



**RZESZOW UNIVERSITY
OF TECHNOLOGY**



**THE FACULTY OF
MECHANICAL ENGINEERING
AND AERONAUTICS**
RZESZOW UNIVERSITY OF TECHNOLOGY

GAS TURBINES

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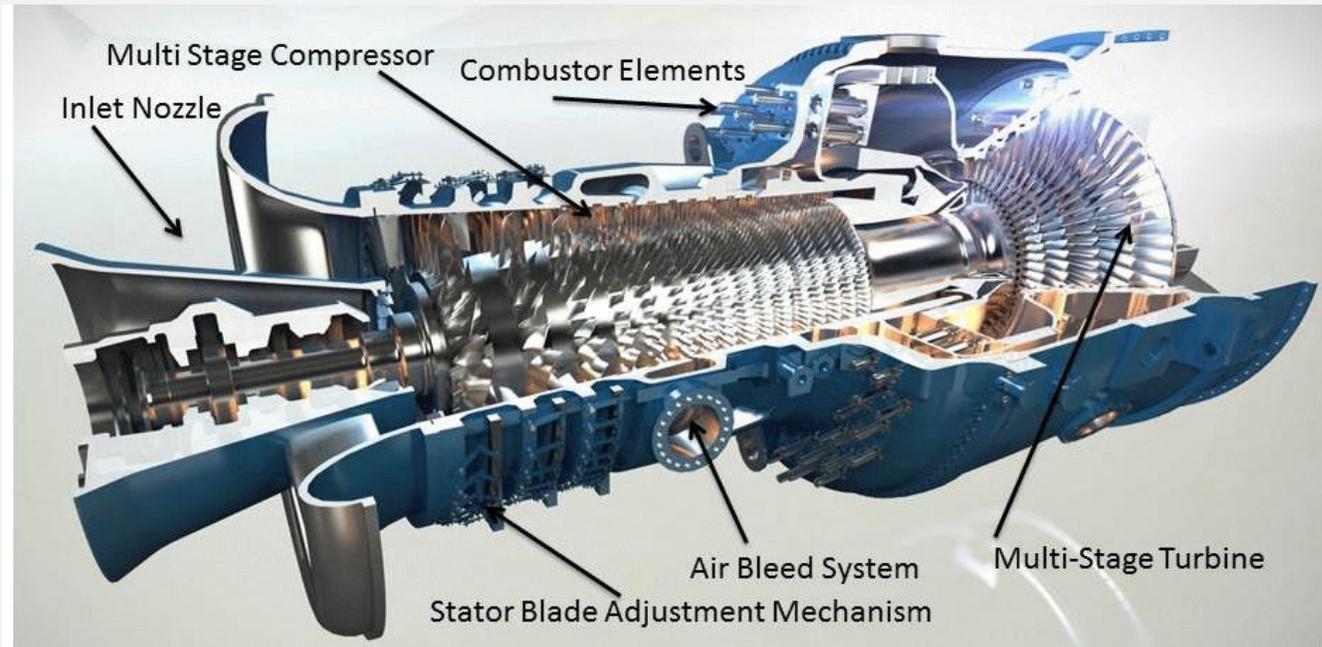
<https://robert-jakubowski.v.prz.edu.pl/en/>

GAS TURBINES

Gas turbines (GT) are power plants within which the chemical energy of the fuel is converted either into mechanical energy in terms of shaft power or into kinetic energy in jet engines.

They produce a great amount of energy for its size and weight

SHAFT POWER GAS TURBINE



Alstom heavy duty power generation gas turbine GT13E2
 Output power 202.7 MW (M.T. Schobeiri, Gas Turbine Design,
 Components and System Design Interation, DOI 10.1007/978-3-319-85378-5_1)

Output Shaft Power:

$$P_{OSH} = EF_R (P_T - P_C)$$

Thermal efficiency:

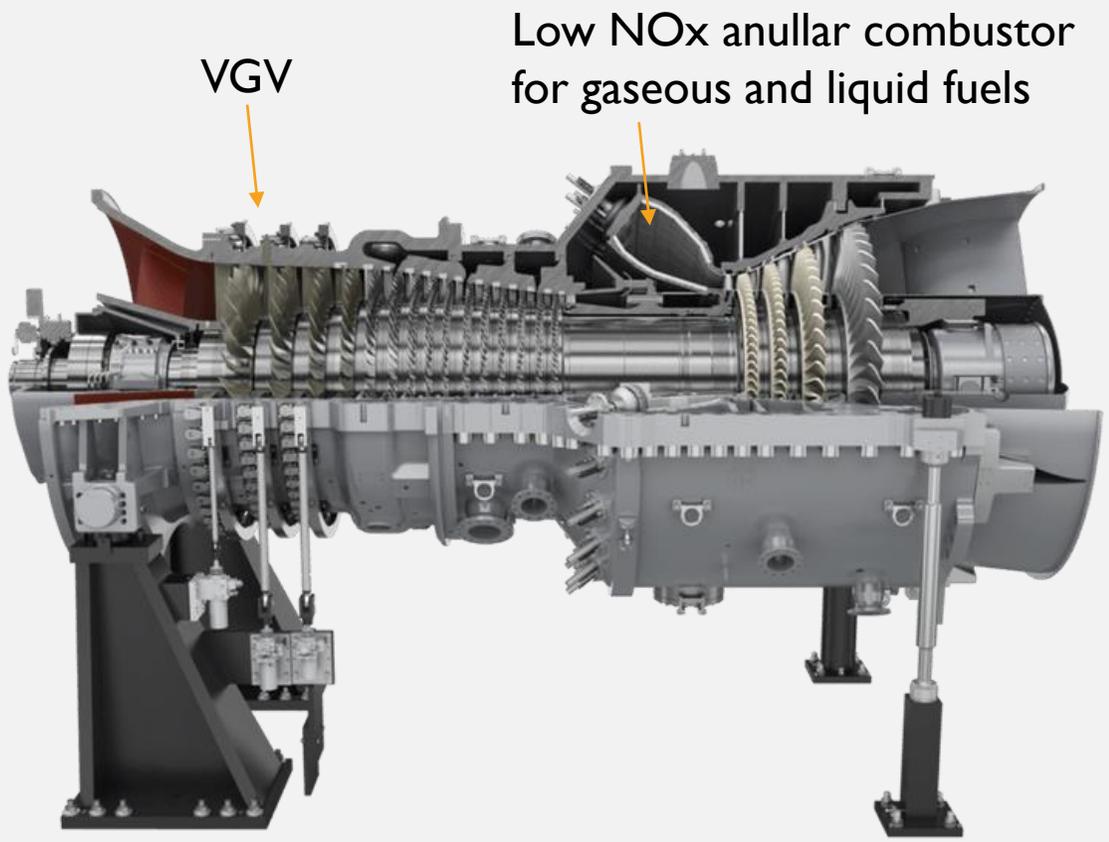
$$\eta_{th} = P_{OSH} / Q_{ad}$$

Heat added:

$$Q_{add} = \dot{m}_f FHV$$

\dot{m}_f - Fuel mass flow
 FHV - Fuel heat value

SIEMENS POWER GAS TURBINE SGT5-4000F



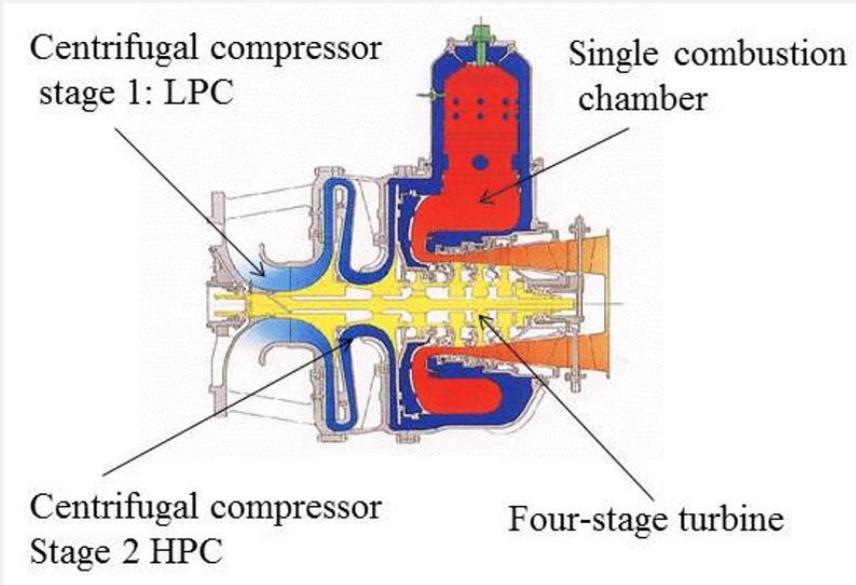
Output shaft power 329 MW
Efficiency 41%
Compressor: 15 stages
CPR= 20.1,
Turbine: 4 stages
Turbine speed 3000 rpm
Mass flow rate = 724 kg/s
Exhaust gas temperature 599° C

VGV – variable guide vane

Source:

<https://www.siemens-energy.com/global/en/home/products-services/product/sgt5-4000f.html#Simple-cycle-power-generation-tab-6>

KAWASAKI GAS TURBINE GBI5D



Gas turbine with single stage side type combustor

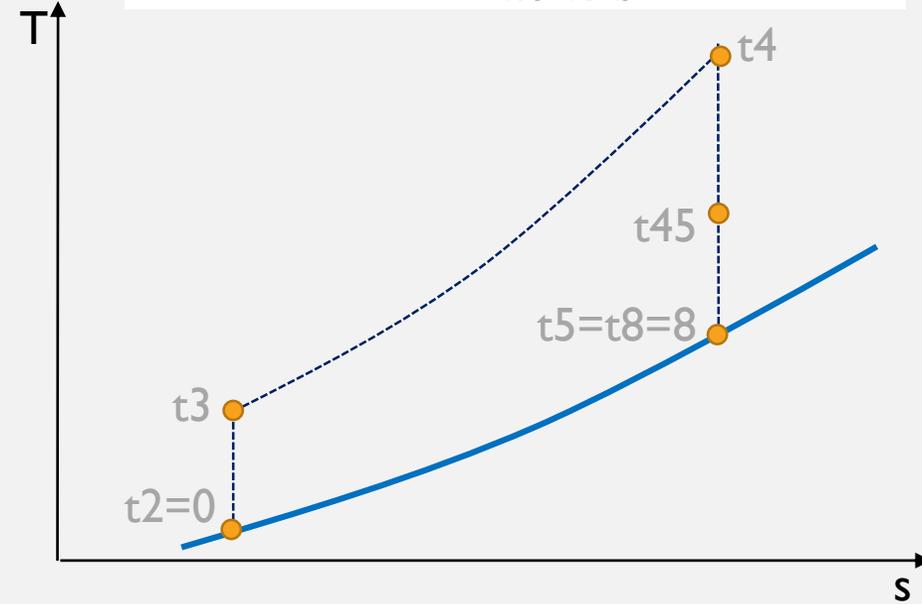
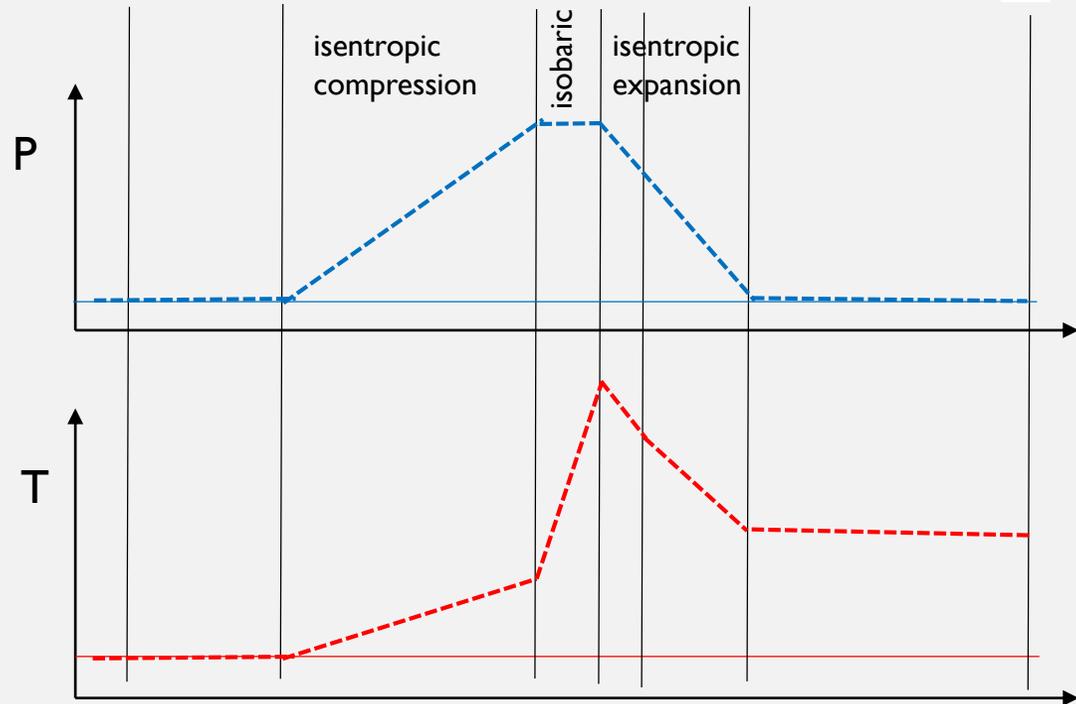
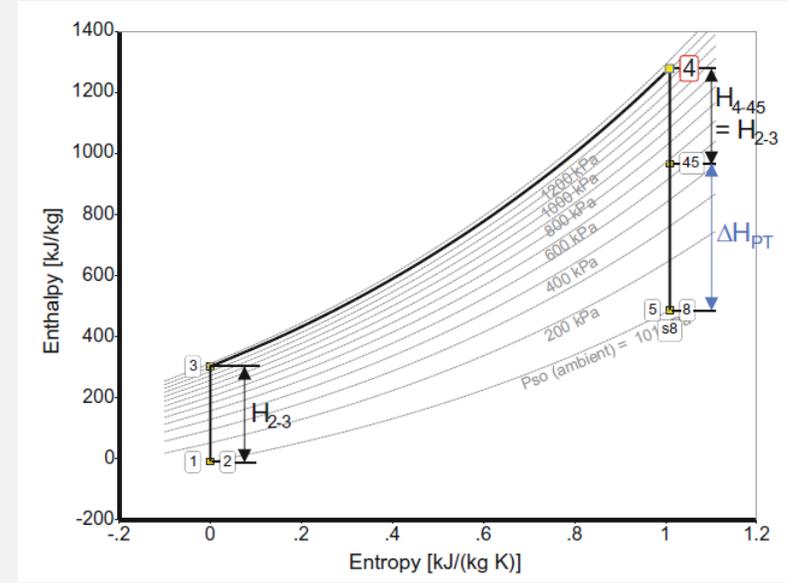
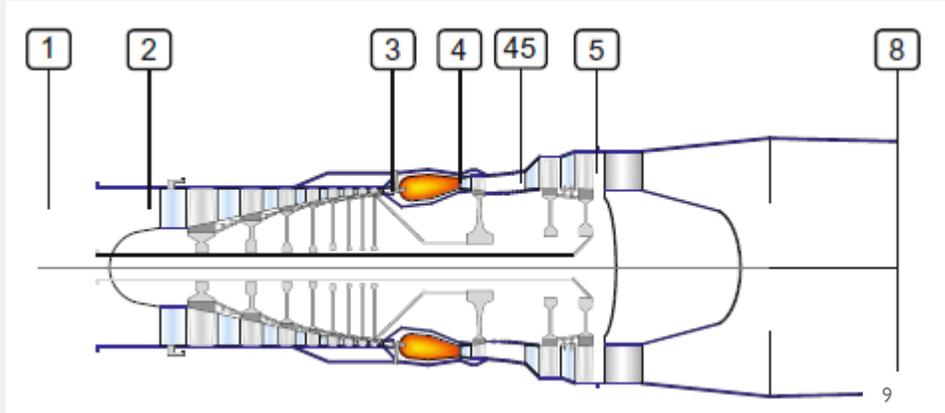
Specifications	Output
Electrical Output	1,450 kW
Heat Rate	15,280 kW
Exhaust Gas Temperature	534 °C
Exhaust Gas Mass Flow	28.5 x10 ³ kg/hr

