

IDEAL TURBOJET

Robert Jakubowski PhD
Rzeszow University of Technology
Aerospace Engineering Department

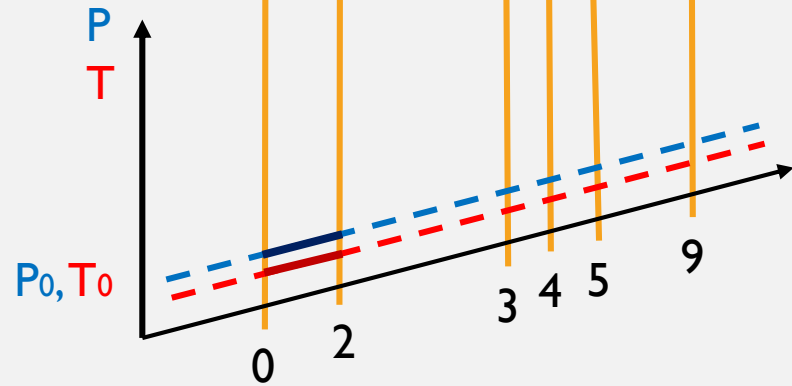
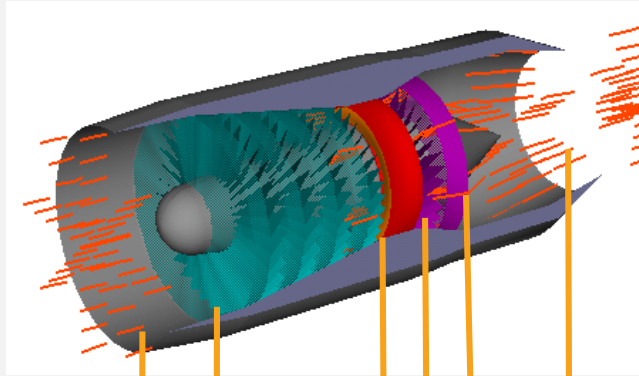
LITERATURE:

- **Jack D. Mattingly, Elements of Propulsion: Gas Turbines and Rockets, AIAA Education Series 2006 (Chapter 5)**
- **Jack D. Mattingly, Elements of Gas Turbine Propulsion, Tata McGraw Hill Education Private Limited, 2013 (Chapter 5)**
- **Gordon C. Oates, Aerothermodynamics of Gas Turbine and Rocket Propulsion, AIAA Education Series, 1997 (Chapter 5)**

ASSUMPTION

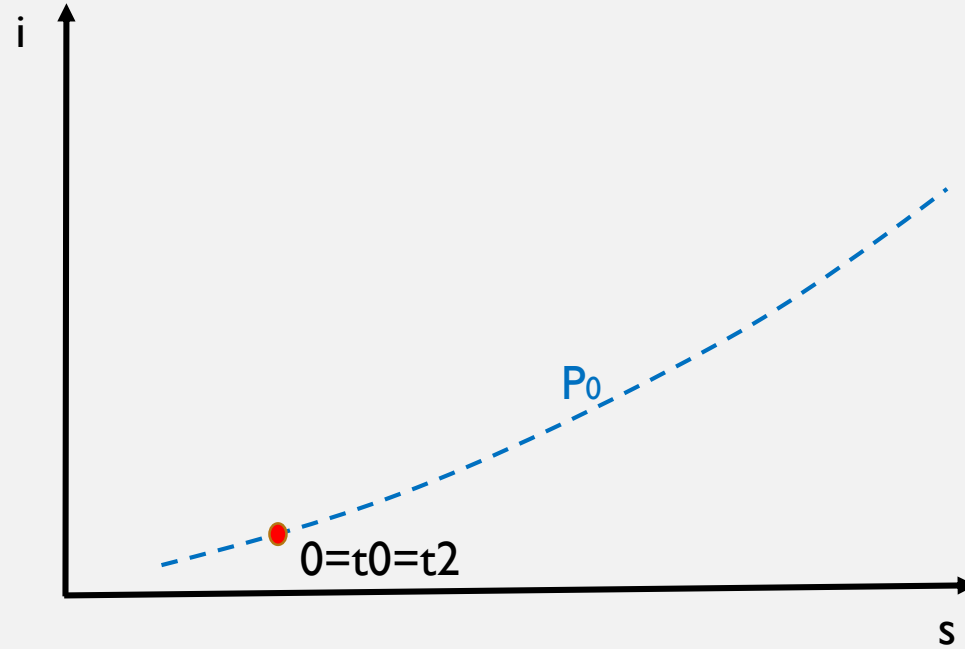
- The gas flowing through the engine is treated as a calorically perfect gas.
- The processes in the engine are considered reversible (flow, mechanical, and thermal losses in the engine are neglected).
- The change in the mass of the working fluid flowing through the engine, e.g., due to fuel addition, is neglected.
- The parameters of the flowing stream are represented at control points by averaged values (0D-1D model).

TURBOJET ENGINE AND BRAYTON CYCLE



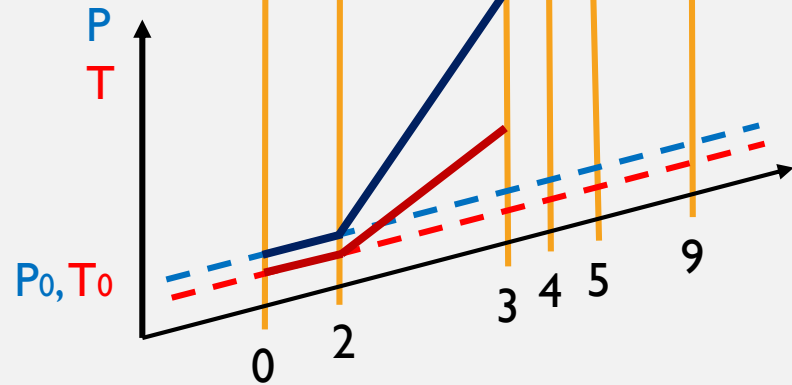
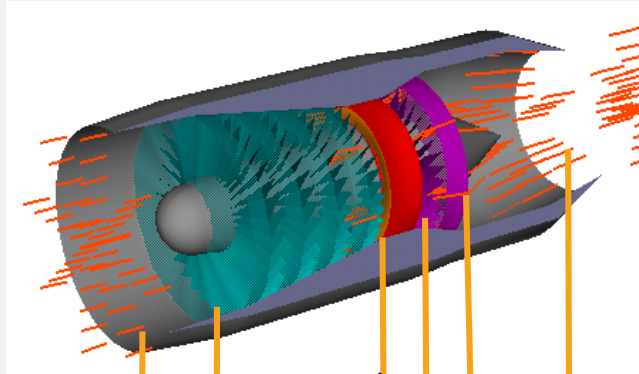
$$P_{t0} = P_0 = P_{t2}, \quad T_{t0} = T_0 = T_{t2}$$

Flight speed $V_H = 0$

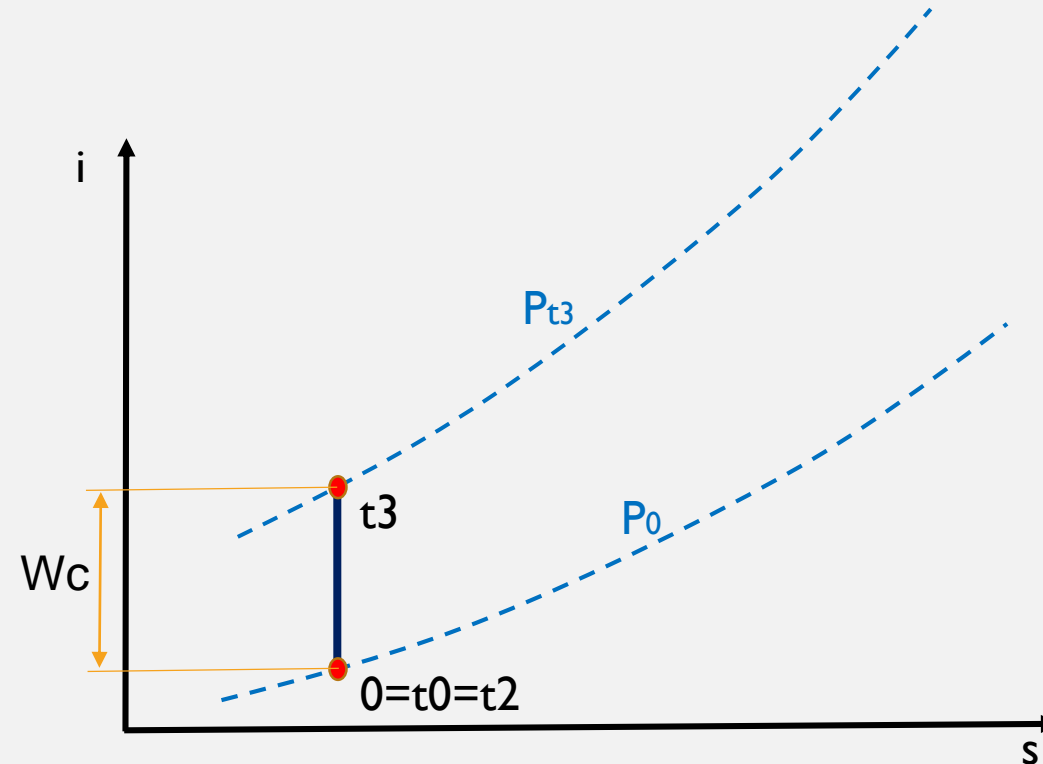


INLET

TURBOJET ENGINE AND BRAYTON CYCLE



Flight speed $V_H = 0$

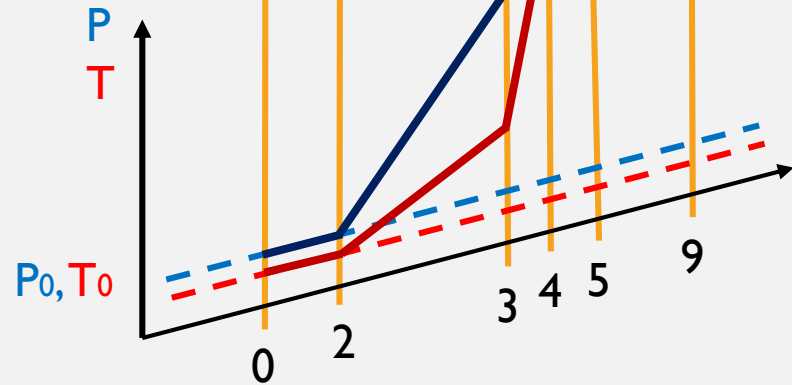
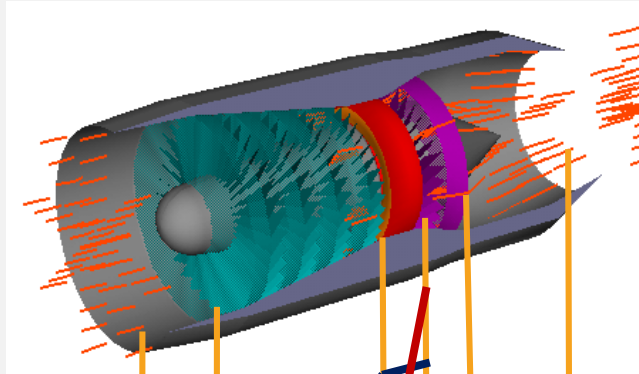


