

$$\psi = \frac{W}{U^2} = 179800/300^2 = 2$$

Turbine isentropic efficiency:

$$\eta_{Tt-t} = \frac{1 - T_{03}/T_{01}}{1 - (p_{03}/p_{01})^{(k-1)/k}} = \frac{1 - 1150/1300}{1 - (567524/10^6)^{(1.33-1)/1.33}} = 0.94$$

Turbine politropic efficiency:

$$\eta_{Tp} = \frac{\ln(T_{03}/T_{01})}{ln(p_{03}/p_{01})^{(k-1)/k}} = \frac{ln(1150/1300)}{ln\left((567524/10^6)^{(1.33-1)/1.33}\right)} = 0.938$$

# Data to calculation:

Given to turbine stage design:

| Task no. | 1 | 2 | 3 |
|---|---|---|---|
| Work: | 179800 J/kg | 200000 J/kg | 160000 J/kg |
| Inlet total temperature: | 1200 K | 1300 K | 1300 K |
| Inlet total pressure: | 1.2*10^6 Pa | 1.2*10^6 Pa | 1.2*10^6 Pa |
| Inlet flow speed c1: | 180 m/s | 230 m/s | 190 m/s |
| Inlet flow angle: | 6 | 30 | 12 |
| Axial velocity is constant | | | |
| Outlet flow angle: | 12 | 30 | 3 |
| Rotation speed U: | 300 m/s | 320 m/s | 300 m/s |
| Pressure losses in stator: | 0.985 | 0.98 | 0.985 |
| Pressure losses in rotor: | 0.98 | 0.97 | 0.975 |