Sources:
<https://kga.com.my/product/gbp15d/>

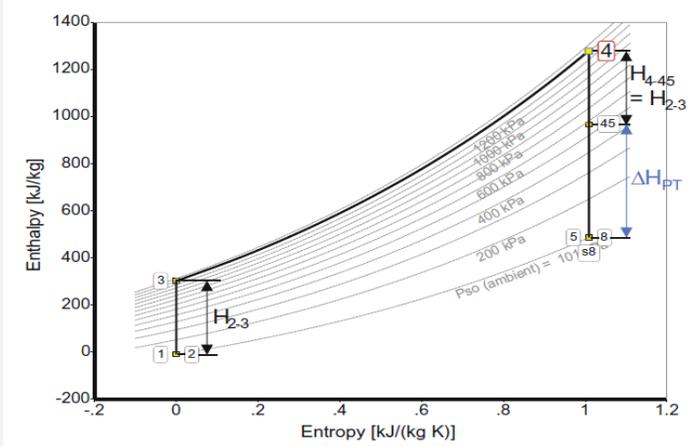
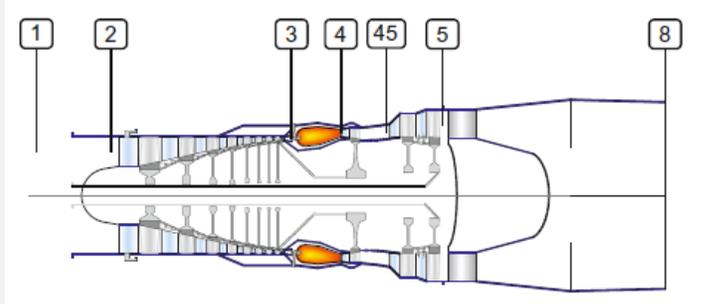
ASSUMPTIONS FOR IDEAL GAS TURBINE

- The gas flowing through the engine is treated as a calorically perfect.
- The processes in the engine are considered reversible (flow, mechanical, and thermal losses in the engine are neglected).
- The change in the mass of the working fluid flowing through the engine is neglected.
- The parameters of the flowing stream are represented at control points by averaged values.

IDEAL GAS-TURBINE



IDEAL GAS TURBINE ANALYSIS (I)



No **INLET** losses $\rightarrow P_{t2} = P_0$ and $T_{t2} = T_0$

COMPRESSOR (2 – 3)

Compressor work is isentropic

$$\frac{T_{t3}}{T_{t2}} = \left(\frac{P_{t3}}{P_{t2}}\right)^{(k-1)/k} = CPR^{(k-1)/k}$$

Compressor work:

$$W_C = H_{23} = Cp(T_{t3} - T_{t2})$$

Compressor power:

$$P_C = \dot{m}_C W_C = \dot{m}_0 Cp(T_{t3} - T_{t2})$$

COMBUSTOR (3 – 4)

Energy balance

$$\dot{m}_f FHV = \dot{m}_0 Cp_B (T_{t4} - T_{t3})$$

$$\dot{m}_f = \frac{\dot{m}_0 Cp_B (T_{t4} - T_{t3})}{FHV}$$

Fuel air ratio:

$$f = \frac{\dot{m}_f}{\dot{m}_0} = \frac{Cp_B (T_{t4} - T_{t3})}{FHV}$$

$$\pi_B = 1 \rightarrow P_{t4} = P_{t3}$$

TURBINE (4 - 45)

Compressor turbine power balance equation

$$P_C = \dot{m}_0 Cp (T_{t3} - T_{t2}) = P_T = \dot{m}_T Cp_T (T_{t45} - T_{t5})$$

Turbine mass flow: $\dot{m}_T = \dot{m}_0 + \dot{m}_f$

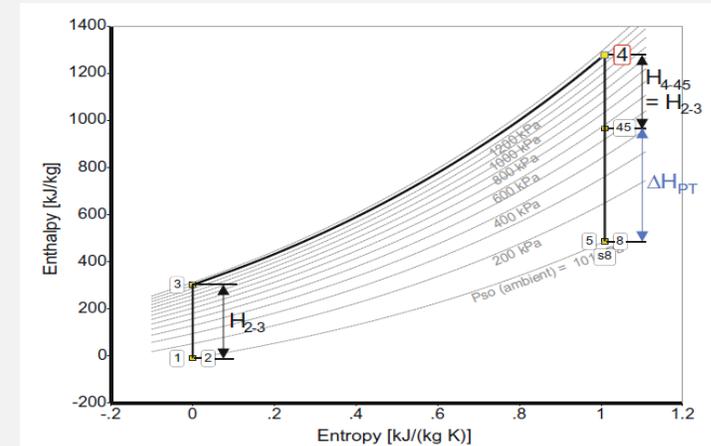
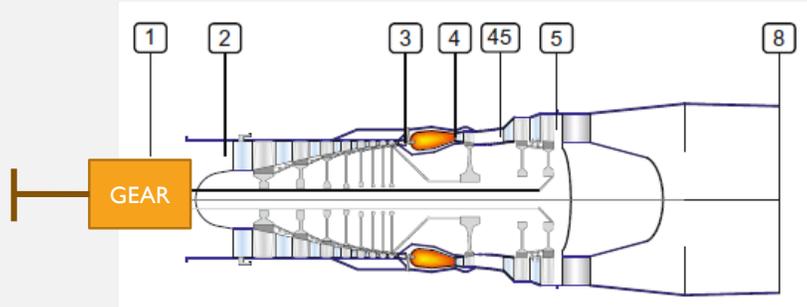
Turbine outlet temperature:

$$T_{t45} = T_{t4} - \frac{Cp (T_{t3} - T_{t2})}{Cp_T (1 + f)}$$

Turbine outlet pressure:

$$P_{t45} = P_{t4} \left(\frac{T_{t45}}{T_{t4}}\right)^{k_t/(k_t-1)}$$

IDEAL GAS TURBINE ANALYSIS (2)



Turbine expansion to ambient pressure $\rightarrow P_{t5} = P_0$

TURBINE (45 - 5)

Power Turbine outlet temperature:

$$T_{t5} = T_{t45} \left(\frac{P_{t5}}{P_{t45}} \right)^{(k_t - 1)/k_t}$$

Power Turbine work:

$$W_{PT} = H_{PT} = C p_T (T_{t45} - T_{t5})$$

Power Turbine power:

$$P_{PT} = \dot{m}_T W_{PT} = \dot{m}_0 C p_T (1 + f) (T_{t45} - T_{t5})$$

Output Shaft Power:

$$P_{OSH} = E F_G P_{PT} = E F_R \dot{m}_{45} C p_T (T_{t45} - T_{t5})$$

$E F_G$ - Gear efficiency

Thermal efficiency:

$$\eta_{th} = P_{OSH} / Q_{add}$$

Heat added:

$$Q_{add} = \dot{m}_f FHV$$

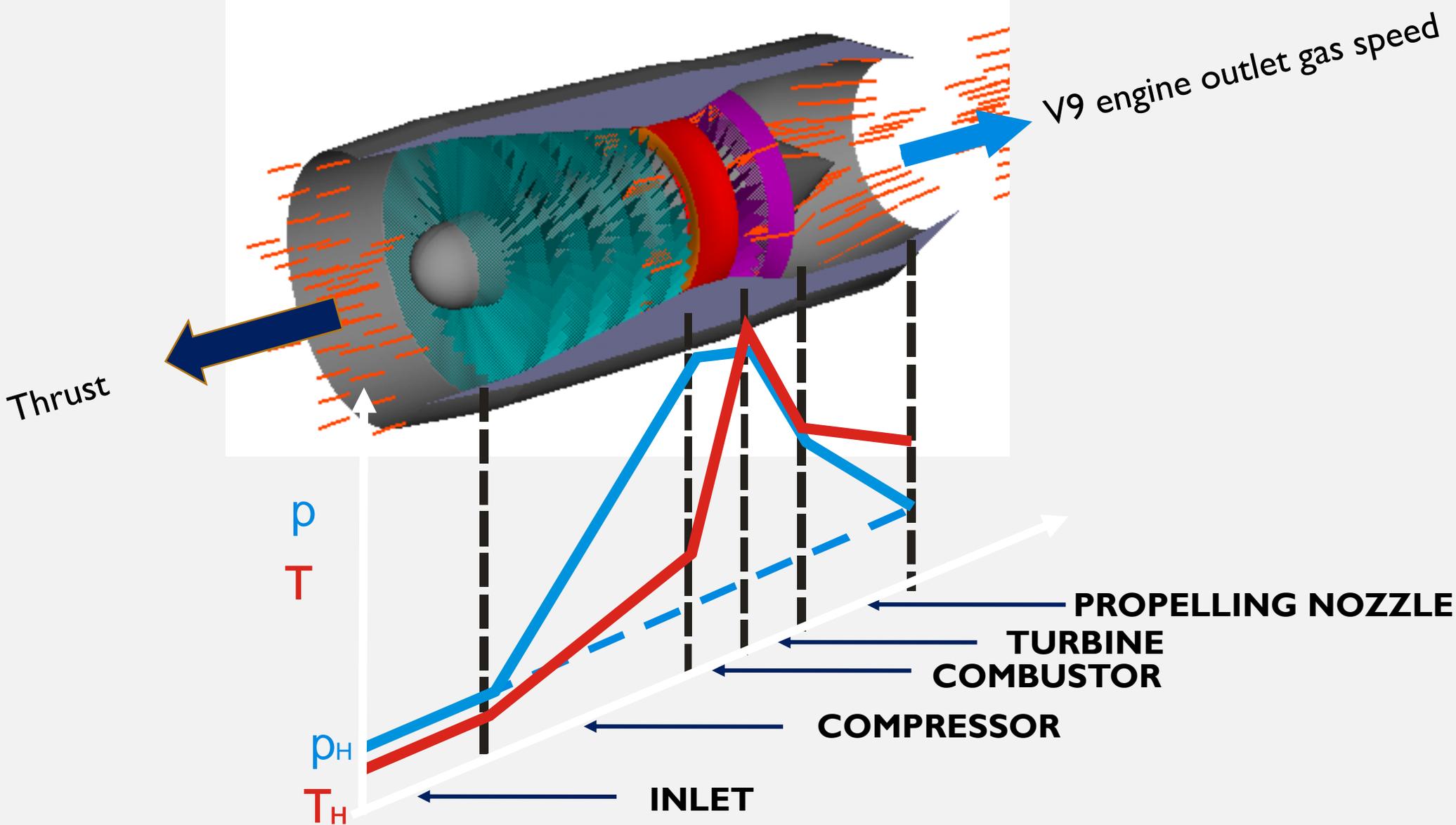
Specific fuel consumption:

$$SFC = \dot{m}_f / P_{OSH} = f / W_{OSH}$$

Work of output shaft:

$$W_{OSH} = P_{OSH} / \dot{m}_0$$

TURBOJET ENGINE

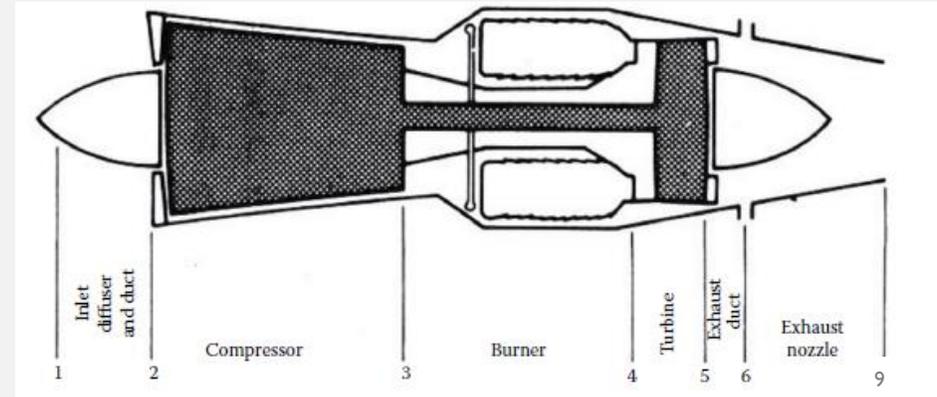


IDEAL TURBOJET ENGINE

GIVEN: H – altitude, CPR compressor pressure ratio,
TIT – turbine inlet temperature (T_{t4})

Engine work in static conditions $V_0=0 \rightarrow P_{t0} = P_0, T_{t0} = T_0$

No **INLET** losses $\rightarrow P_{t2} = P_{t0}$ and $T_{t2} = T_{t0}$



COMPRESSOR (2 – 3)

Compressor work is isentropic

$$\frac{T_{t3}}{T_{t2}} = \left(\frac{P_{t3}}{P_{t2}} \right)^{(k-1)/k} = \pi_C^{(k-1)/k}$$

Compressor work:

$$W_C = C_p(T_{t3} - T_{t2})$$

Compressor power:

$$P_C = \dot{m}_C W_C = \dot{m}_0 C_p(T_{t3} - T_{t2})$$

COMBUSTOR (3 – 4)