$$P_{t3} = CPR * P_{t2}, \quad T_{t3} = T_{t2} * CPR^{(k-1)/k}$$

Compressor work: $W_C = Cp(T_{t3} - T_{t2})$

COMPRESSOR

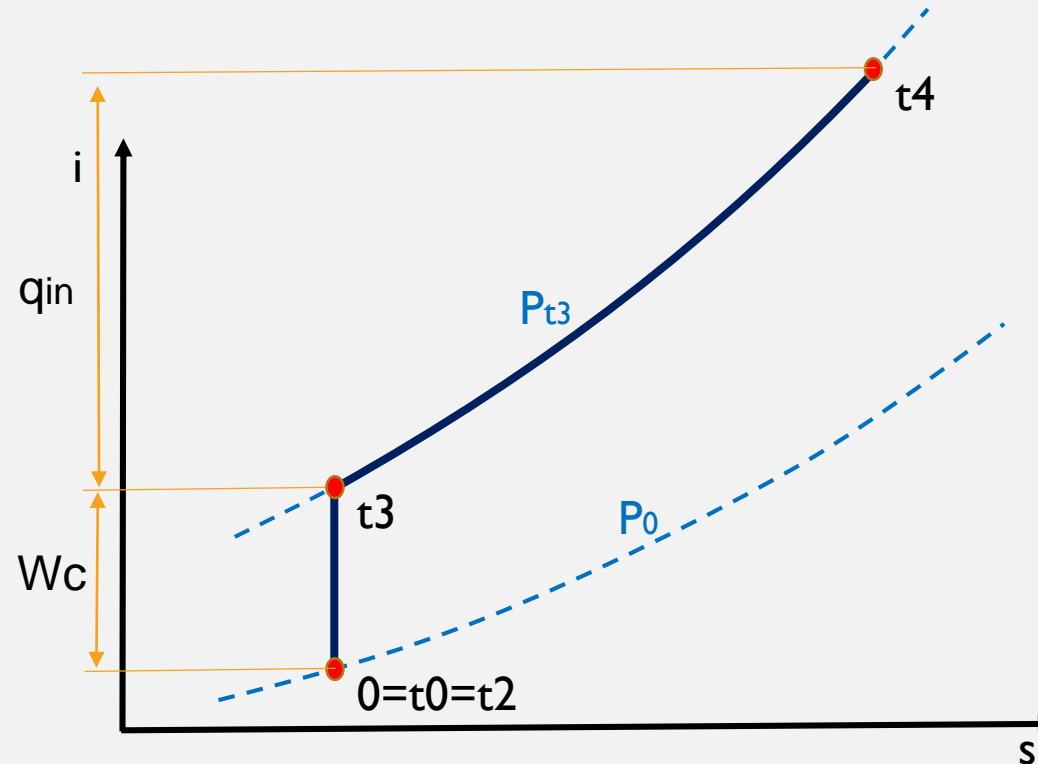
TURBOJET ENGINE AND BRAYTON CYCLE



$$P_{t4} = P_{t3}, \quad T_{t4} = T_{t3} + \frac{q_{in}}{Cp}$$

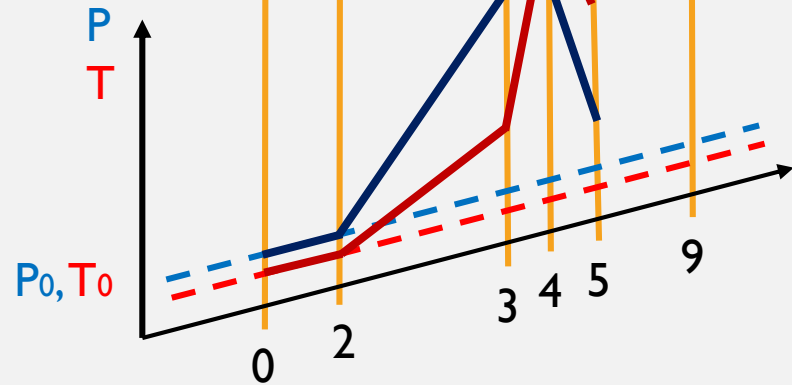
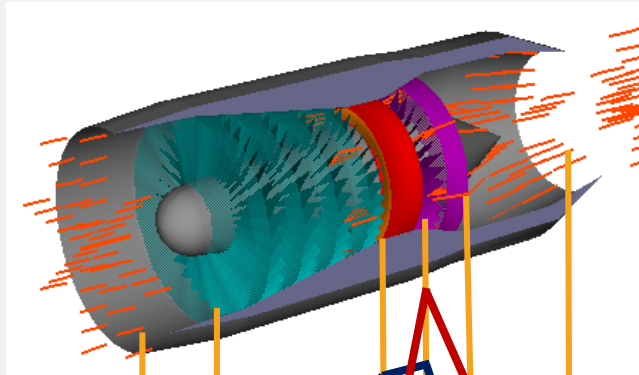
Heat added: $q_{in} = Cp(T_{t4} - T_{t3})$

Flight speed $V_H = 0$



COMBUSTOR

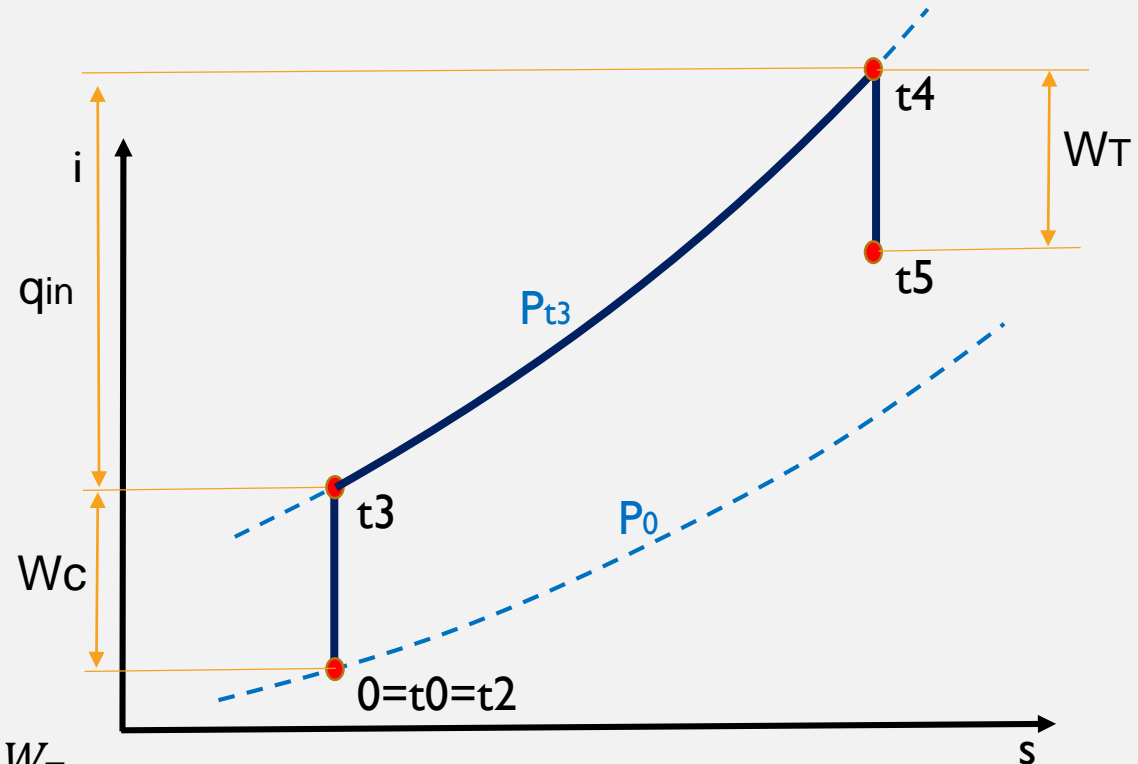
TURBOJET ENGINE AND BRAYTON CYCLE



$$P_{t5} = P_{t4} \left(\frac{T_{t5}}{T_{t4}} \right)^{k/(k-1)}, \quad T_{t5} = T_{t4} - \frac{W_T}{C_p}$$

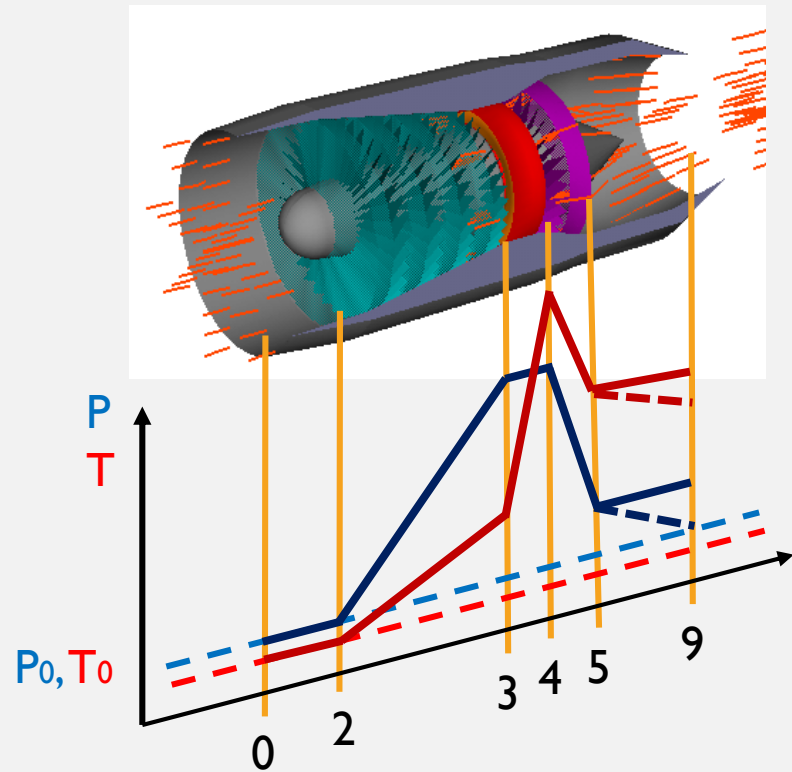
Turbine work: $W_T = C_p(T_{t4} - T_{t5}) = W_C$

Flight speed $V_H = 0$



TURBINE

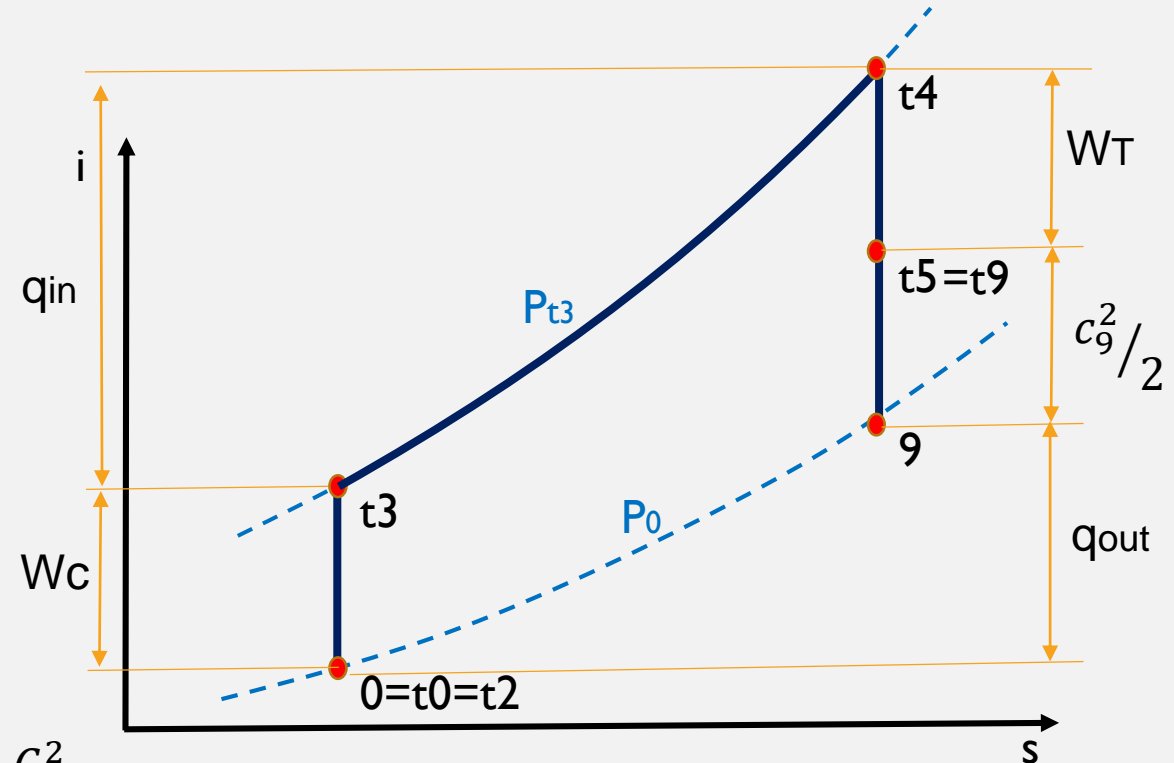
TURBOJET ENGINE AND BRAYTON CYCLE



$$P_{t9} = P_{t5}, \quad P_9 = P_0, \quad T_{t9} = T_{t5} = T_9 - \frac{C_9^2}{2 C_p}$$