Energy balance

$$\dot{m}_f FHV = \dot{m}_0 C_{pB}(T_{t4} - T_{t3})$$

$$\dot{m}_f = \frac{\dot{m}_0 C_{pB}(T_{t4} - T_{t3})}{FHV}$$

Fuel air ratio:

$$f = \frac{\dot{m}_f}{\dot{m}_0} = \frac{C_{pB}(T_{t4} - T_{t3})}{FHV}$$

$$\pi_B = 1 \rightarrow P_{t4} = P_{t3}$$

TURBINE (4 - 5)

Compressor turbine power balance equation

$$P_C = \dot{m}_0 C_p(T_{t3} - T_{t2}) = P_T = \dot{m}_T C_{pT}(T_{t4} - T_{t5})$$

Turbine mass flow: $\dot{m}_T = \dot{m}_0 + \dot{m}_f$

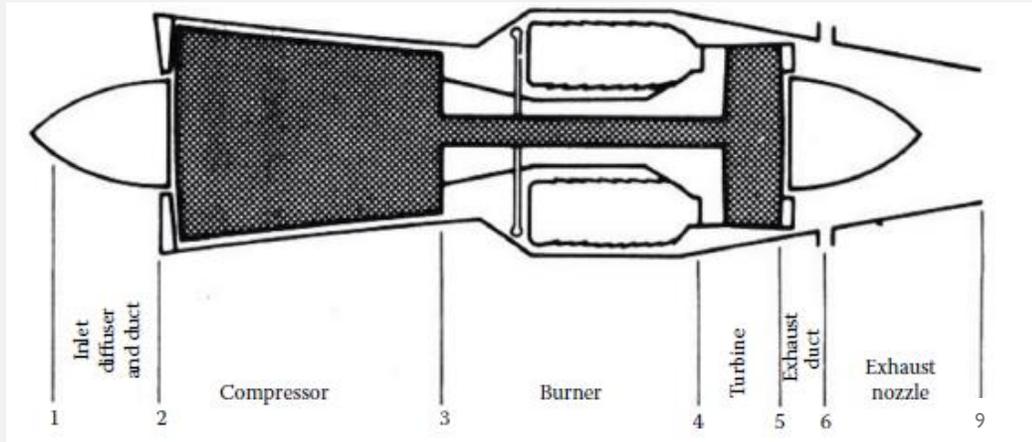
Turbine outlet temperature:

$$T_{t5} = T_{t4} - \frac{C_p(T_{t3} - T_{t2})}{C_{pT}(1 + f)}$$

Turbine outlet pressure:

$$P_{t5} = P_{t4} \left(\frac{T_{t5}}{T_{t4}} \right)^{k_t/(k_t-1)}$$

IDEAL TURBOJET ENGINE – GASS FULL EXPANSION IN THE NOZZLE



NOZZLE (5-9)

No losses: $\pi_N = 1 \rightarrow P_{t9} = P_{t5}$ and $T_{t9} = T_{t5}$

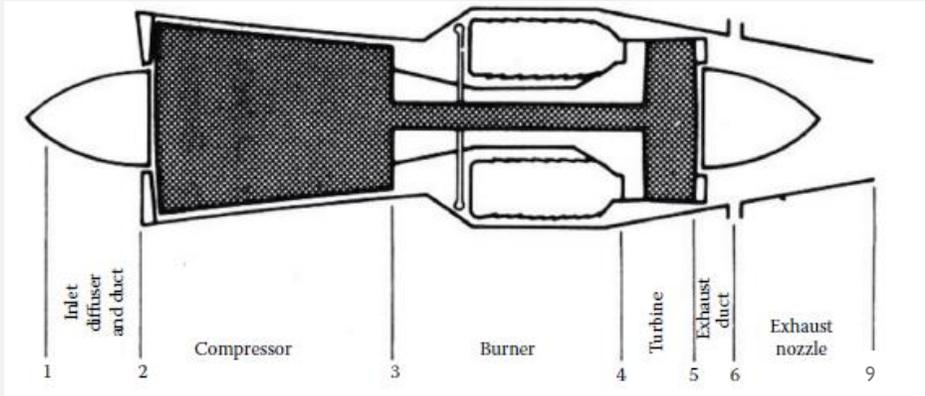
Full expansion: $P_9 = P_0$

$$c_9 = \sqrt{2Cp_t(T_{9t} - T_9)} \quad - \text{ for incompressible flow}$$

$$c_9 = a_9 M_9 = \sqrt{k_t R_t T_9} * \sqrt{\frac{2}{k_t - 1} \left(\frac{T_{t9}}{T_9} - 1 \right)} \quad - \text{ for compressible flow}$$

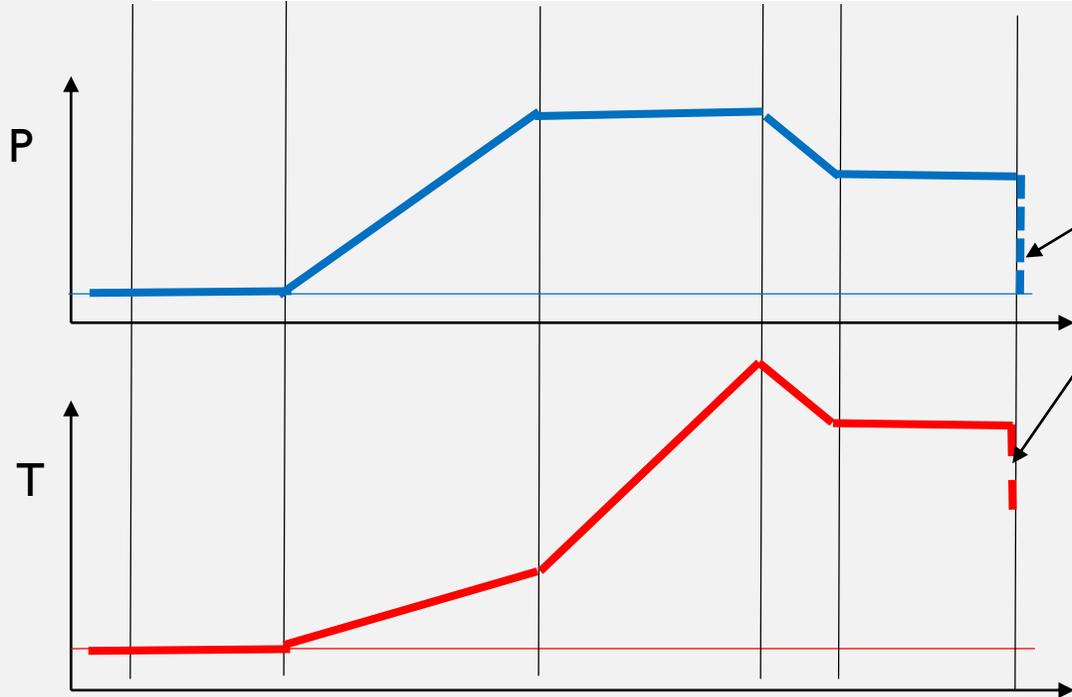
$$\frac{T_{t9}}{T_9} = \frac{P_{t9}^{(k_t-1)/k_t}}{P_9}$$

IDEAL TURBOJET ENGINE

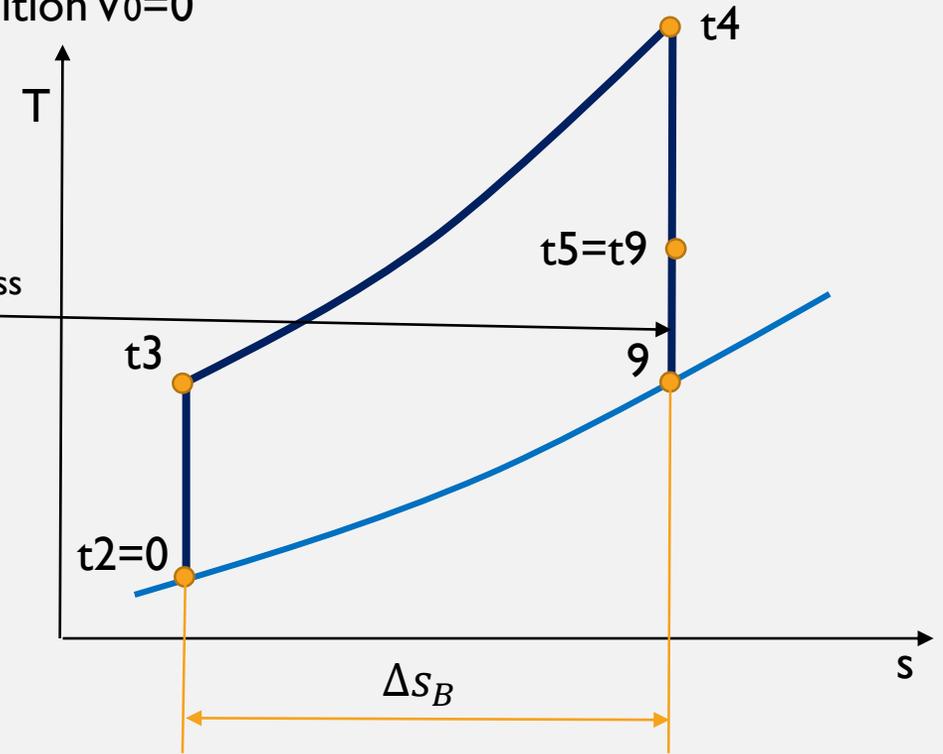


Processes in the inlet (diffuser) burner and nozzle are ideal – no pressure losses $\pi_D = \pi_B = \pi_N = 1$

Static condition $V_0=0$

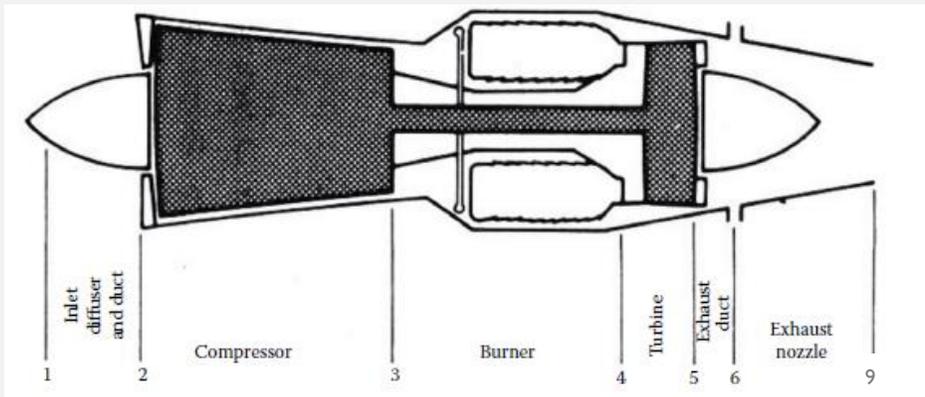


total to static process



Entropy increase in a burner $\Delta s_B = c_{p_B} \ln \frac{T_{t4}}{T_{t3}}$

IDEAL TURBOJET ENGINE

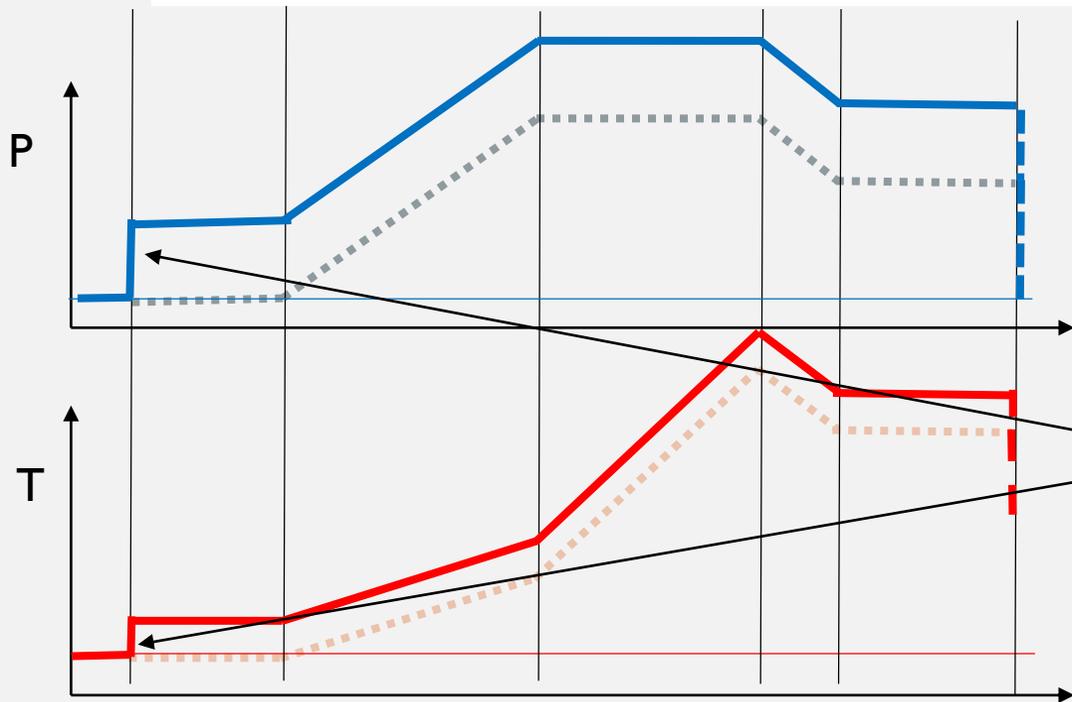


Flight condition $V_0 > 0$

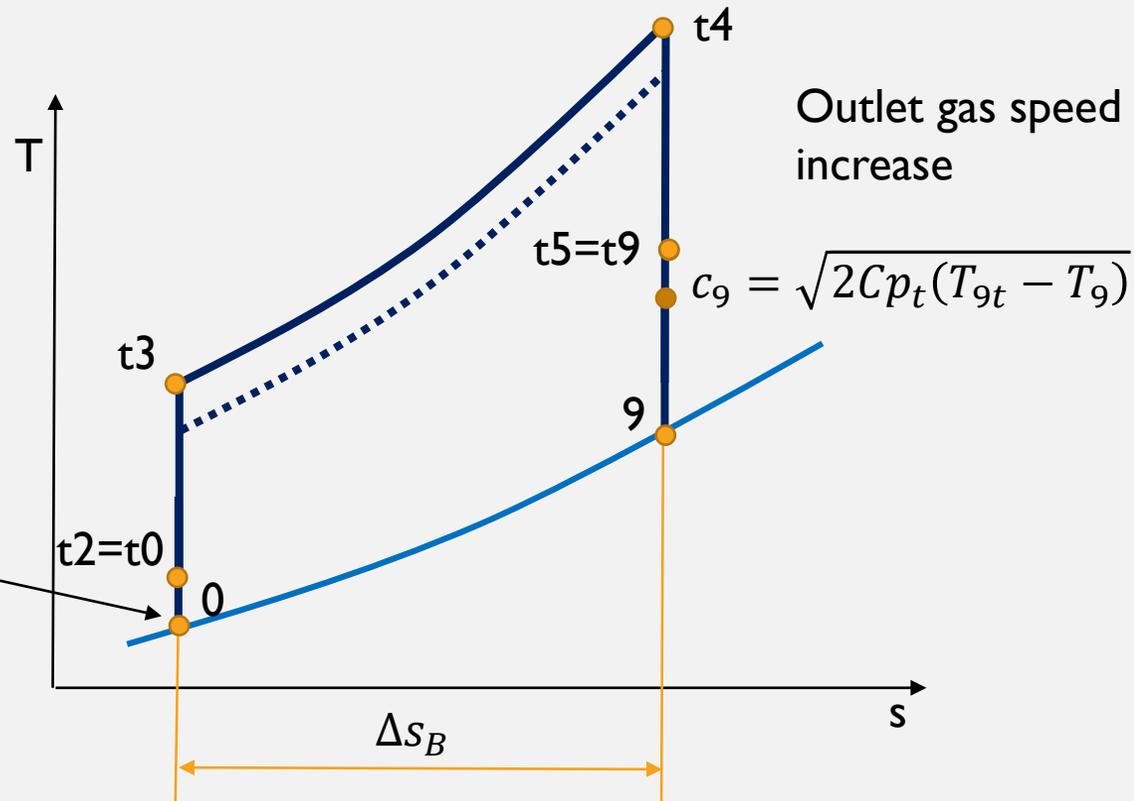
Ram pressure recovery for flight condition ($M_0 > 0$)

$$P_{t0} = P_0 \left(1 + \frac{k-1}{2} M_0^2 \right)^{k/(k-1)}$$

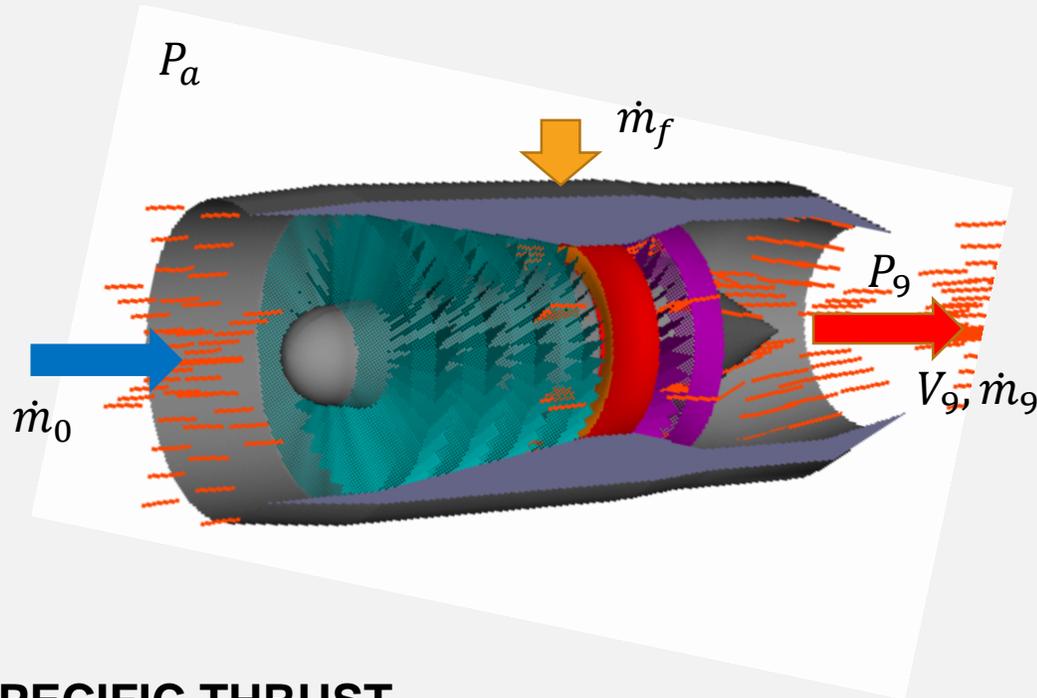
$$T_{t0} = T_0 \left(1 + \frac{k-1}{2} M_0^2 \right)$$



Ram effect



ENGINE THRUST AND SPECIFIC PARAMETERS



THRUST / GROSS THRUST

$$T = \dot{m}_9 V_9 + A_9 (P_9 - P_a)$$

effective exhaust velocity

$$V_{eff} = V_9 + A_9 (P_9 - P_a) / \dot{m}_9$$

$$T = \dot{m}_9 V_{eff}$$

Exit pressure = ambient pressure

$$T = \dot{m}_9 V_9$$

Flight speed > 0

THRUST / NET THRUST

$$T = \dot{m}_9 V_9 + A_9 (P_9 - P_a) - \dot{m}_0 V_0 = \dot{m}_9 V_{eff} - \dot{m}_0 V_0$$

Net thrust = Gross thrust – Momentum drag

SPECIFIC THRUST

$$ST = T / \dot{m}_0$$

SPECIFIC FUEL CONSUMPTION

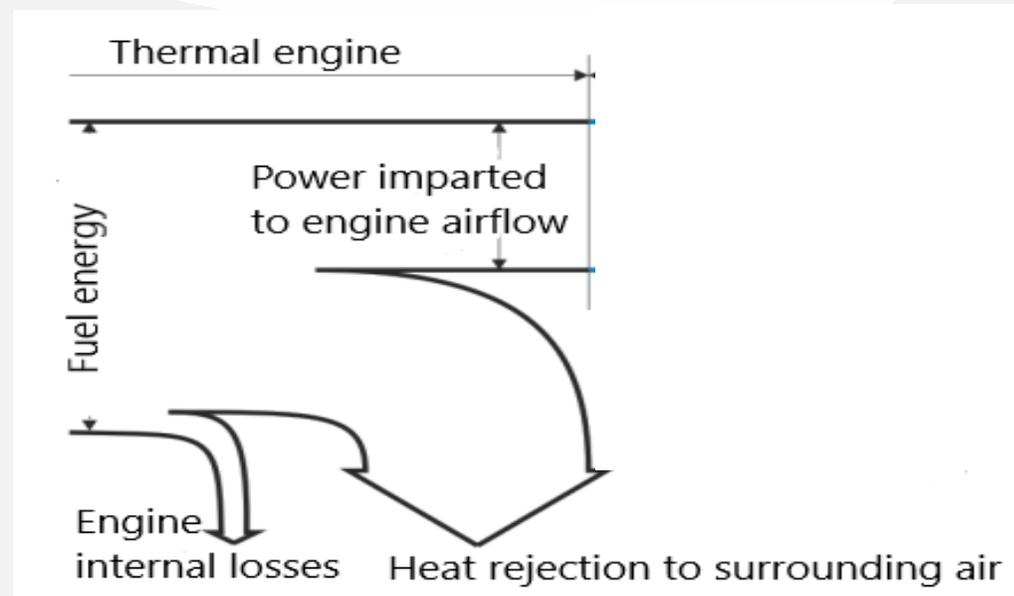
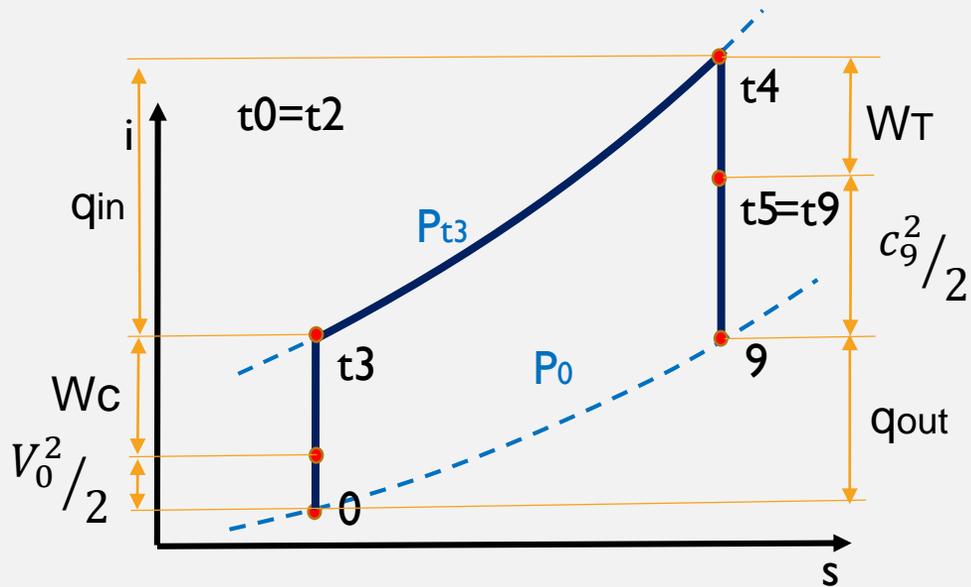
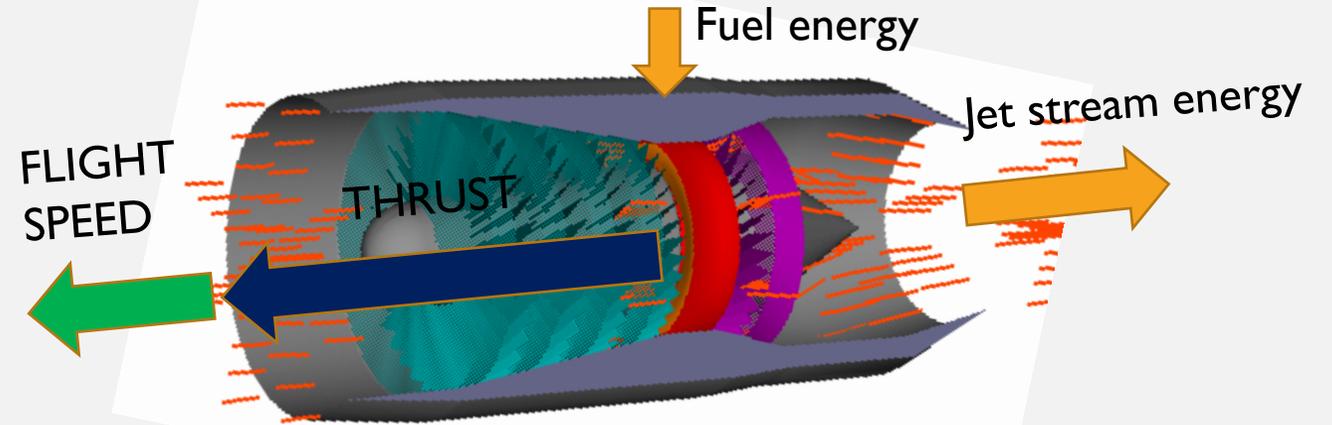
$$SFC = \dot{m}_f / T$$

ENGINE EFFICIENCIES

Thermal efficiency

$$\eta_{TH} = \frac{\text{Power imparted to engine airflow}}{\text{Rate of energy supplied in the fuel}}$$

$$\eta_{TH} = \frac{0,5 * (\dot{m}_9 V_{9e}^2 - \dot{m}_0 V_0^2)}{\dot{m}_f FHV}$$

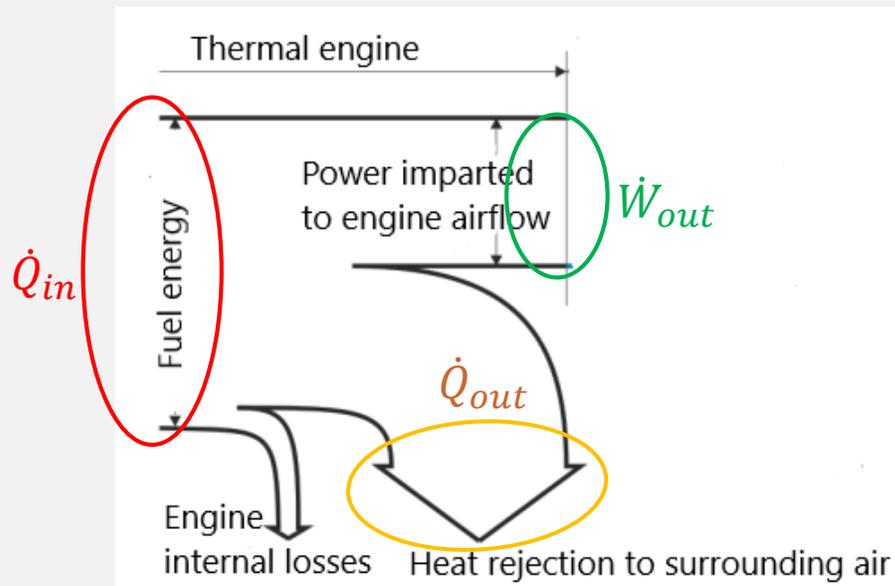


THERMAL EFFICIENCY

$$\eta_{TH} = \frac{\dot{W}_{out}}{\dot{Q}_{in}} \quad \longrightarrow \quad \eta_{TH} = \frac{\dot{Q}_{in} - \dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} \quad \text{where, } \dot{Q}_{out} = \dot{m}_9 c_p (T_9 - T_0)$$

- \dot{W}_{out} = net power out of engine (engine work)
- \dot{Q}_{in} = rate of thermal energy released/supplied in the fuel)

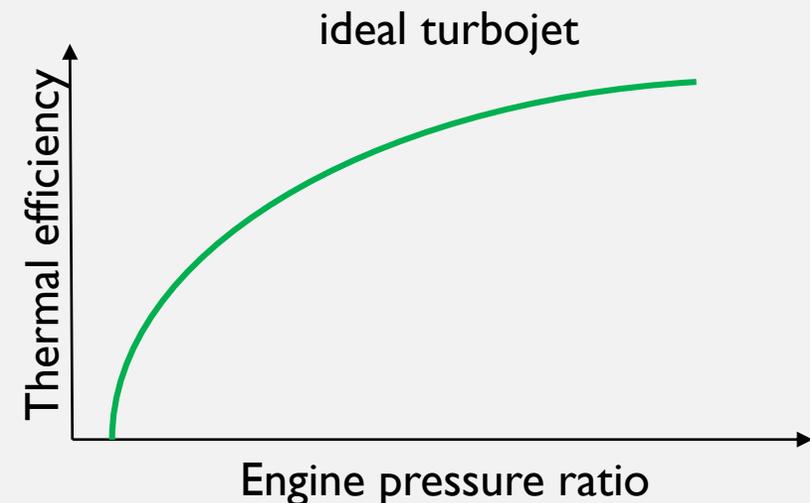
Higher thermal efficiency causes lower specific fuel consumption
Thermal efficiency is higher when exhaust gas temperature is closer to the ambient temperature