Outlet gass velocity: $C_9 = \sqrt{2 C_p (T_{t9} - T_9)}$

Flight speed $V_H = 0$



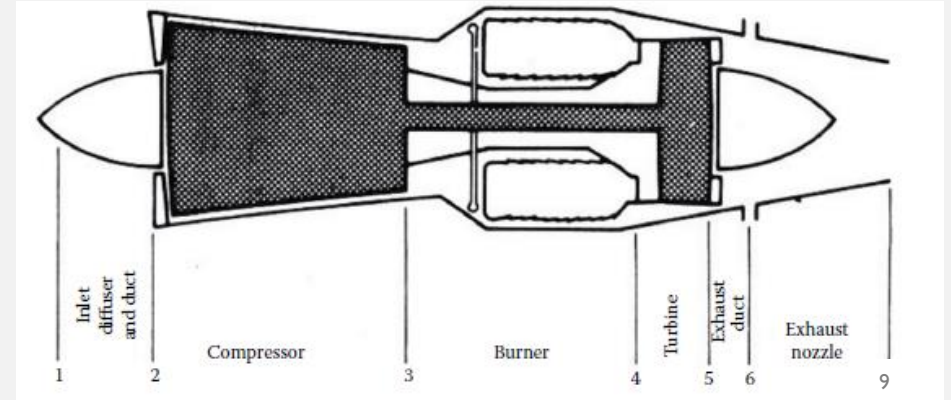
PROPELLING NOZZLE

IDEAL TURBOJET ENGINE

GIVEN: H – altitude, CPR compressor pressure ratio,
TIT – turbine inlet temperature (T_{t4})

Engine work in static conditions $V_0=0 \rightarrow P_{t0} = P_0, T_{t0} = T_0$

No **INLET** losses $\rightarrow P_{t2} = P_{t0}$ and $T_{t2} = T_{t0}$



COMPRESSOR (2 – 3)

Compressor work is isentropic

$$\frac{T_{t3}}{T_{t2}} = \left(\frac{P_{t3}}{P_{t2}} \right)^{(k-1)/k} = \pi_C^{(k-1)/k}$$

Compressor work:

$$W_C = C_p(T_{t3} - T_{t2})$$

Compressor power:

$$P_C = \dot{m}_C W_C = \dot{m}_0 C_p(T_{t3} - T_{t2})$$

COMBUSTOR (3 – 4)

Energy balance

$$\dot{m}_f FHV = \dot{m}_0 C_p(T_{t4} - T_{t3})$$

$$\dot{m}_f = \frac{\dot{m}_0 C_p(T_{t4} - T_{t3})}{FHV}$$

Fuel air ratio:

$$f = \frac{\dot{m}_f}{\dot{m}_0} = \frac{C_p(T_{t4} - T_{t3})}{FHV}$$

$$\pi_B = 1 \rightarrow P_{t4} = P_{t3}$$

TURBINE (4 - 5)

Compressor turbine power balance equation

$$P_C = \dot{m}_0 C_p(T_{t3} - T_{t2}) = P_T = \dot{m}_T C_p(T_{t4} - T_{t5})$$

Turbine mass flow: $\dot{m}_T = \dot{m}_0 + \dot{m}_f$

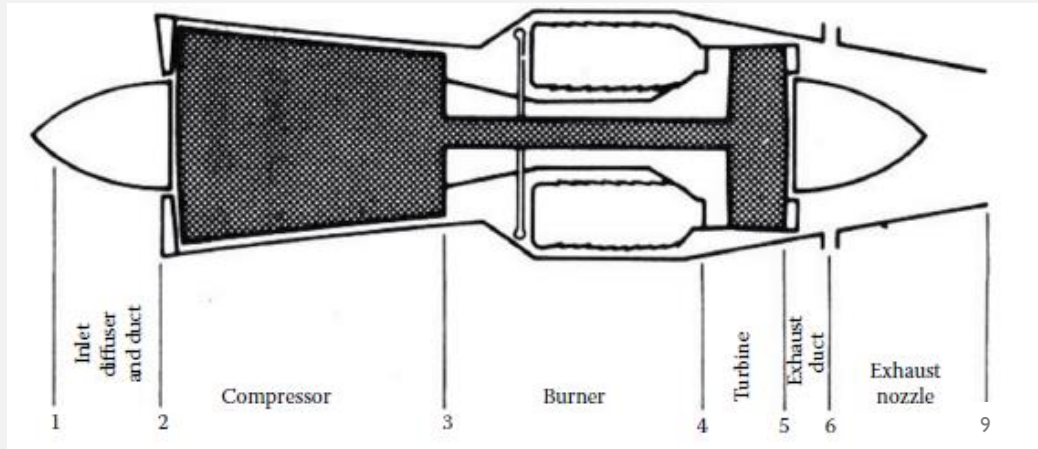
Turbine outlet temperature:

$$T_{t5} = T_{t4} - \frac{C_p(T_{t3} - T_{t2})}{C_p(1 + f)}$$

Turbine outlet pressure:

$$P_{t5} = P_{t4} \left(\frac{T_{t5}}{T_{t4}} \right)^{k_t/(k_t-1)}$$

IDEAL TURBOJET ENGINE – GASS FULL EXPANSION IN THE NOZZLE



NOZZLE (5-9)

No losses: $\pi_N = 1 \rightarrow P_{t9} = P_{t5}$ and $T_{t9} = T_{t5}$

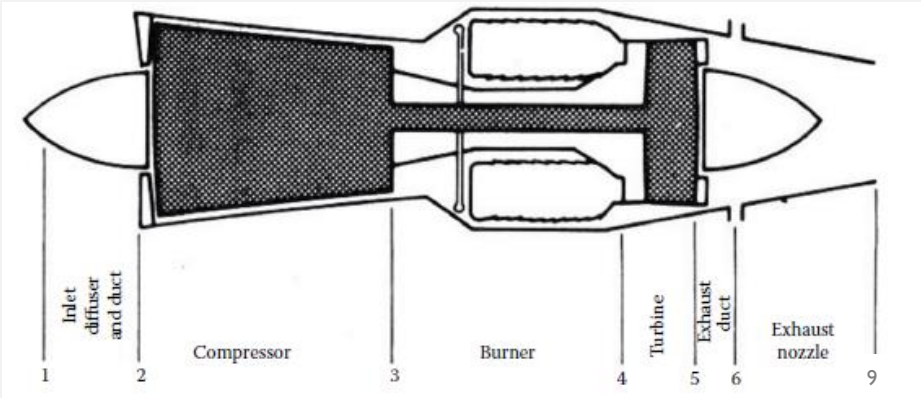
Full expansion: $P_9 = P_0$

$$c_9 = \sqrt{2Cp_t(T_{9t} - T_9)} \quad - \text{ for incompressible flow}$$

$$c_9 = a_9 M_9 = \sqrt{k_t R_t T_9} * \sqrt{\frac{2}{k_t - 1} \left(\frac{T_{t9}}{T_9} - 1 \right)} \quad - \text{ for compressible flow}$$

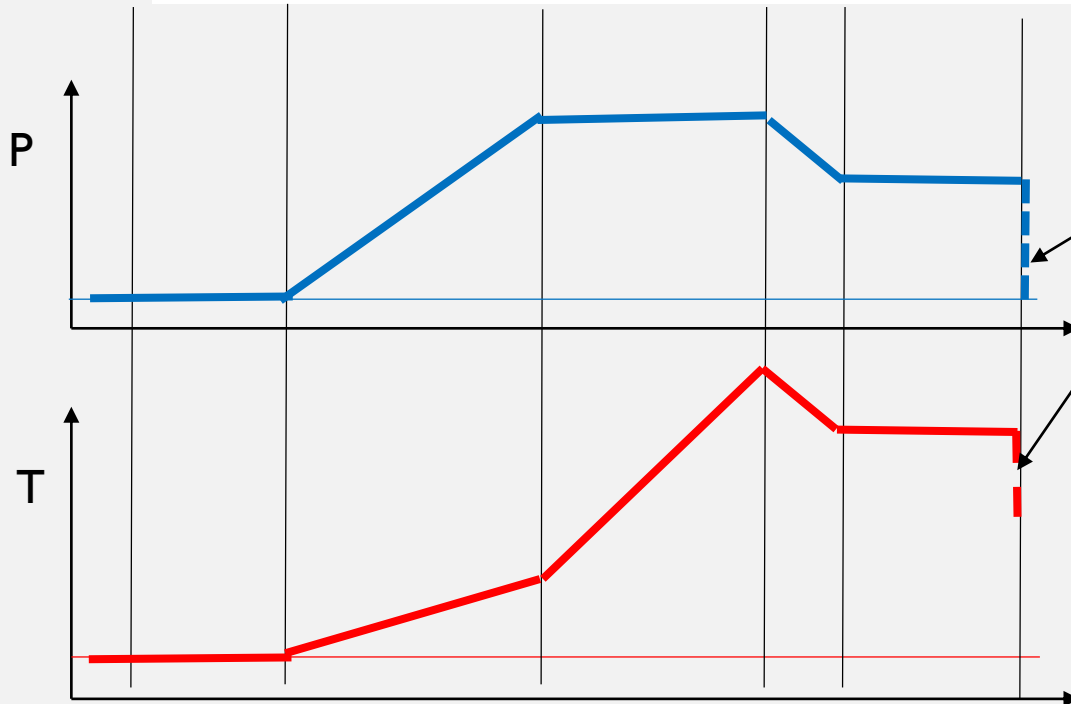
$$\frac{T_{t9}}{T_9} = \frac{P_{t9}^{(k_t-1)/k_t}}{P_9}$$

IDEAL TURBOJET ENGINE

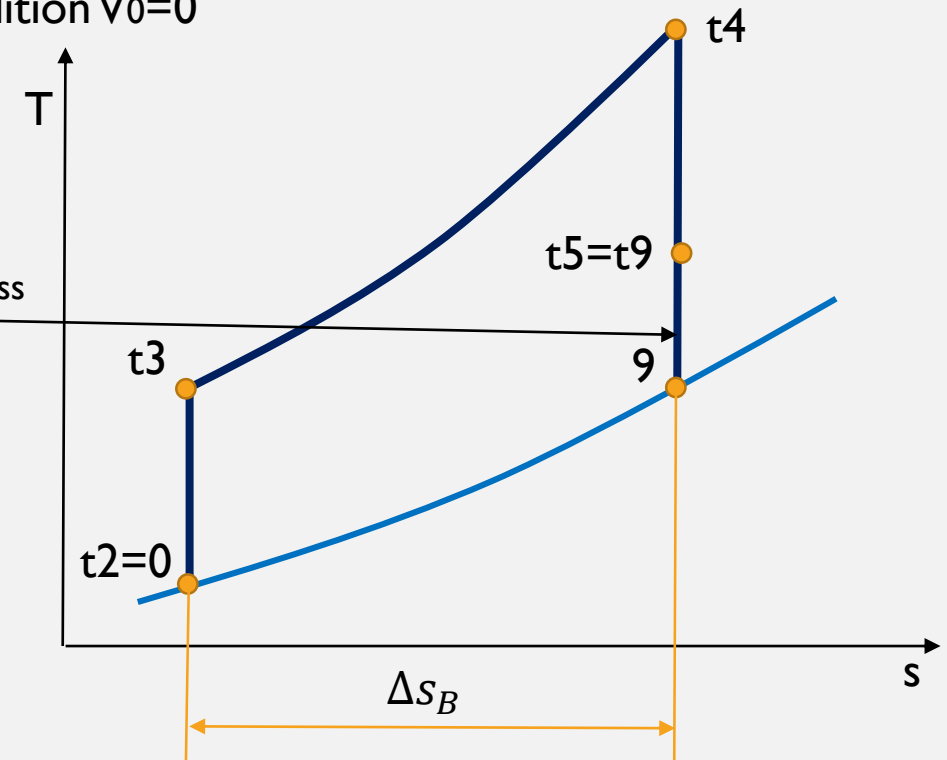


Processes in the inlet (diffuser) burner and nozzle are ideal – no pressure losses $\pi_D = \pi_B = \pi_N = 1$