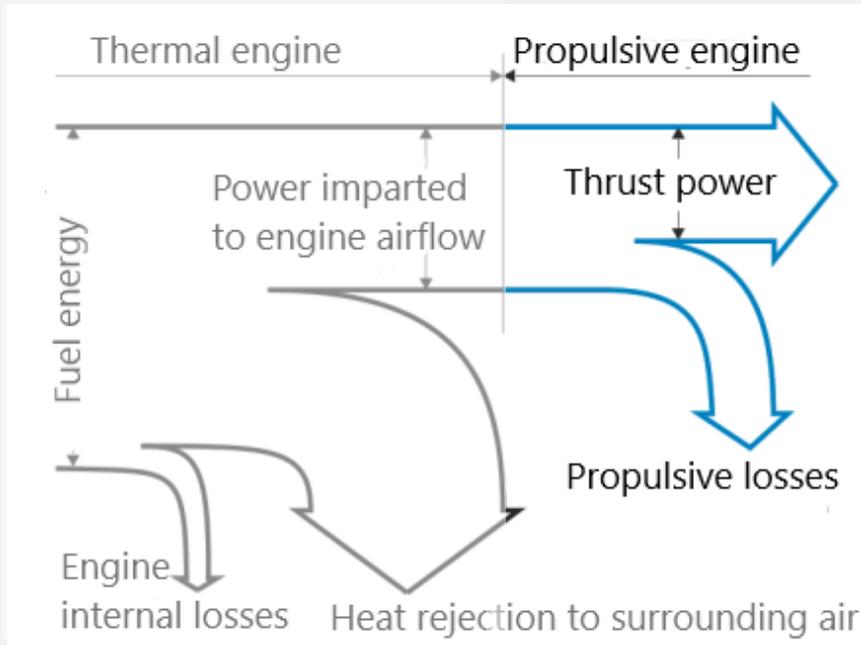
For ideal turbojet engine:

$$\eta_{TH} = 1 - \frac{1}{\pi^{(k-1)/k}},$$

where π – engine compression pressure ratio, k – isentropic exponent



PROPULSIVE EFFICIENCY

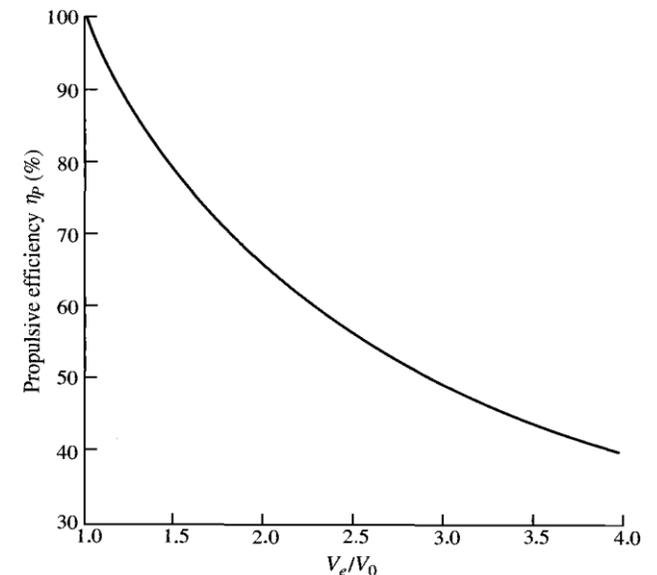


$$\eta_P = \frac{V_0 * T}{0,5 * (\dot{m}_9 V_{9e}^2 - \dot{m}_0 V_0^2)}$$

$$T = \dot{m}_9 V_{9e} - \dot{m}_0 V_0 \quad \text{and} \quad \dot{m}_9 = \dot{m}_0$$

$$\eta_P = \frac{V_0(V_{9e} - V_0)}{0,5 * (V_{9e}^2 - V_0^2)} = \frac{2V_0(V_{9e} - V_0)}{(V_{9e} - V_0)(V_{9e} + V_0)} = \frac{2}{1 + V_{9e}/V_0}$$

$$\eta_P \Rightarrow 1, \text{ for } V_{9e} \Rightarrow V_0$$



Propulsive efficiency define the thrust produced for specific flight speed from kinetic energy added to engine airflow

ENGINE EFFICIENCIES

Thermal efficiency

$$\eta_{TH} = \frac{\text{Power imparted to engine airflow}}{\text{Rate of energy supplied in the fuel}}$$

$$\eta_{TH} = \frac{0,5 * (\dot{m}_9 V_{9e}^2 - \dot{m}_0 V_0^2)}{\dot{m}_f FHV}$$

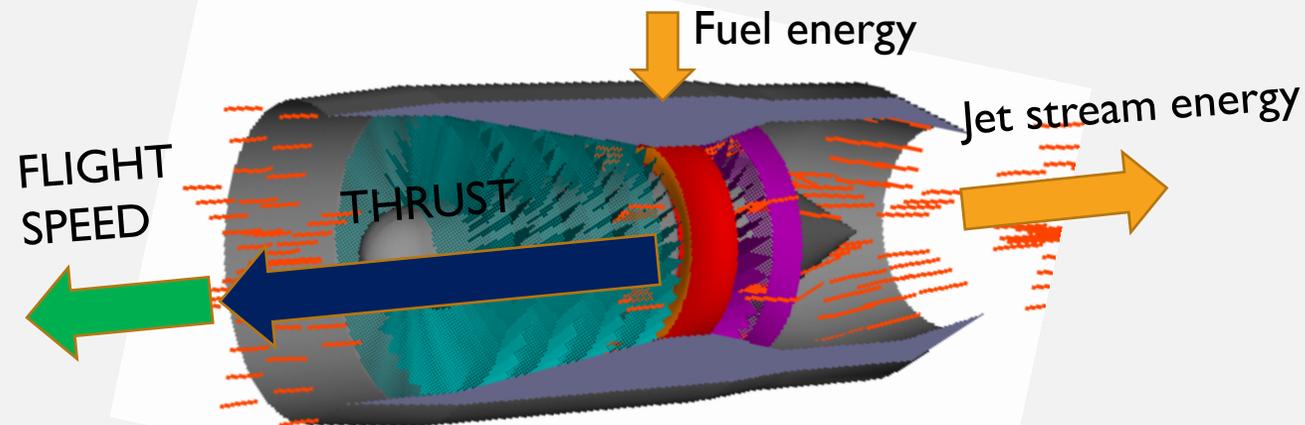
Propulsive efficiency

$$\eta_P = \frac{\text{Thrust power}}{\text{Power imparted to engine airflow}}$$

$$\eta_P = \frac{V_0 * T}{0,5 * (\dot{m}_9 V_{9e}^2 - \dot{m}_0 V_0^2)}$$

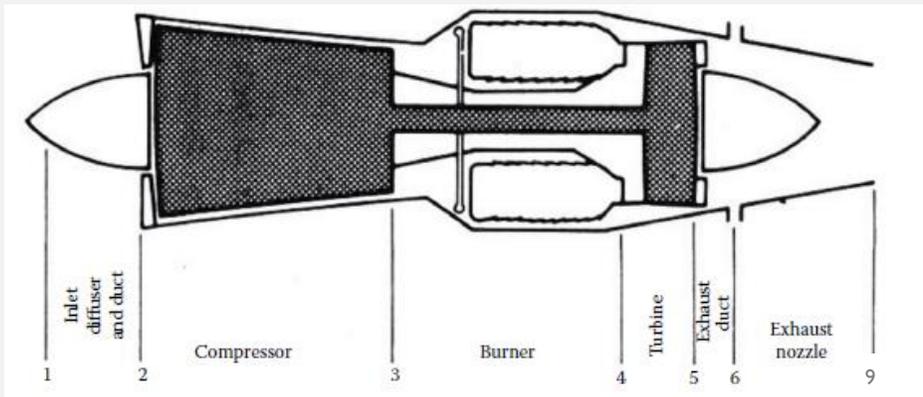
Overall efficiency

$$\eta_O = \eta_{TH} * \eta_P = \frac{V_0 * T}{\dot{m}_f FHV}$$

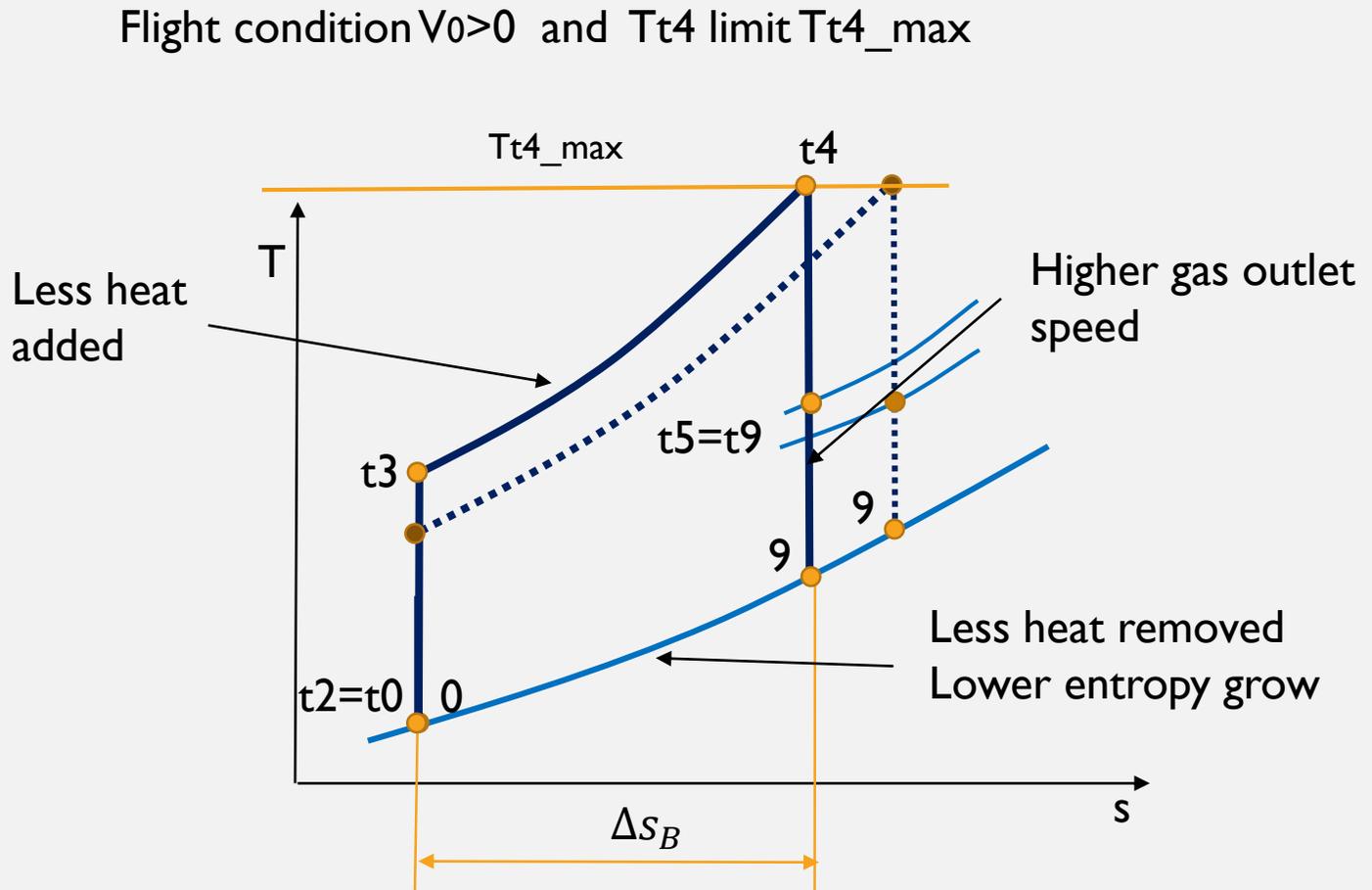
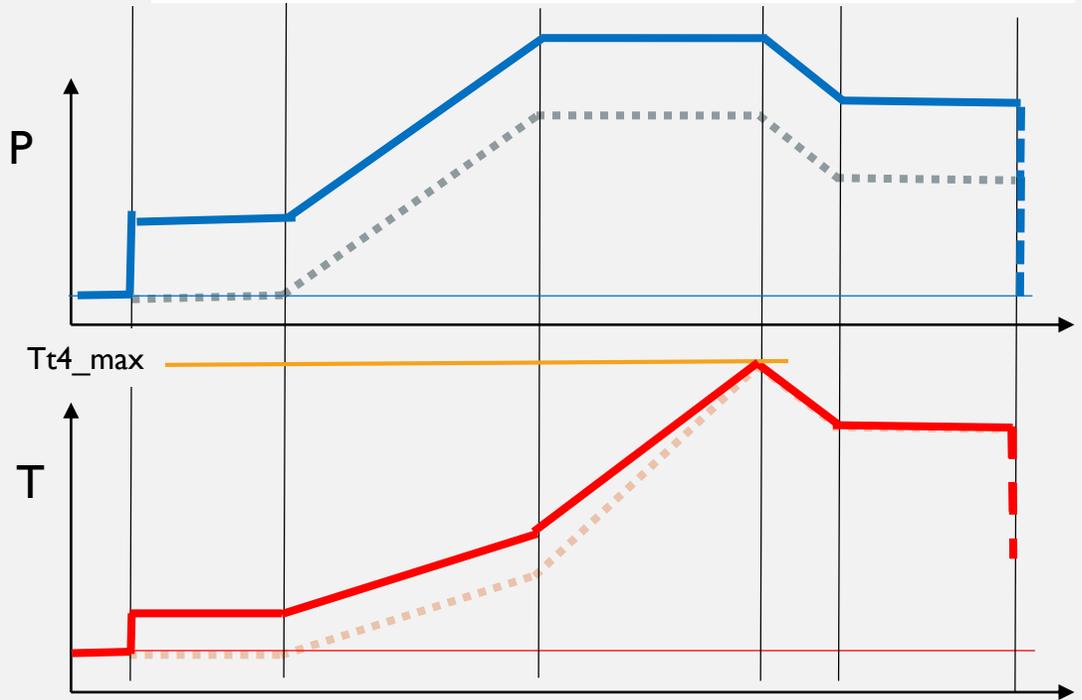


For $V_0=0$	$\eta_O = 0$ $\eta_P = 0$
For $V_0>0$	$\eta_O < \eta_P$ $\eta_O < \eta_{TH}$