Static condition $V_0=0$

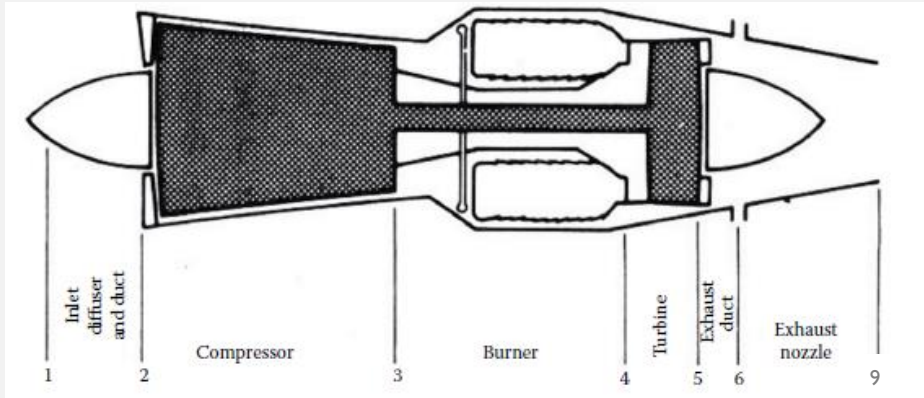


total to static process



Entropy increase in a burner $\Delta S_B = c_{p_B} \ln \frac{T_{t4}}{T_{t3}}$

IDEAL TURBOJET ENGINE

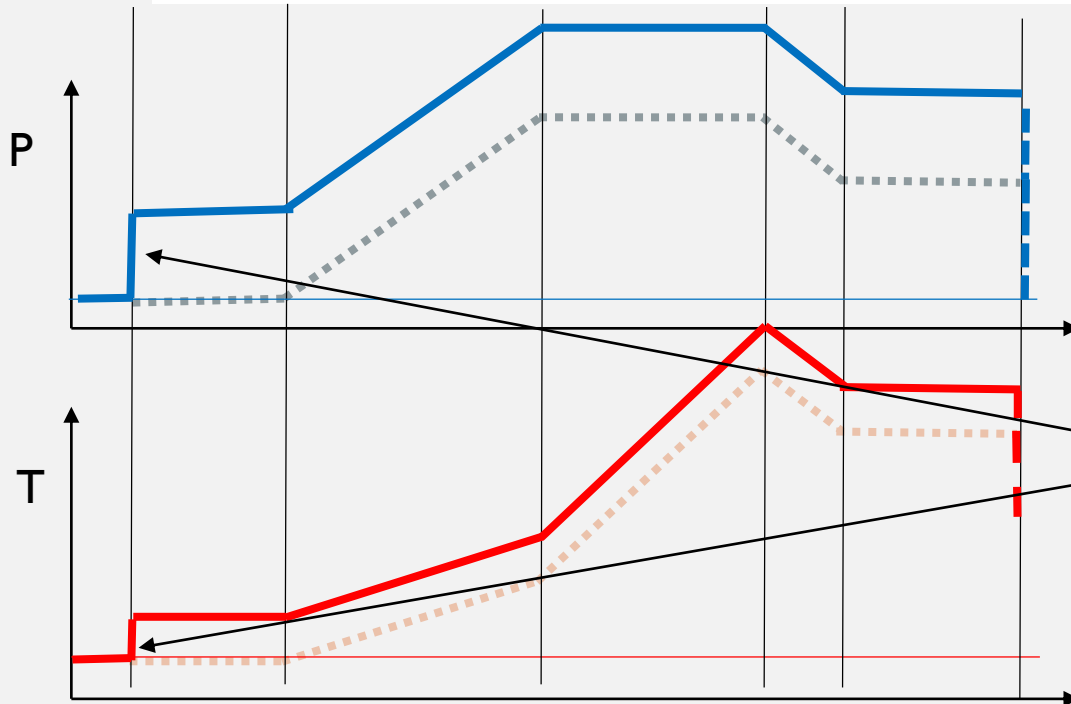


Flight condition $V_0 > 0$

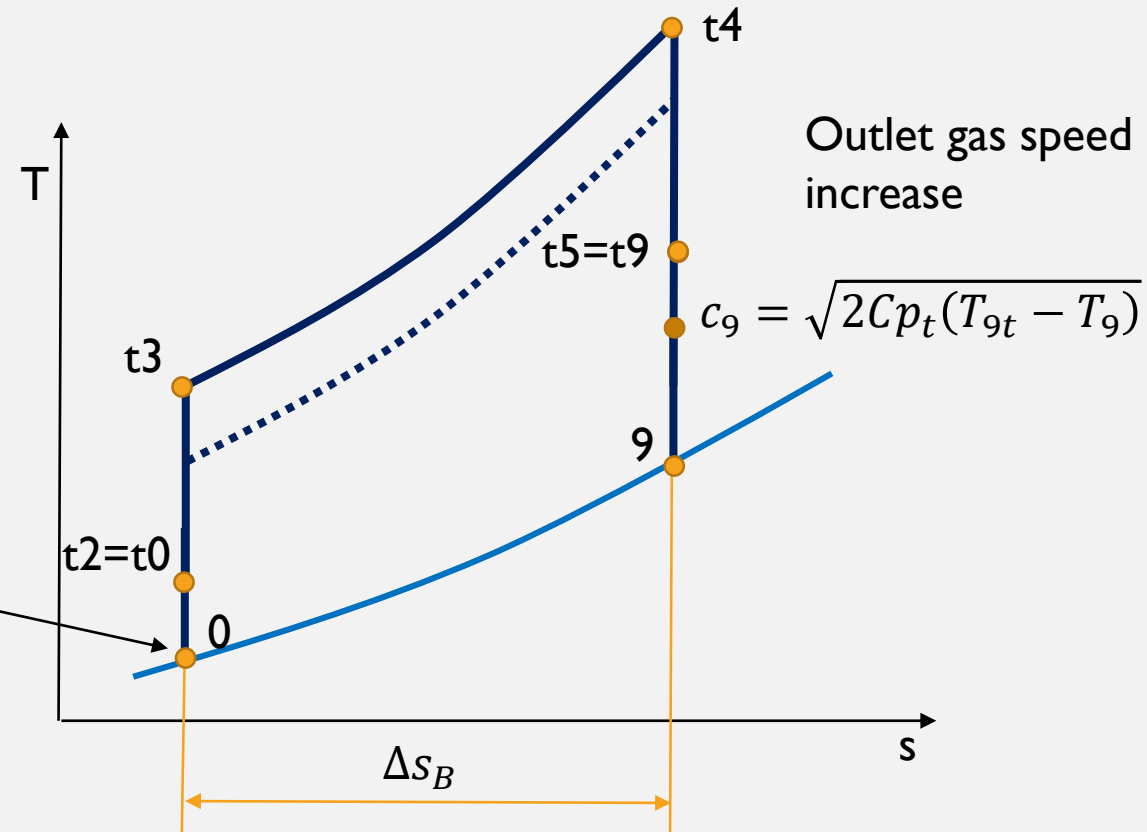
Ram pressure recovery for flight condition ($M_0 > 0$)

$$P_{t0} = P_0 \left(1 + \frac{k-1}{2} M_0^2 \right)^{k/(k-1)}$$

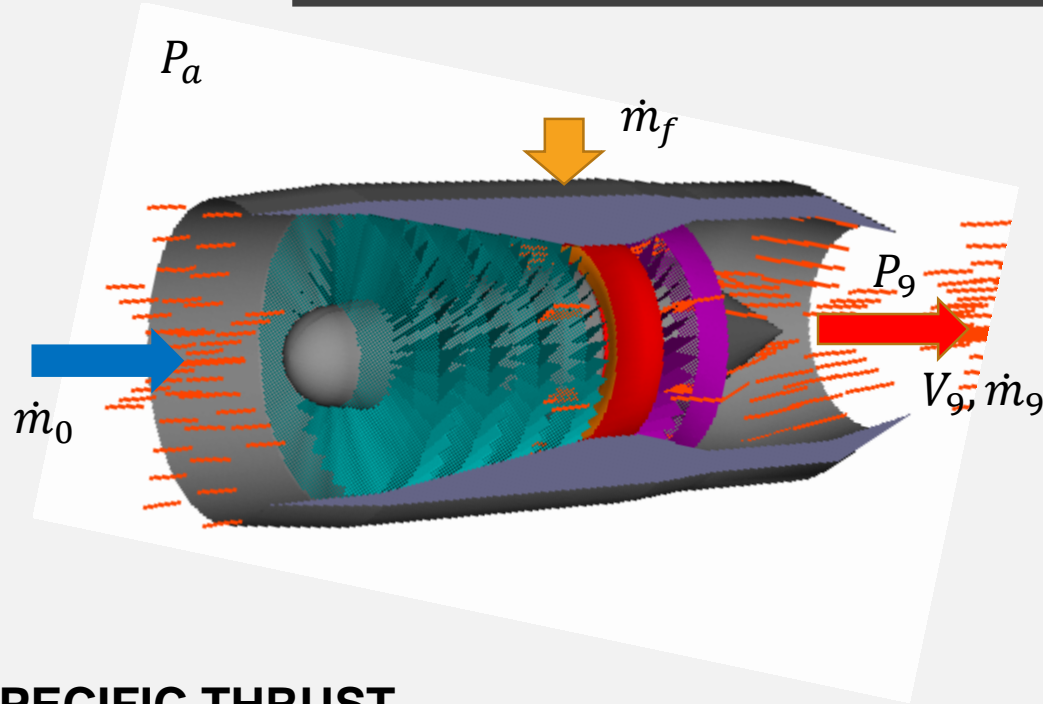
$$T_{t0} = T_0 \left(1 + \frac{k-1}{2} M_0^2 \right)$$



Ram effect



ENGINE THRUST AND SPECIFIC PARAMETERS



Flight speed is 0

THRUST / GROSS THRUST

$$T = \dot{m}_9 V_9 + A_9 (P_9 - P_a)$$

effective exhaust velocity

$$V_{eff} = V_9 + A_9 (P_9 - P_a) / \dot{m}_9$$

$$T = \dot{m}_9 V_{eff}$$

Exit pressure = ambient pressure

$$T = \dot{m}_9 V_9$$

Flight speed > 0

THRUST / NET THRUST

$$T = \dot{m}_9 V_9 + A_9 (P_9 - P_a) - \dot{m}_0 V_0 = \dot{m}_9 V_{eff} - \dot{m}_0 V_0$$

Net thrust = Gross thrust – Momentum drag

SPECIFIC THRUST

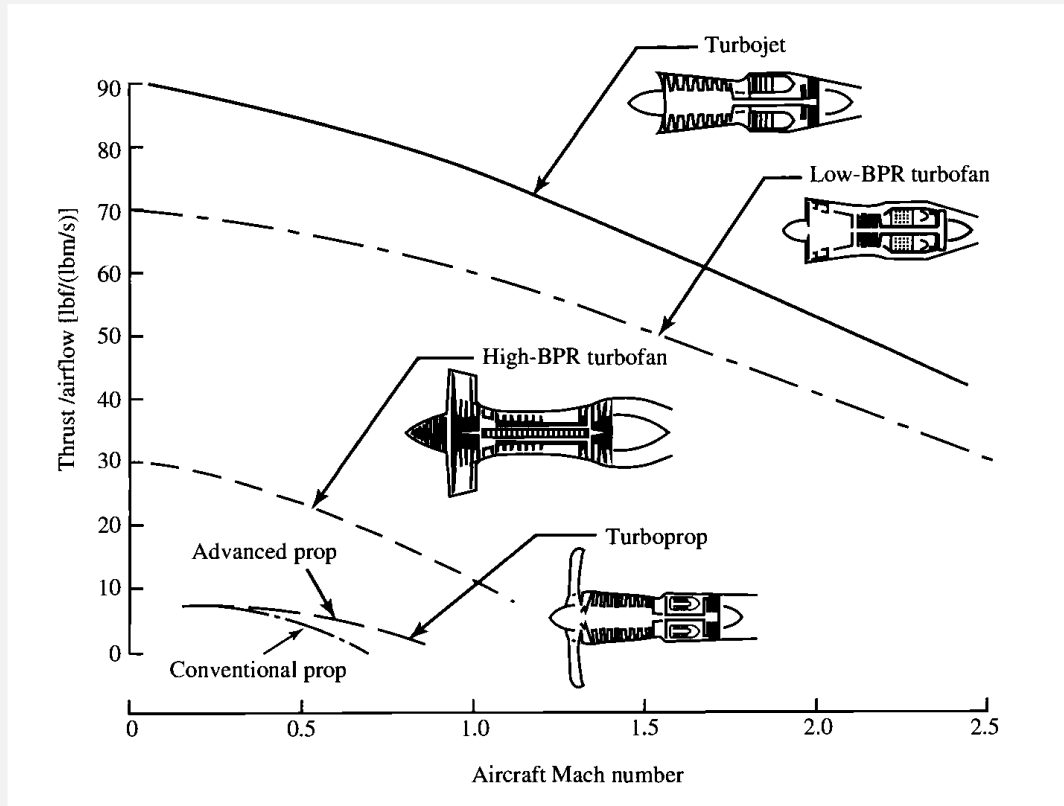
$$ST = T / \dot{m}_0$$

SPECIFIC FUEL CONSUMPTION

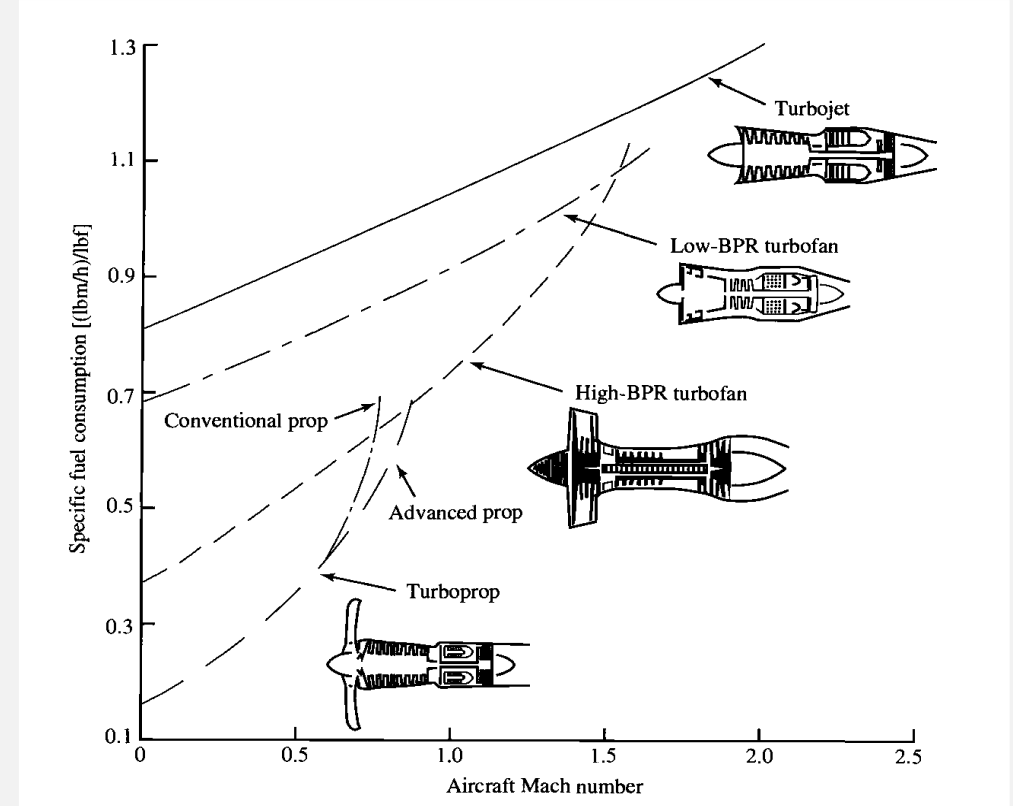
$$SFC = \dot{m}_f / T$$

ENGINES PERFORMANCE

Specific thrust vs. flight speed



Specific fuel consumption vs. flight speed



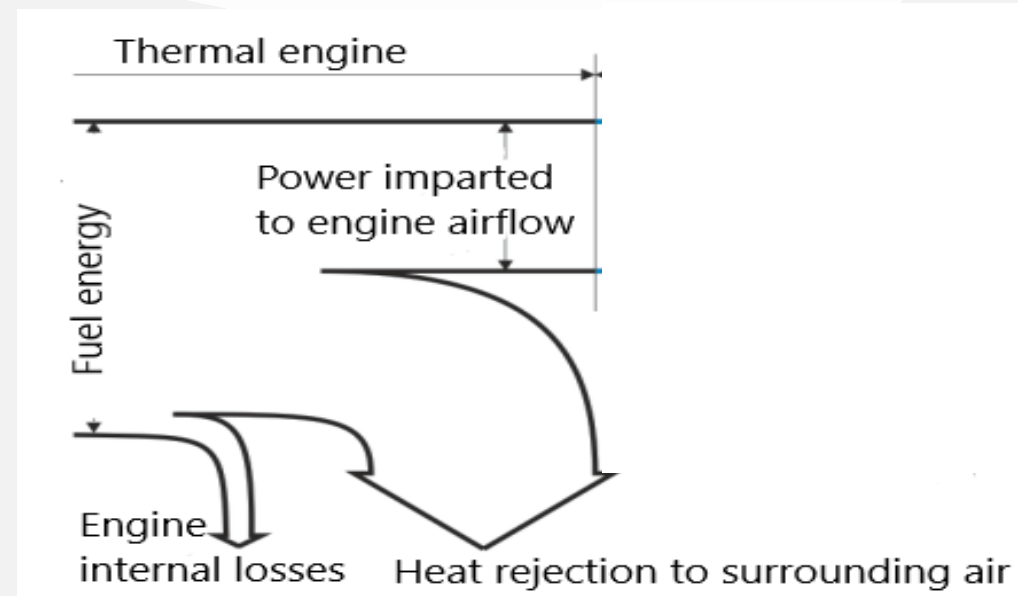
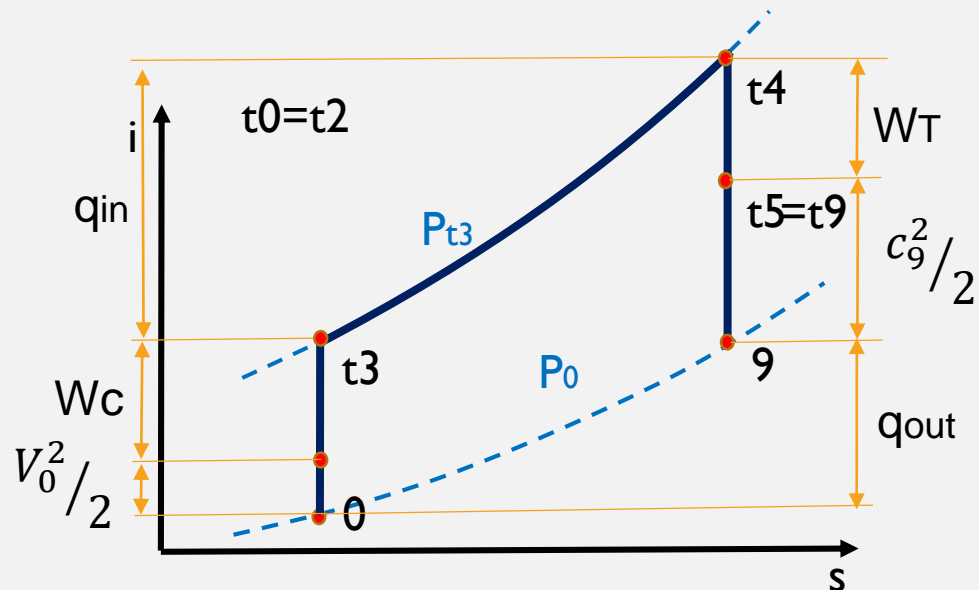
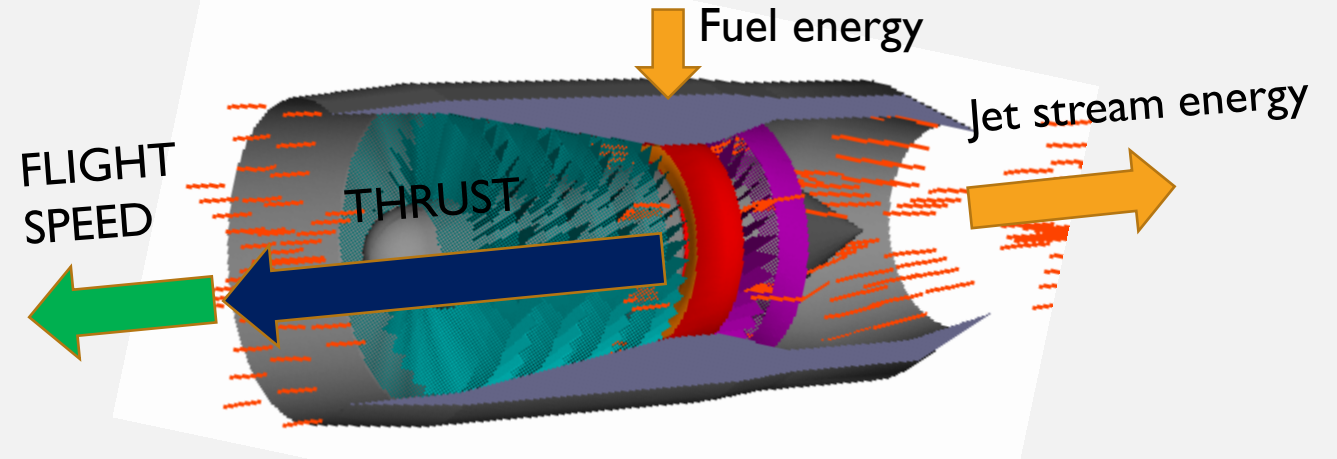
Propulsions of high specific thrust have got high specific fuel consumption
Low specific fuel consumption is characteristic for propulsions dedicated for low speed

ENGINE EFFICIENCIES

Thermal efficiency

$$\eta_{TH} = \frac{\text{Power imparted to engine airflow}}{\text{Rate of energy supplied in the fuel}}$$

$$\eta_{TH} = \frac{0,5 * (\dot{m}_9 V_{9e}^2 - \dot{m}_0 V_0^2)}{\dot{m}_f FHV}$$

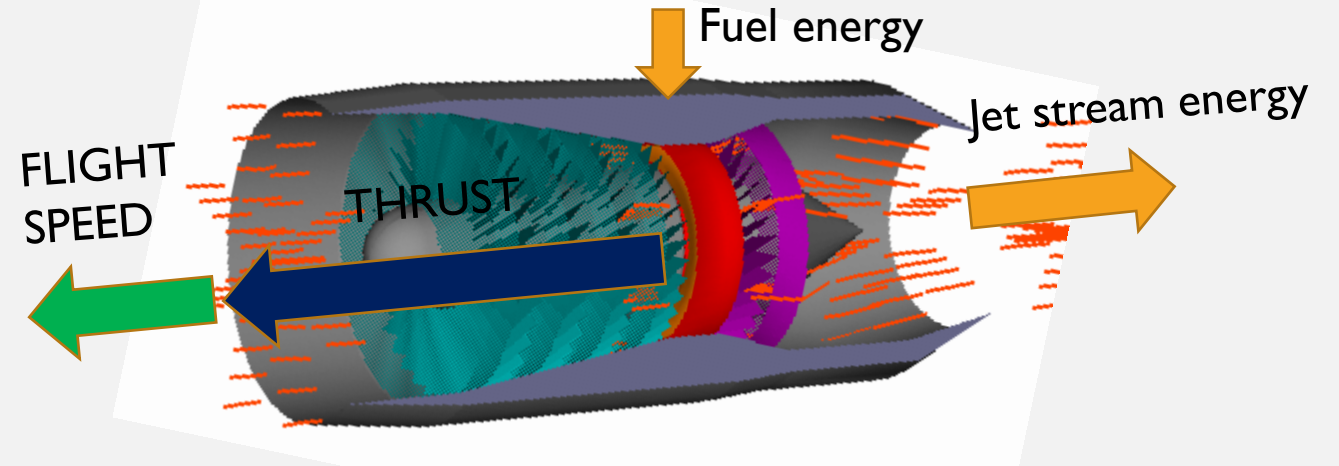


ENGINE EFFICIENCIES

Thermal efficiency

$$\eta_{TH} = \frac{\text{Power imparted to engine airflow}}{\text{Rate of energy supplied in the fuel}}$$

$$\eta_{TH} = \frac{0,5 * (\dot{m}_9 V_{9e}^2 - \dot{m}_0 V_0^2)}{\dot{m}_f FHV}$$