IDEAL TURBOJET ENGINE – FLIGHT SPEED INFLUENCE

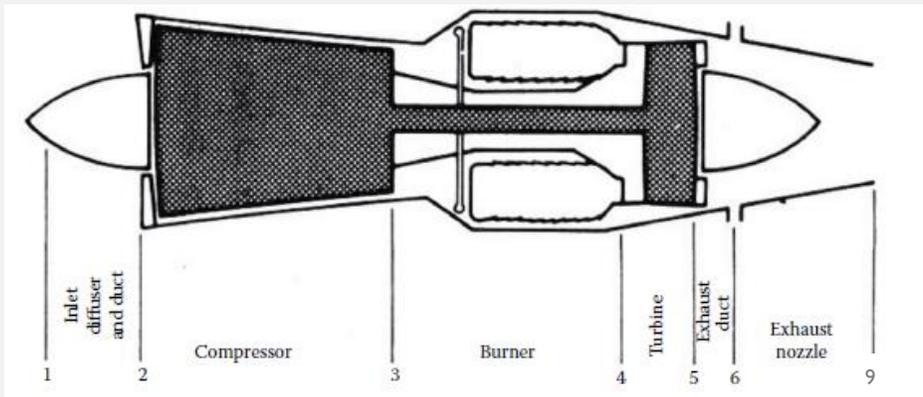


Flight condition $V_0 > 0$ and T_{t4} limit T_{t4_max}

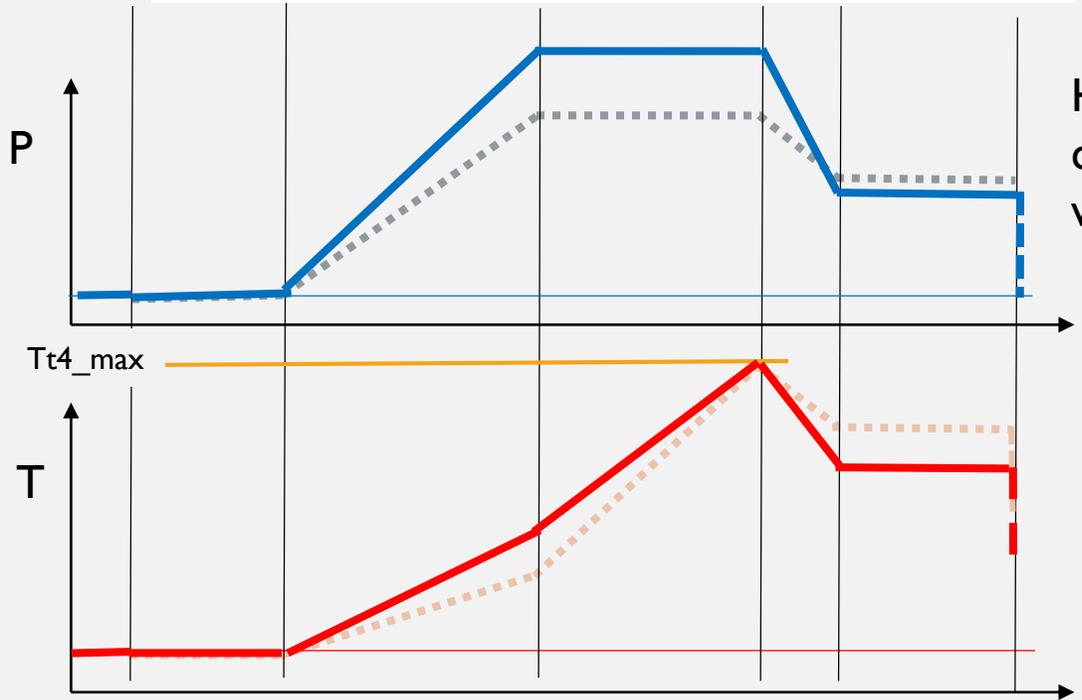


Engine thermal efficiency grow (less heat removed)
Lower fuel consumption (less heat added)

IDEAL TURBOJET ENGINE – COMPRESSOR PRESSURE RATIO INFLUENCE

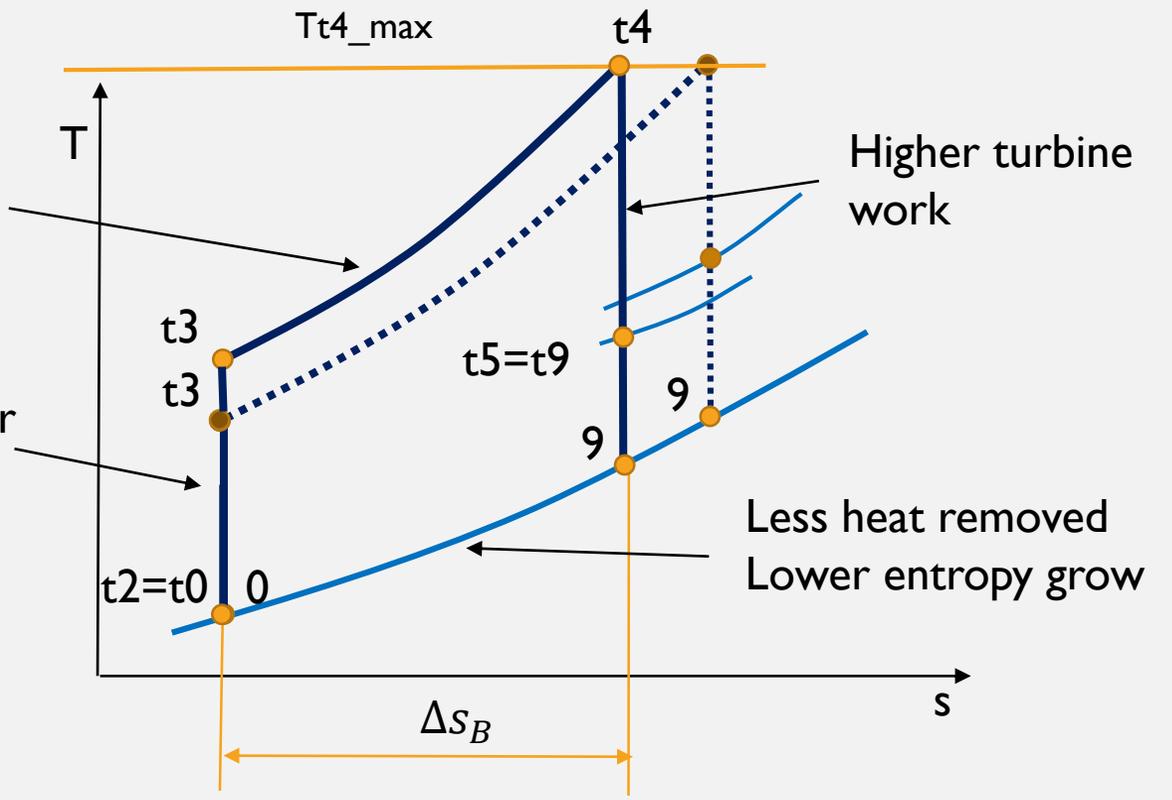


CPR (compressor pressure ratio) growing and T_{t4} is limited T_{t4_max}



Less heat added

Higher compressor work

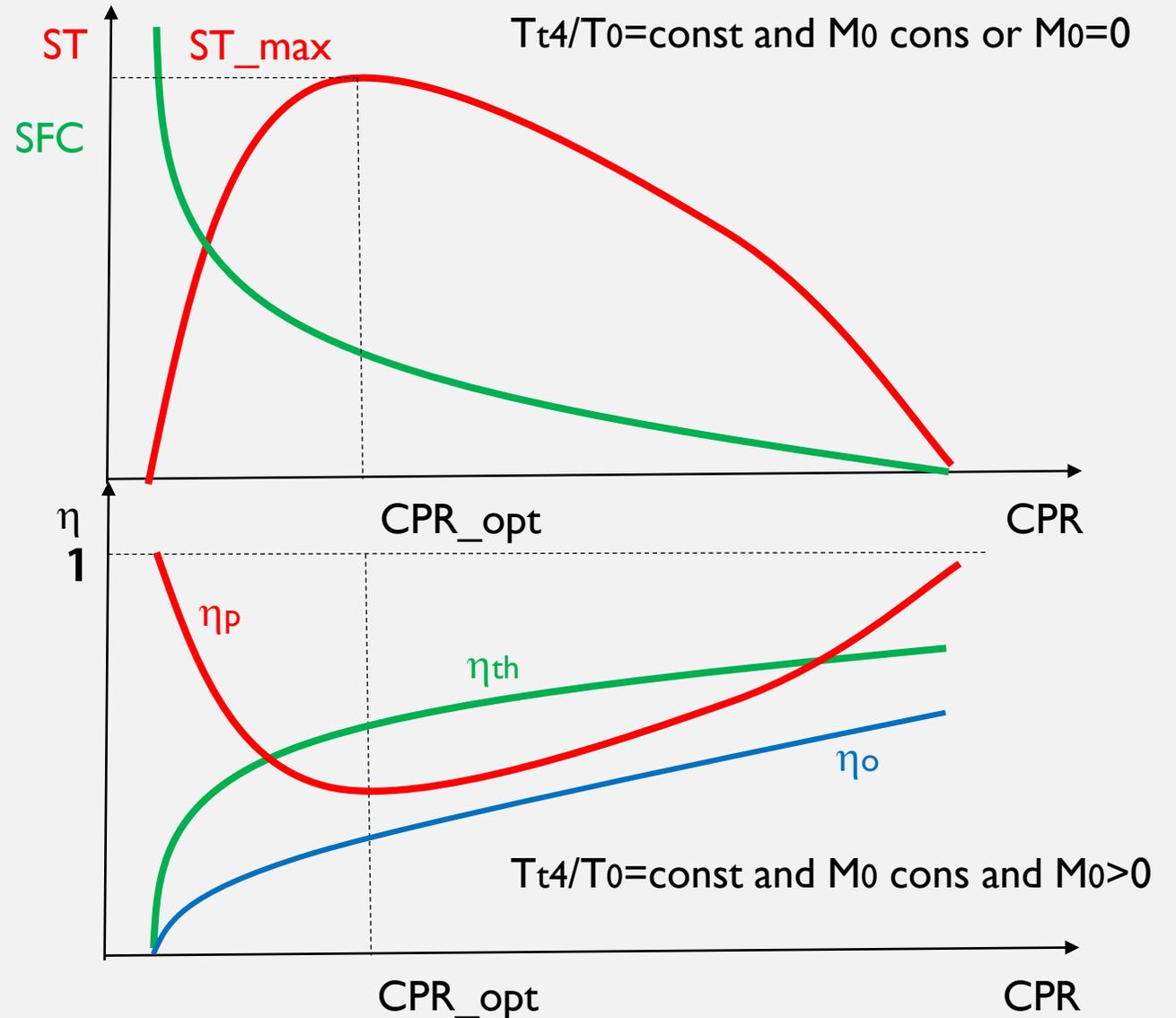


**Engine thermal efficiency grow (less heat removed)
Lower fuel consumption (less heat added) and lower SFC**

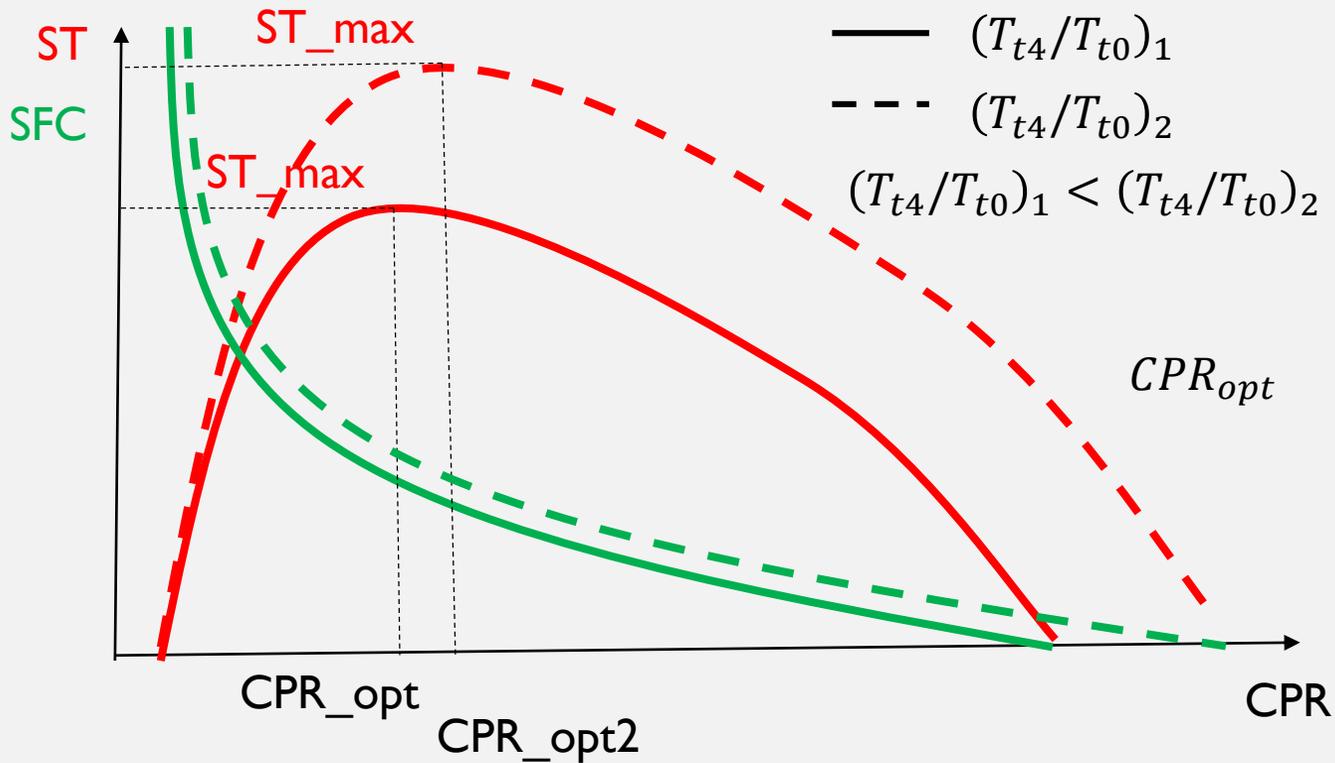
IDEAL TURBOJET CYCLE OPTIMISATION

SUMMARY:

- Specific thrust (ST) grows with compressor pressure ratio increasing, achieves maximum for optimal CPR than it goes down
- Specific fuel consumption decreases with CPR growing
- Propulsive efficiency as a function of CPR represents opposite relation to ST, it is minimal for optimal CPR and achieves 1 for ST=0.
- Thermal and overall efficiencies grow with CPR
- The presented relationship between efficiencies and CPR are right for $M_0 > 0$, for $M_0 = 0$, propulsive and overall efficiency are 0



IDEAL TURBOJET CYCLE OPTIMISATION FOR DIFFERENT ENGINE TEMPERATURE RATIO



SUMMARY:

- Specific thrust (ST) is higher for higher engine temperature ratio T_{t4}/T_{t0} and achieve ST_{max} for higher CPR (higher CPR_{opt})
- Specific fuel consumption decreases with CPR growing, but for high T_{t4}/T_{t0} is higher
- Range of available CPR increases for higher T_{t4}/T_{t0}