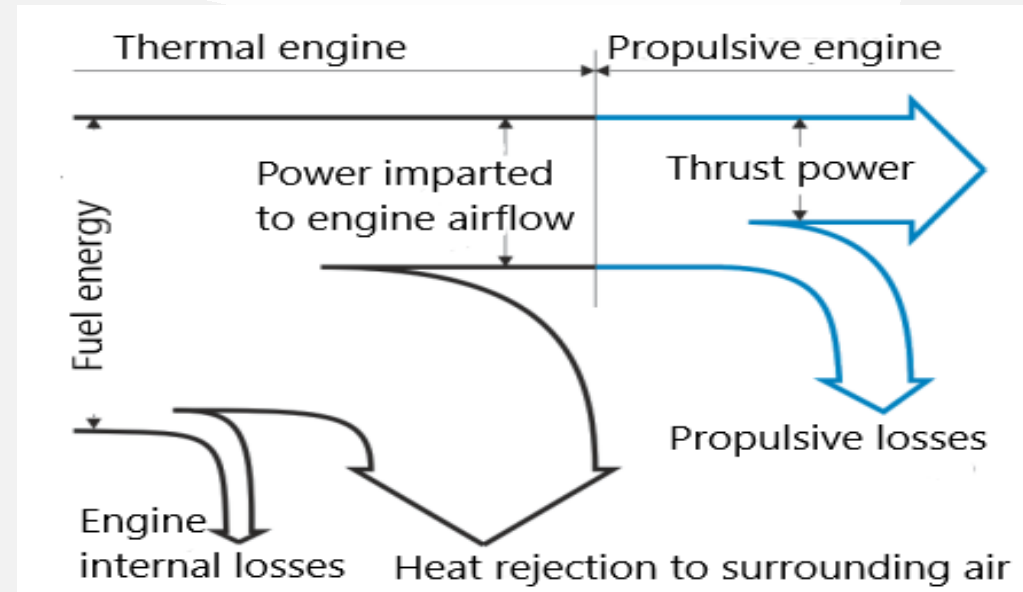
Propulsive efficiency

$$\eta_P = \frac{\text{Thrust power}}{\text{Power imparted to engine airflow}}$$

$$\eta_P = \frac{V_0 * T}{0,5 * (\dot{m}_9 V_{9e}^2 - \dot{m}_0 V_0^2)}$$

Overall efficiency

$$\eta_O = \eta_{TH} * \eta_P = \frac{V_0 * T}{\dot{m}_f FHV}$$



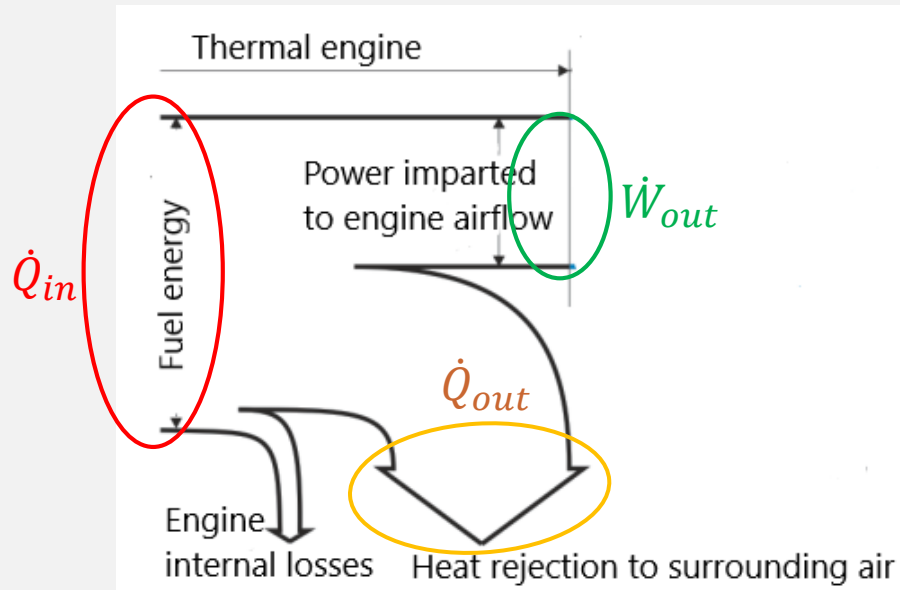
THERMAL EFFICIENCY

$$\eta_{TH} = \frac{\dot{W}_{out}}{\dot{Q}_{in}} \quad \longrightarrow \quad \eta_{TH} = \frac{\dot{Q}_{in} - \dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} \quad \text{where, } \dot{Q}_{out} = \dot{m}_9 c_p (T_9 - T_0)$$

- \dot{W}_{out} = net power out of engine (engine work)
- \dot{Q}_{in} = rate of thermal energy released/supplied in the fuel)

Higher thermal efficiency causes lower specific fuel consumption

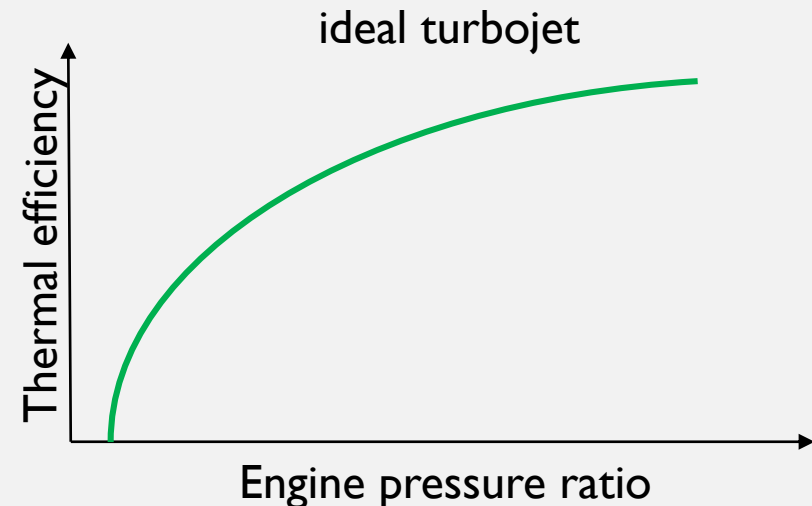
Thermal efficiency is higher when exhaust gas temperature is closer to the ambient temperature



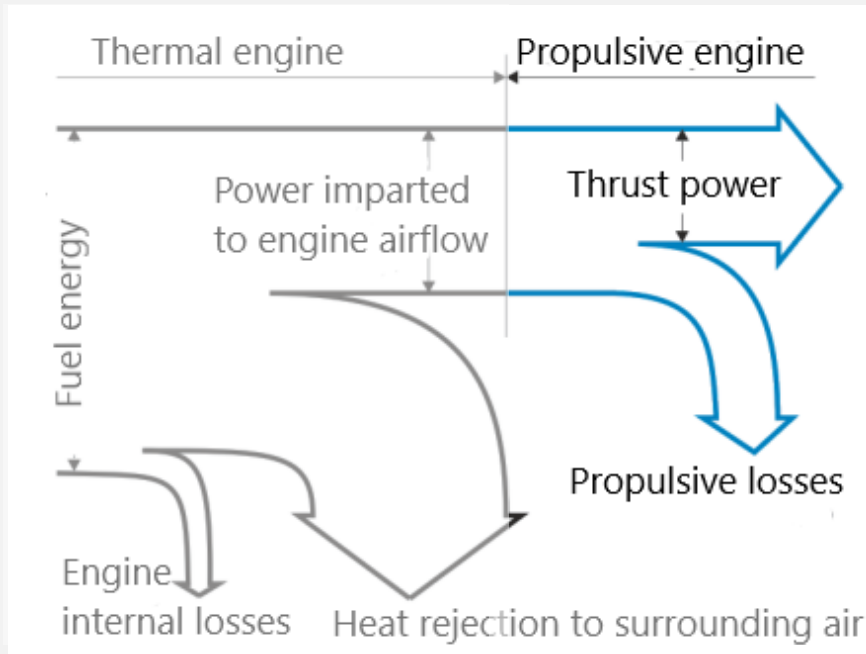
For ideal turbojet engine:

$$\eta_{TH} = 1 - \frac{1}{\pi^{(k-1)/k}},$$

where π – engine compression pressure ratio, k – isentropic exponent



PROPULSIVE EFFICIENCY



$$\eta_P = \frac{V_0 * T}{0,5 * (\dot{m}_9 V_{9e}^2 - \dot{m}_0 V_0^2)}$$

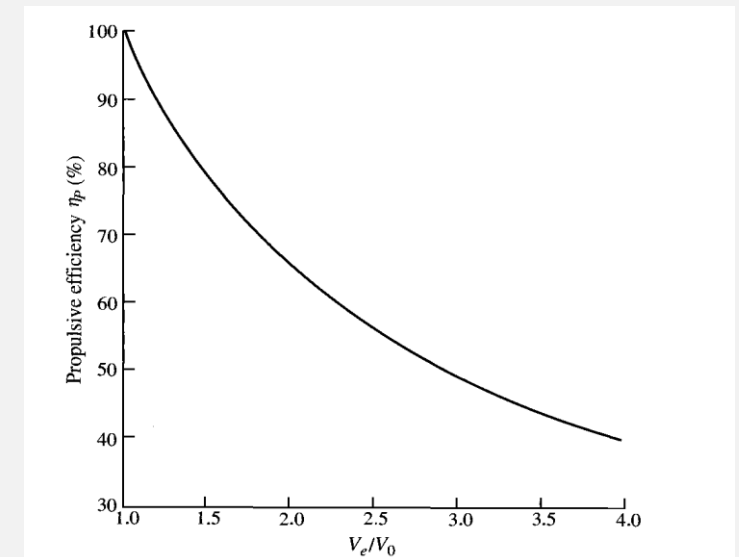
$$T = \dot{m}_9 V_{9e} - \dot{m}_0 V_0$$



$$\text{and} \quad \dot{m}_9 = \dot{m}_0$$

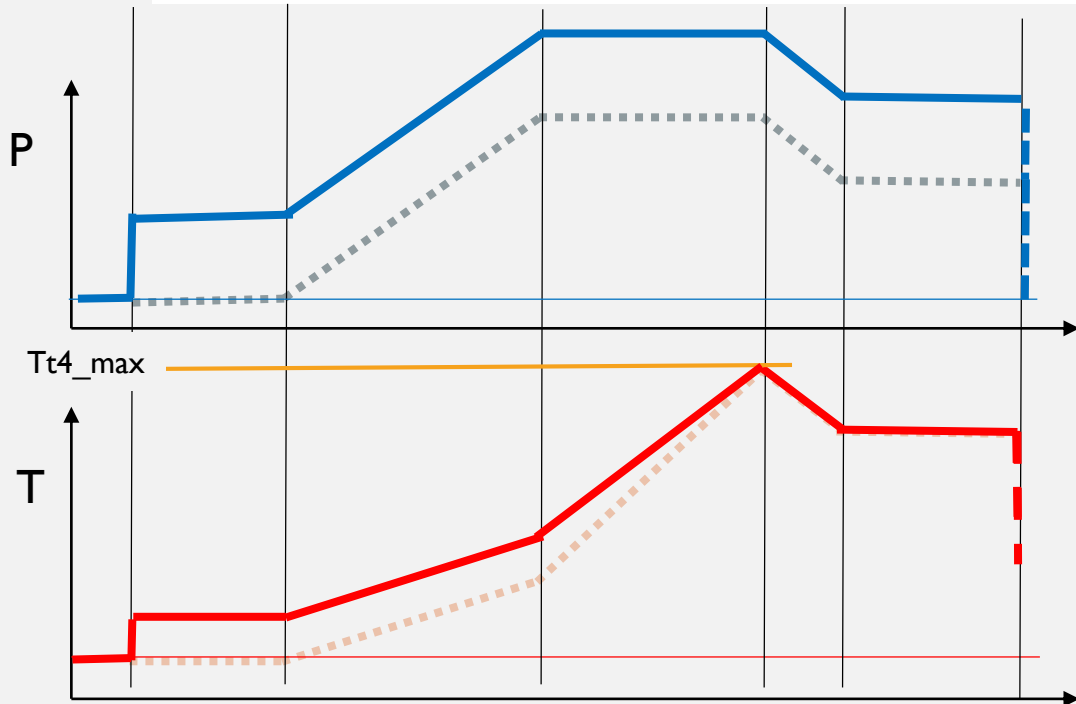
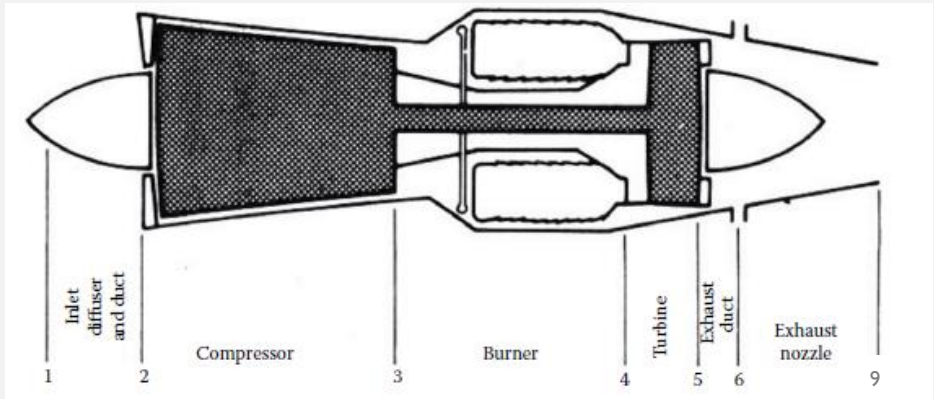
$$\eta_P = \frac{V_0(V_{9e} - V_0)}{0,5 * (V_{9e}^2 - V_0^2)} = \frac{2V_0(V_{9e} - V_0)}{(V_{9e} - V_0)(V_{9e} + V_0)} = \frac{2}{1 + V_{9e}/V_0}$$