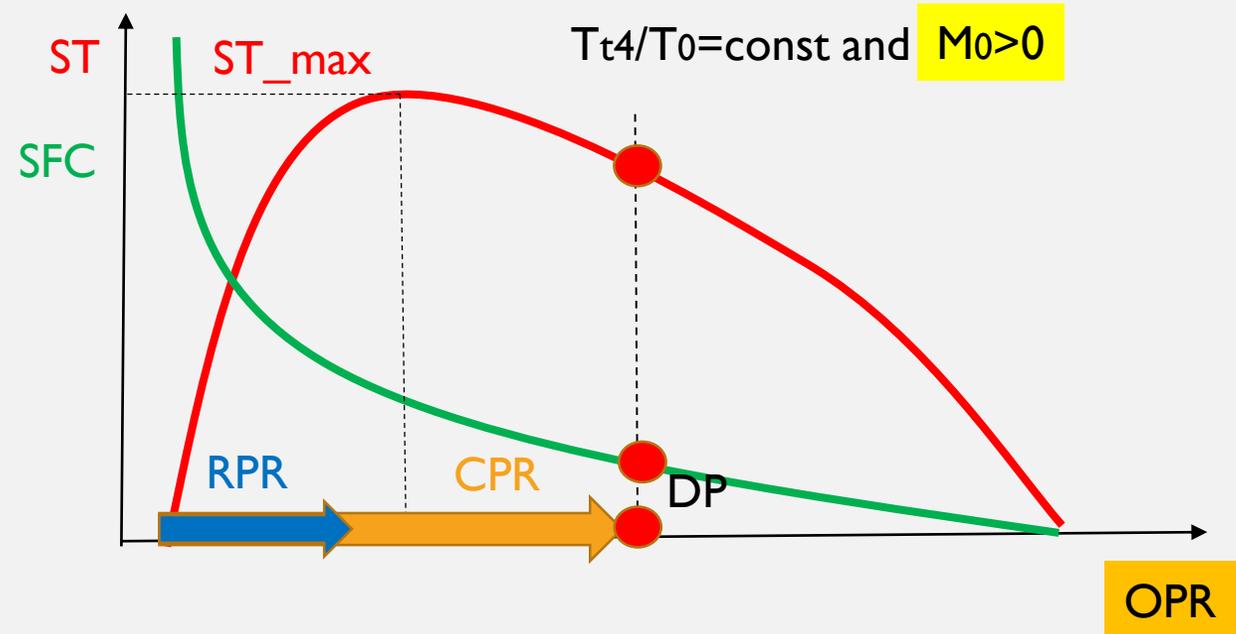
T_{t4}/T_{t0}	CPR_{opt}	CPR_{max}
4	11,3	128
5	16,7	279,5
6	23	529

For ideal cycle:

$$CPR_{opt} = T_{t4}/T_{t0}^{\frac{k}{2(k-1)}} \quad CPR_{max} = CPR_{opt}^2$$

IDEAL TURBOJET CYCLE OPTIMISATION FOR VARIOUS V_0

Flight speed M_0	RPR (ideal)
1	1,89
1,5	3,67
2	7,82
2,5	17,09
3	36,73



SUMMARY:

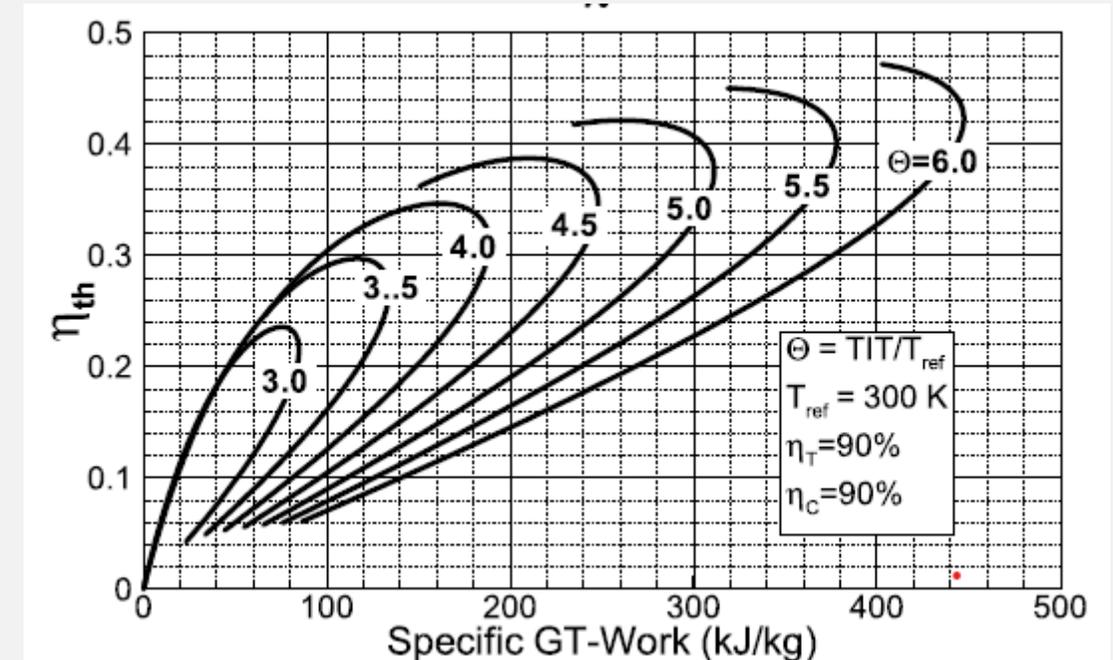
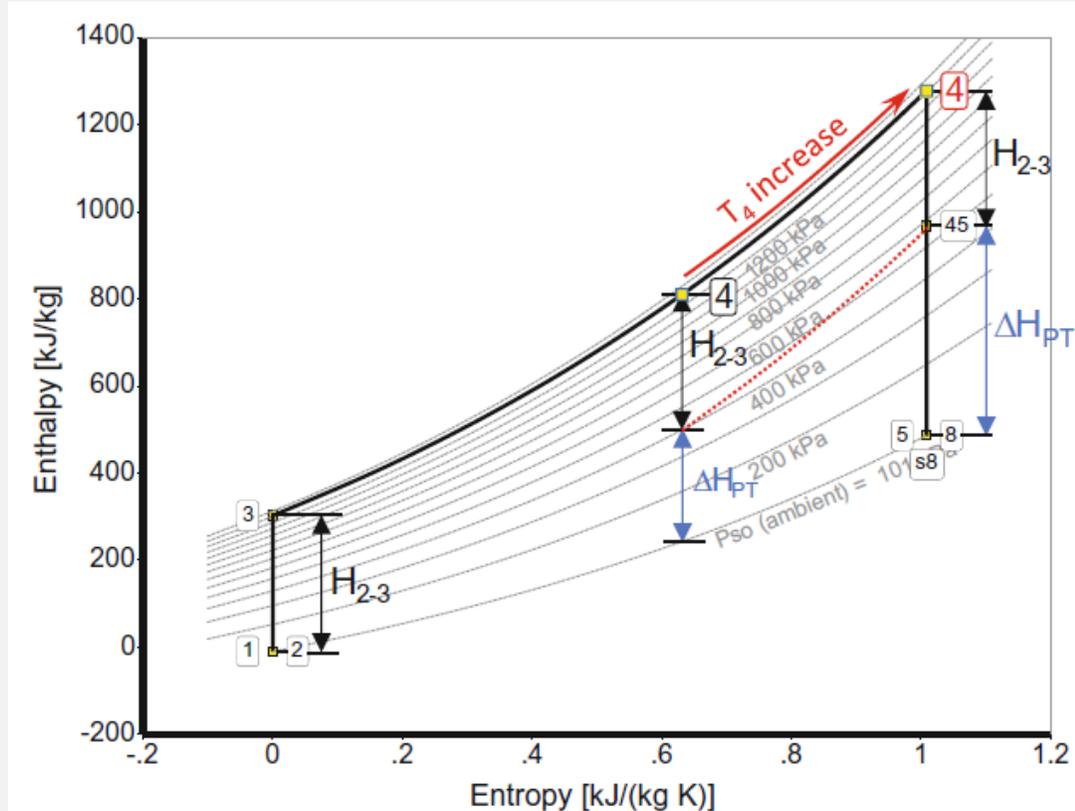
- An increase in flight speed causes an increase in the ram pressure ratio (RPR).
- When optimizing the engine cycle for increasing flight speeds, RPR should be included in OPR, and CPR should be reduced accordingly.

$$OPR = RPR * CPR$$

$$RPR = \left(1 + \frac{k-1}{2} M_0^2 \right)^{\frac{k}{k-1}}$$

INCREASE OF OUTPUT SHAFT POWER

- Increase of turbine inlet temperature (TIT)



Thermal efficiency vs. Specific work of GT

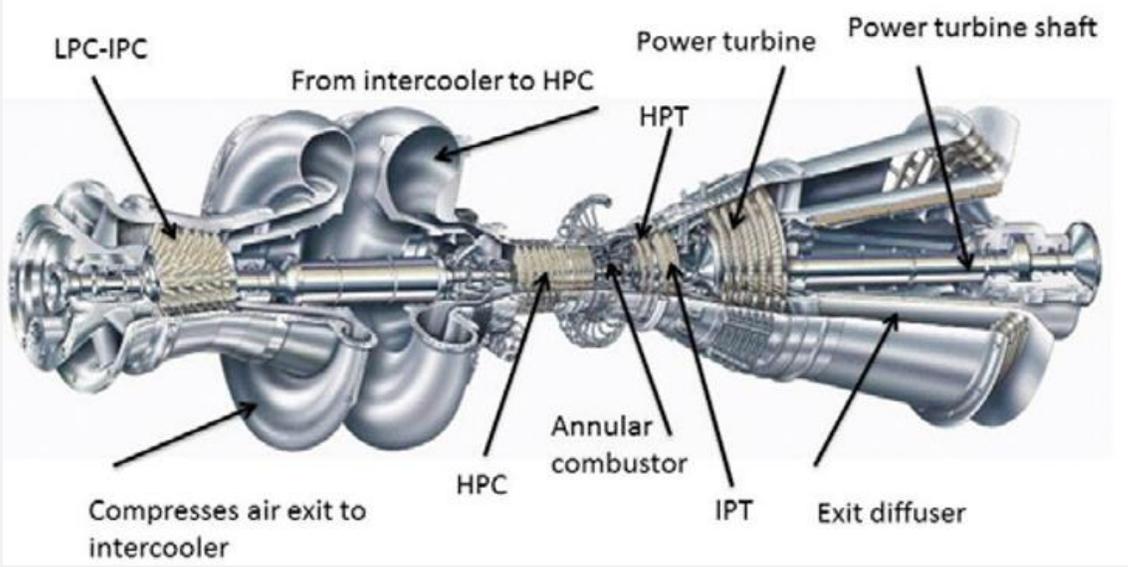
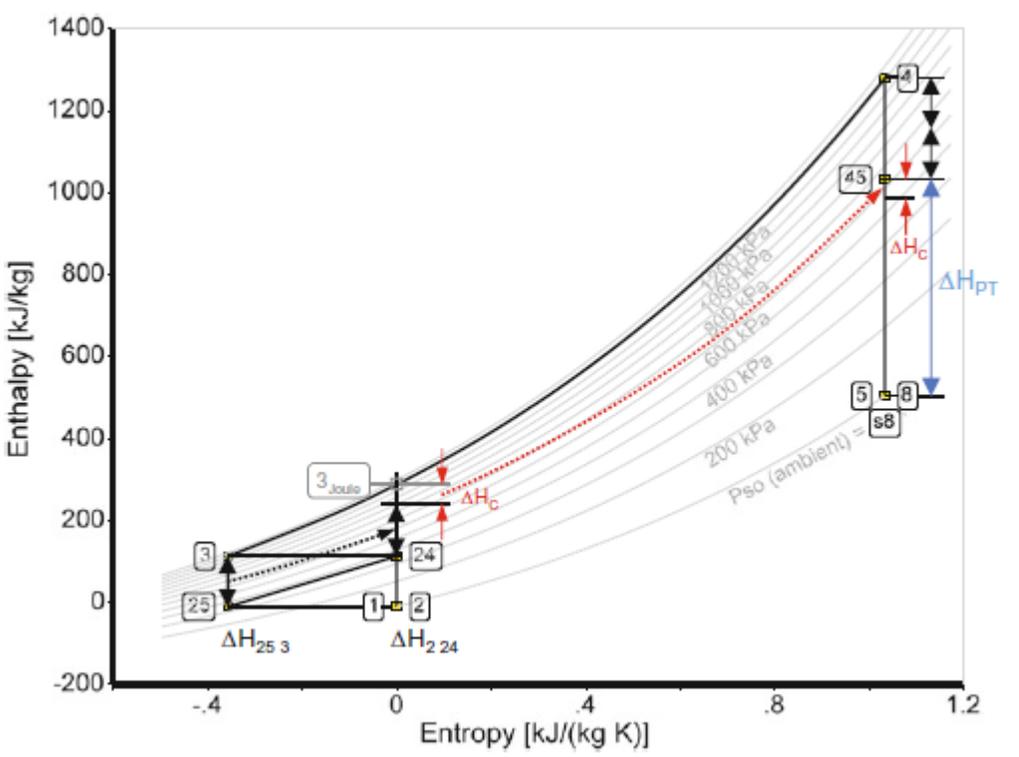
M.T. Schobeiri, Gas Turbine Design, Components and System Design Iteration, DOI 10.1007/978-3-319-85378-5_1

Gas turbine entalpy – entropy diagram for TIT rise

J. Kurzke, I. Halliwell, Propulsion and Power An Exploration of Gas Turbine Performance Modeling, <https://doi.org/10.1007/978-3-319-75979-1>