$$\eta_P \Rightarrow 1, \text{ gdy } V_{9e} \Rightarrow V_0$$

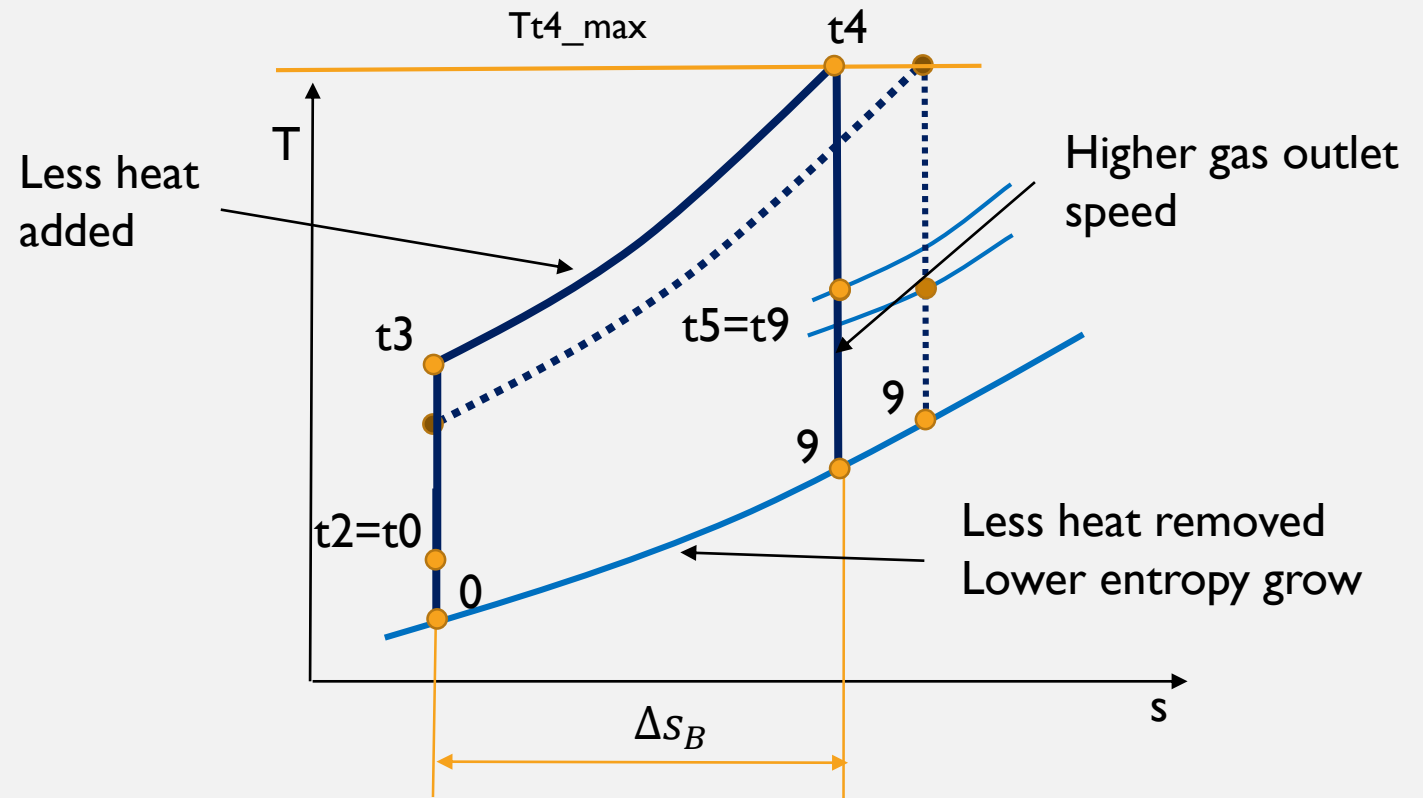


Propulsiv efficiency define the thrust produced for specific flight speed from kinetic energy added to engine airflow

IDEAL TURBOJET ENGINE – FLIGHT SPEED INFLUENCE

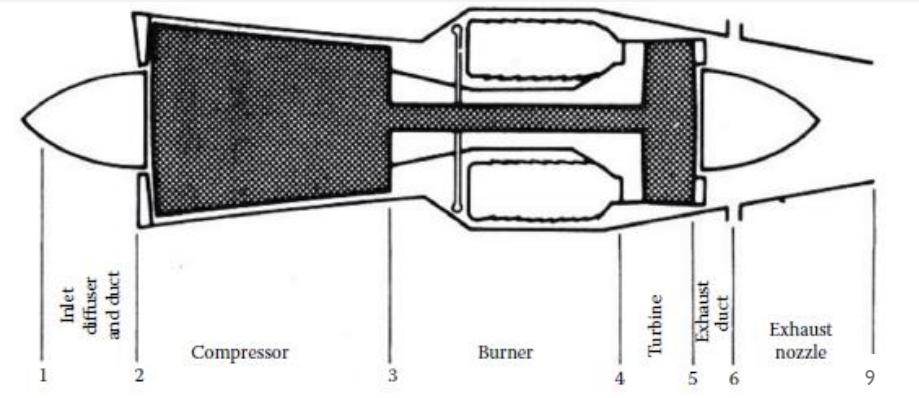


Flight condition $V_0 > 0$ and T_{t4} limit T_{t4_max}

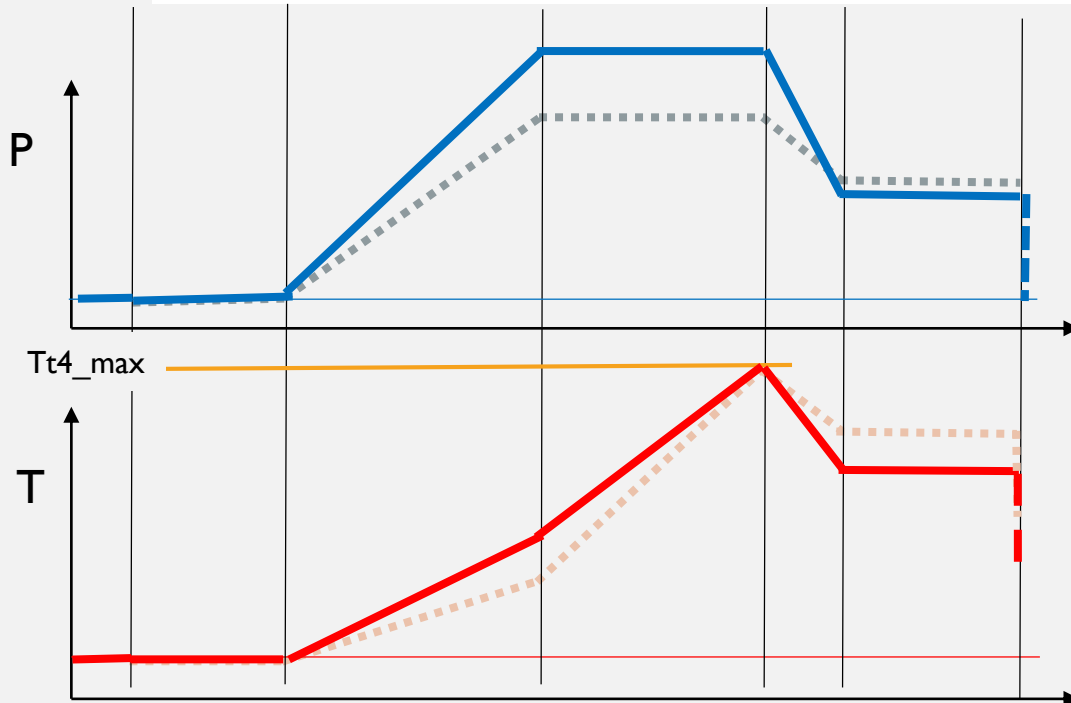


Engine thermal efficiency grow (less heat removed)
Lower fuel consumption (less heat added)