INCREASE OF OUTPUT SHAFT POWER

- Compressor intercooling

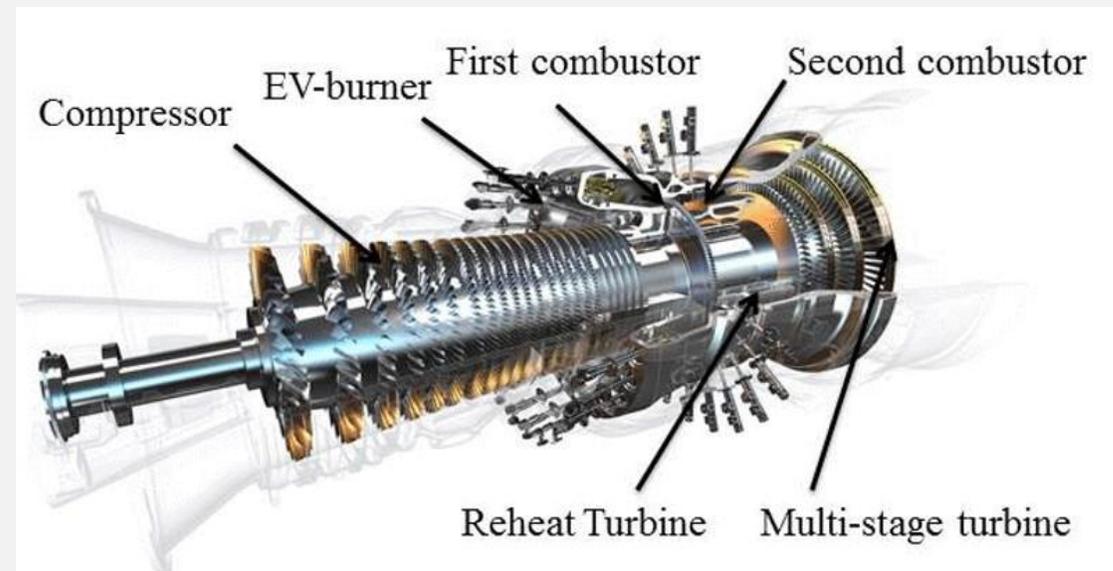
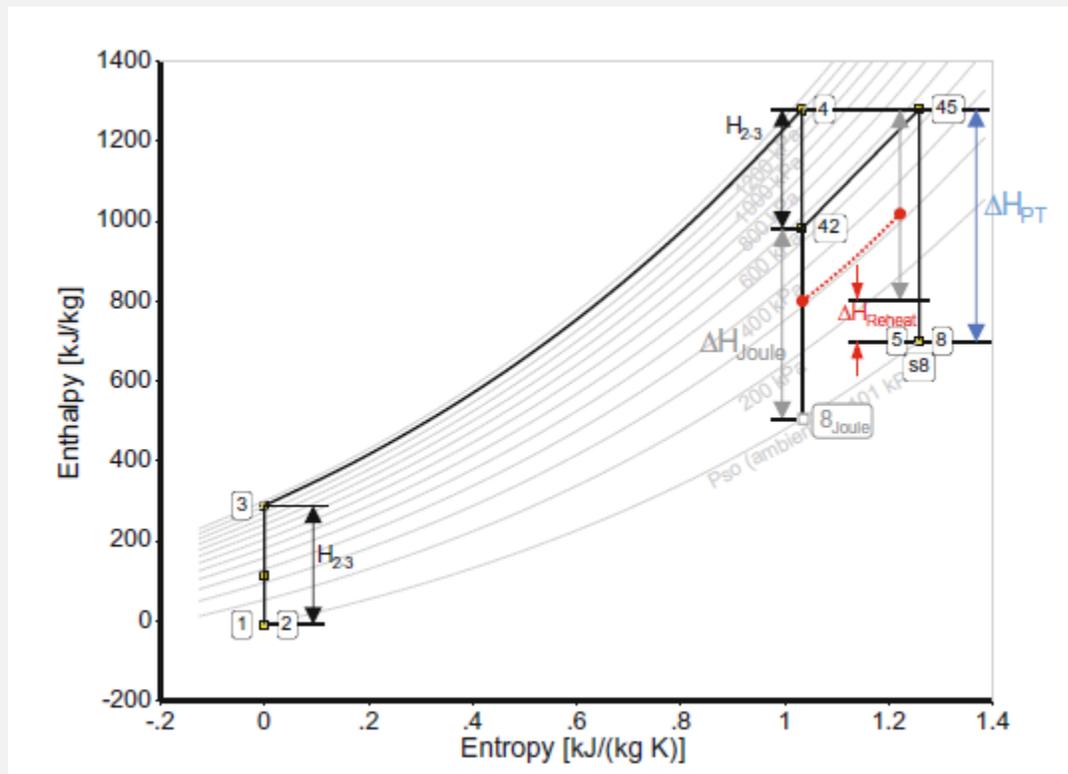


Gas turbine LSI00 (GE)
M.T. Schobeiri, Gas Turbine Design, Components and System Design Interaction, DOI 10.1007/978-3-319-85378-5_1

Gas turbine entalpy – entropy diagram for compressor intercooling
J. Kurzke, I. Halliwell, Propulsion and Power An Exploration of Gas Turbine Performance Modeling, <https://doi.org/10.1007/978-3-319-75979-1>

INCREASE OF OUTPUT SHAFT POWER

- Reheating – inter turbine burner (ITB)



GT with ITB – GT 24/26

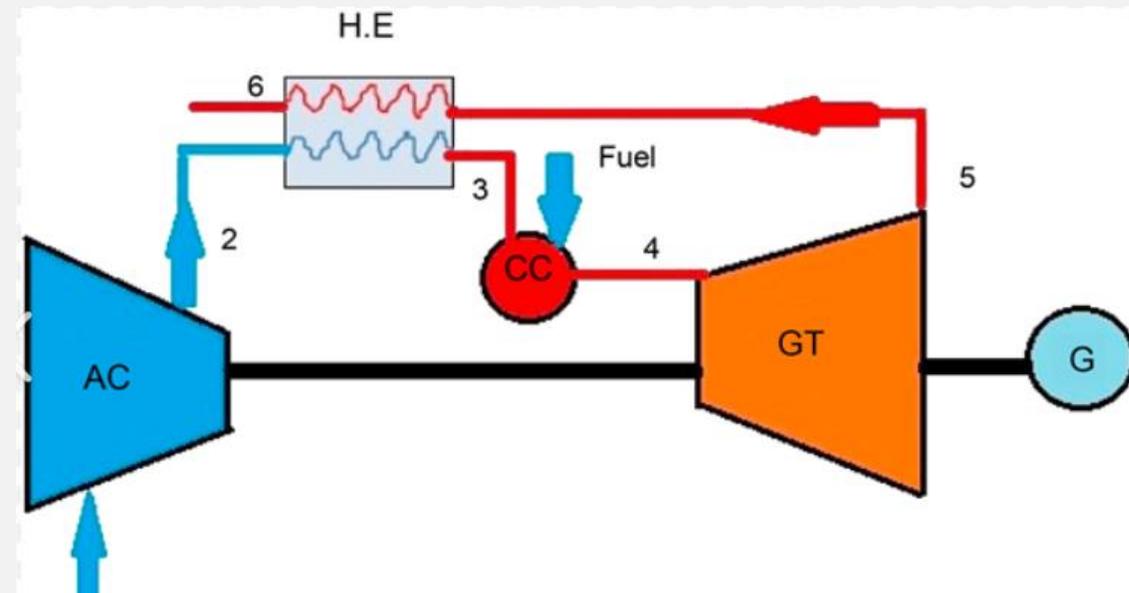
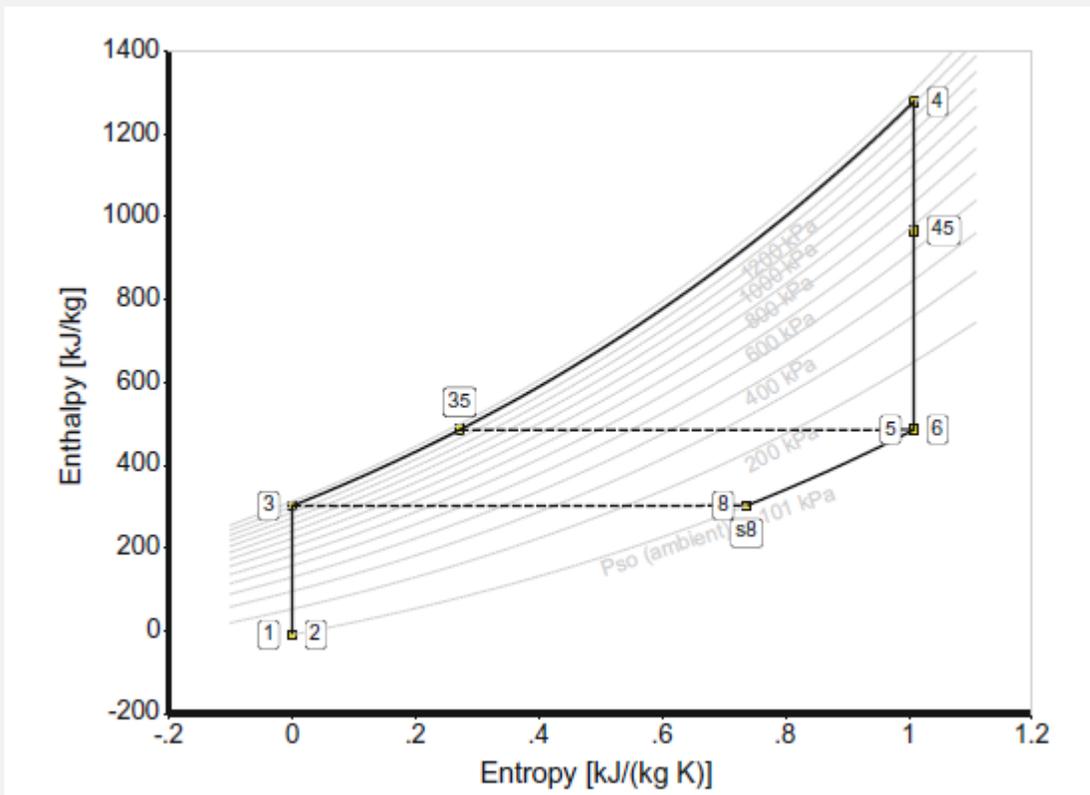
M.T. Schobeiri, Gas Turbine Design, Components and System Design Iteration, DOI 10.1007/978-3-319-85378-5_1

Gas turbine with ITB entalpy – entropy diagram

J. Kurzke, I. Halliwell, Propulsion and Power An Exploration of Gas Turbine Performance Modeling, <https://doi.org/10.1007/978-3-319-75979-1>

REDUCE FUEL CONSUMPTION

- **GT with heat exchanger (HE)**

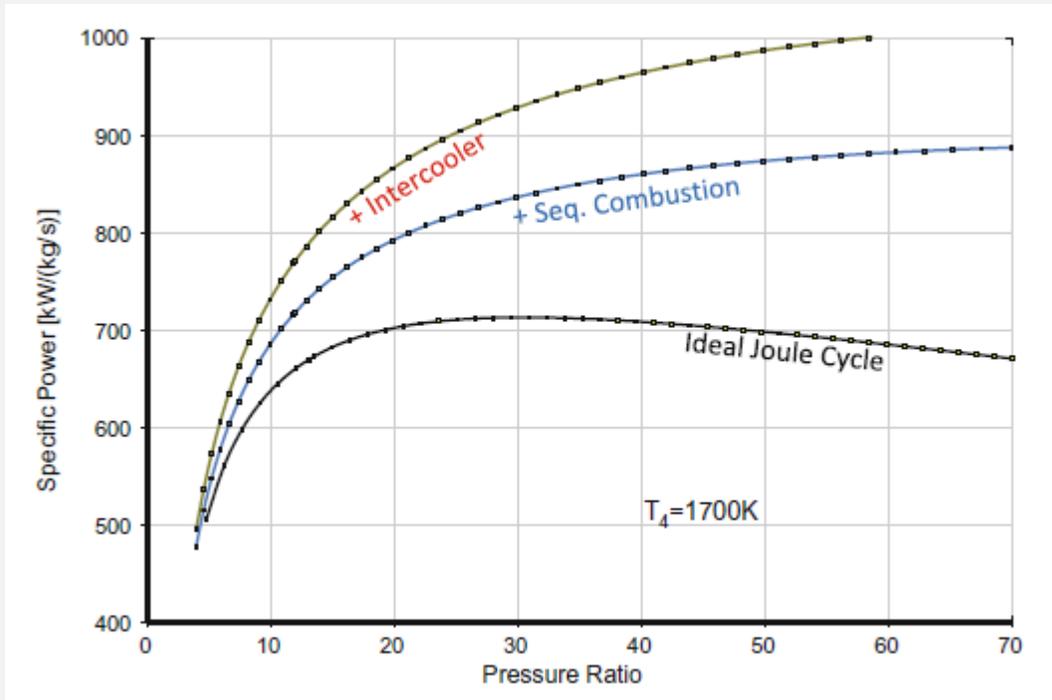


Scheme of GT with HE

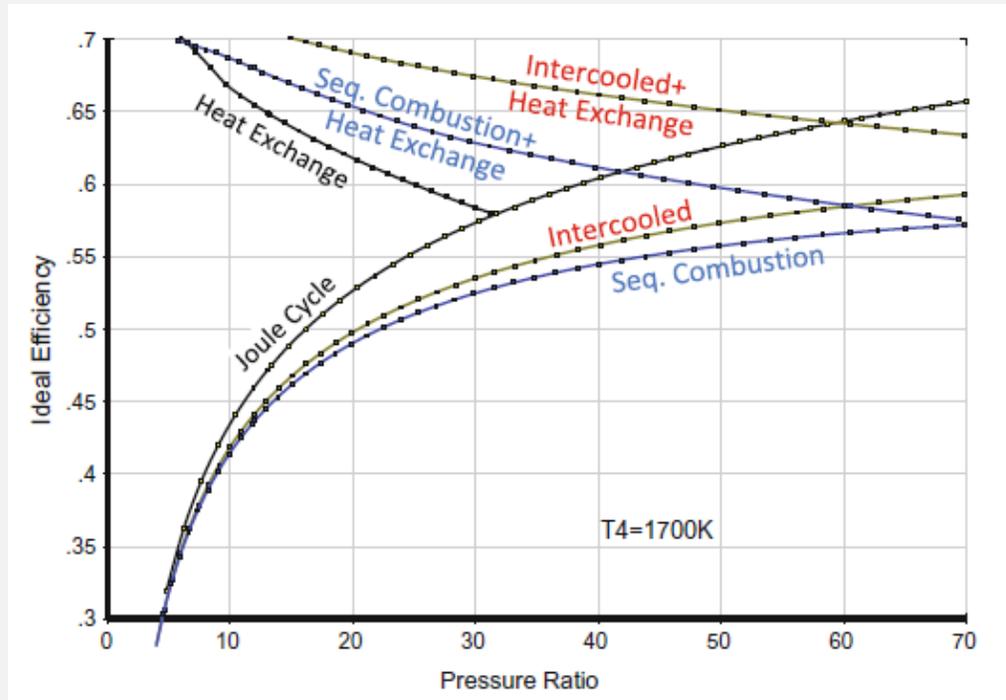
Gas turbine with heat exchanger enthalpy – entropy diagram

J. Kurzke, I. Halliwell, Propulsion and Power An Exploration of Gas Turbine Performance Modeling, <https://doi.org/10.1007/978-3-319-75979-1>

GAS TURBINES AND MODIFIED GT



Specific power for ideal cycles



Efficiency of ideal cycles for shaft power generation

J. Kurzke, I. Halliwell, Propulsion and Power An Exploration of Gas Turbine Performance Modeling, <https://doi.org/10.1007/978-3-319-75979-1>

Higher efficiency means lower specific fuel consumption!

LITERATURE

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THANKS FOR YOUR ATENTION

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