IDEAL TURBOJET ENGINE – COMPRESSOR PRESSURE RATIO INFLUENCE

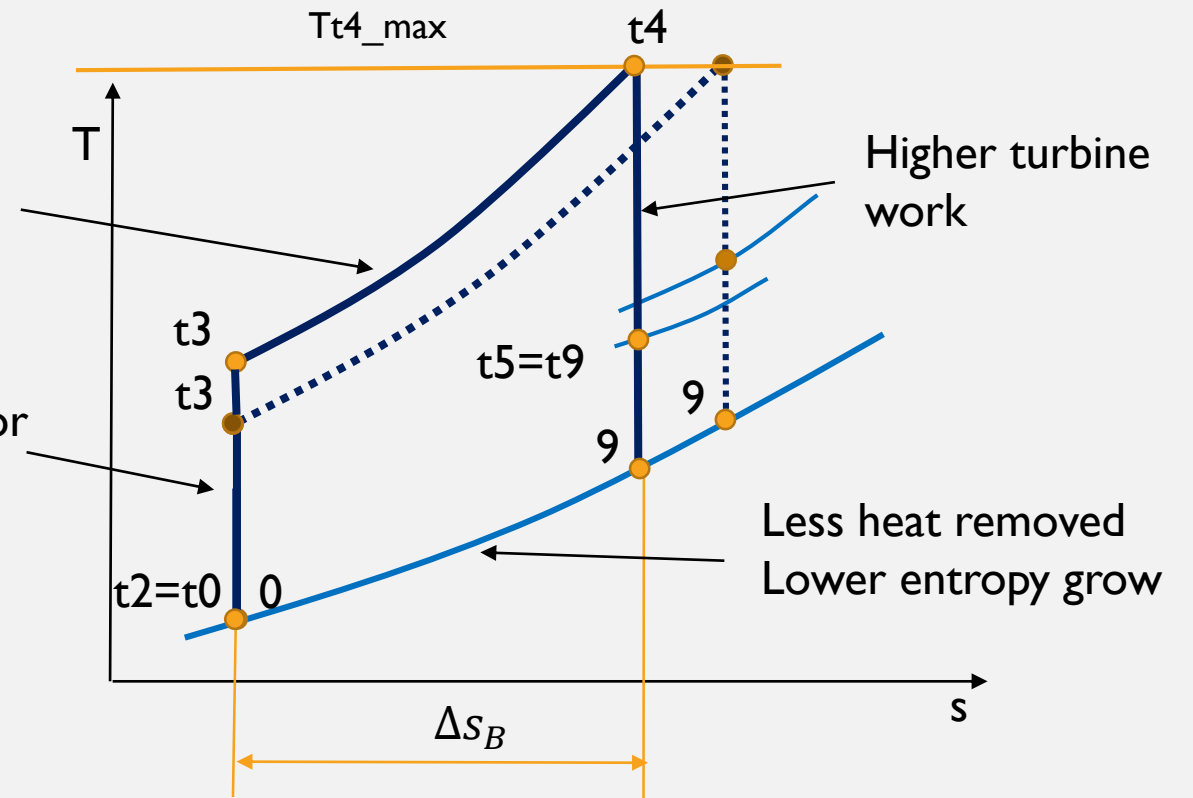


CPR (compressor pressure ratio) growing and T_{t4} is limited T_{t4_max}



Less heat added

Higher compressor work

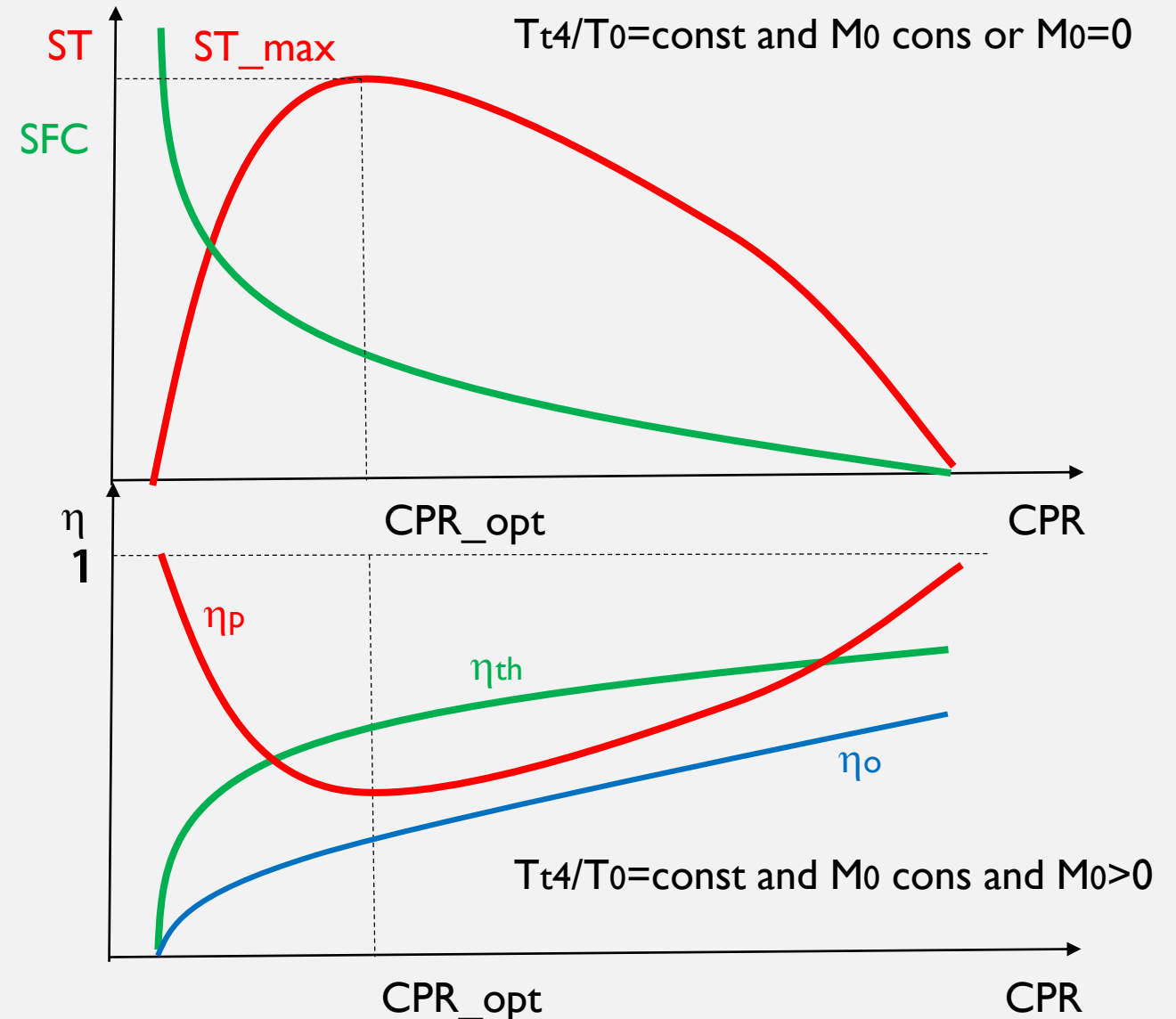


**Engine thermal efficiency grow (less heat removed)
Lower fuel consumption (less heat added) and lower SFC**

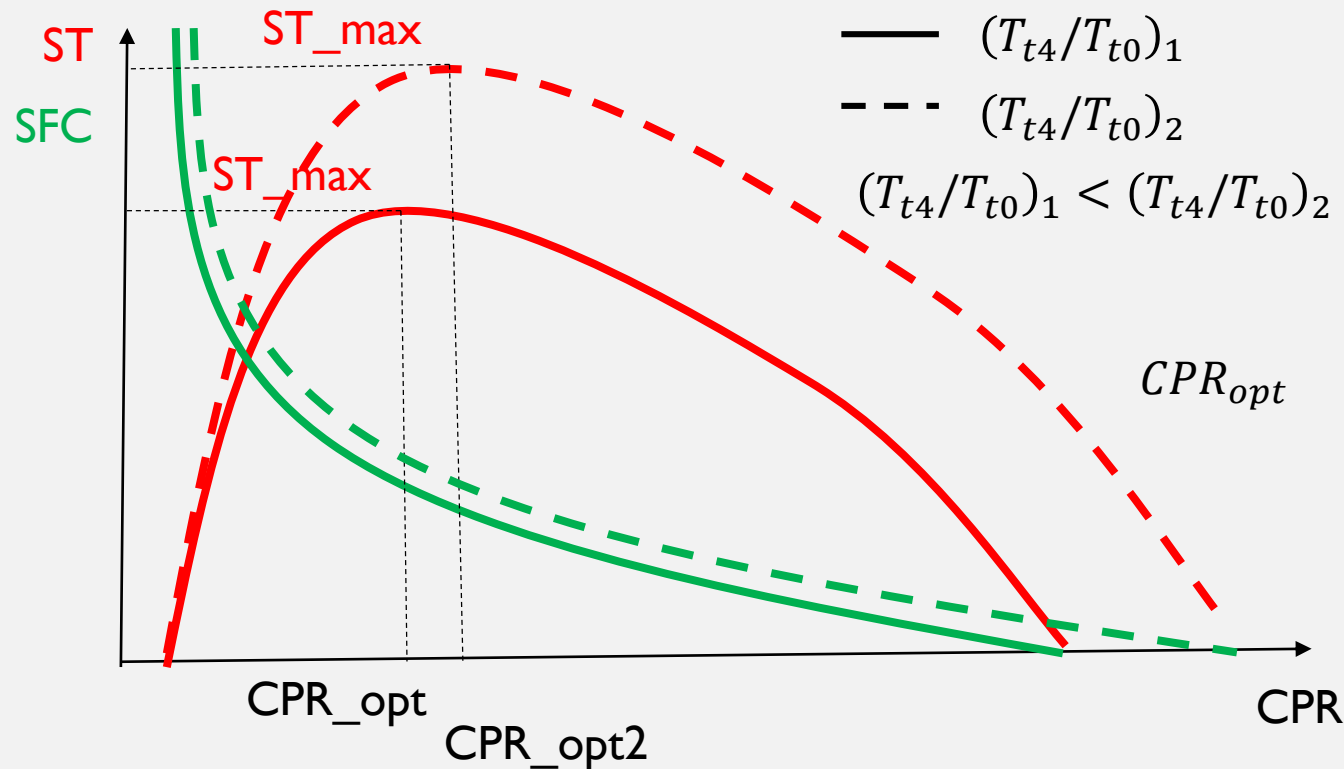
IDEAL TURBOJET CYCLE OPTIMISATION

SUMMARY:

- Specific thrust (ST) grows with compressor pressure ratio increasing, achieves maximum for optimal CPR than is goes down
- Specific fuel consumption decreases with CPR growing
- Propulsive efficiency as a function of CPR represents opposite relation to ST, it is minimal for optimal CPR and achieves 1 for ST=0.
- Thermal and overall efficiencies grow with CPR
- The presented relationship between efficiencies and CPR are right for $M_0 > 0$, for $M_0 = 0$, propulsive and overall efficiency are 0



IDEAL TURBOJET CYCLE OPTIMISATION FOR DIFFERENT ENGINE TEMPERATURE RATIO



SUMMARY:

- Specific thrust (ST) is higher for higher engine temperature ratio T_{t4}/T_{t0} and achieve ST_max for higher CPR (higher CPR_opt)
- Specific fuel consumption decreases with CPR growing, but for high T_{t4}/T_{t0} is higher
- Range of available CPR increases for higher T_{t4}/T_{t0}

T_{t4}/T_{t0}	CPR_opt	CPR_max
4	11,3	128
5	16,7	279,5
6	23	529

For ideal cycle:

$$\text{CPR}_{opt} = T_{t4}/T_{t0}^{\frac{k}{2(k-1)}}$$

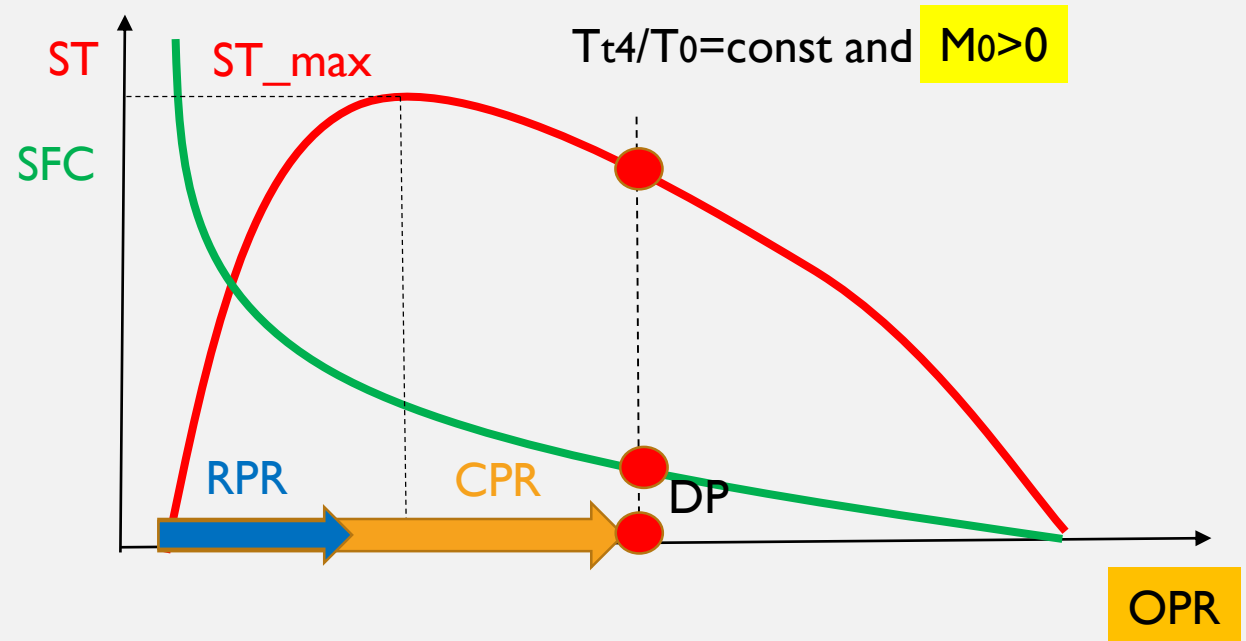
$$\text{CPR}_{max} = \text{CPR}_{opt}^2$$

IDEAL TURBOJET CYCLE OPTIMISATION FOR VARIOUS V_0

Flight speed M_0	RPR (ideal)
1	1,89
1,5	3,67
2	7,82
2,5	17,09
3	36,73

SUMMARY:

- An increase in flight speed causes an increase in the ram pressure ratio (RPR).
- When optimizing the engine cycle for increasing flight speeds, RPR should be included in OPR, and CPR should be reduced accordingly.



$$OPR = RPR * CPR$$

$$RPR = \left(1 + \frac{k-1}{2} M_0^2\right)^{\frac{k}{k-1}}$$

THANKS FOR YOUR ATENTION

Questions and Comments ?

1.

2.

3.