Aircraft Engine Construction - turbofan engine

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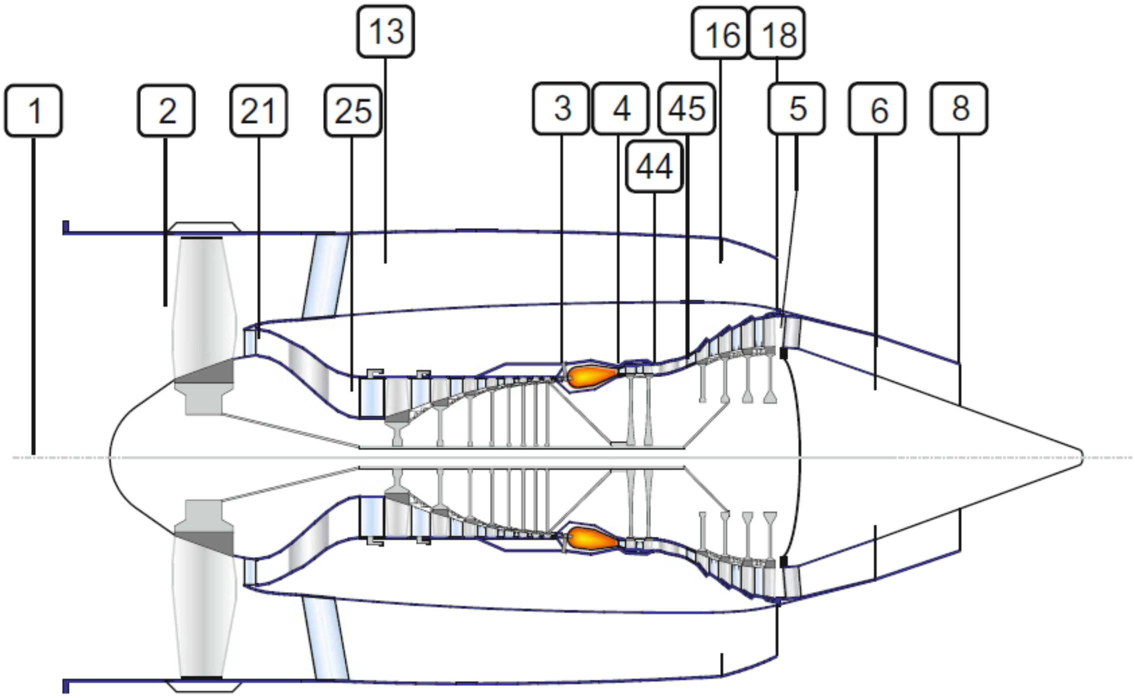
IDEAL TURBOFAN ENGINE

En example of turbofan engin calculation is presented. The assumption is that external and internal engine nozzles expanison is to ambient pressure



Given

Flight conditions: T0=217 K, P0=22 kPa, M0=0.88, bypass ratio 9, fan pressure ratio 1.6, compressor pressure ratio 20, Turbine inlet temperature Tt4=1600 K, mass flow m=100 kg/s.

Gas parameters:

Air: k=1.4; cp=1005 J/kg/K, R=287 J/kg/K,

Fumes in turbine and nozzle kt=1.33, cpt=1170 J/kg/K, Rt=290 J/kg/K, For combustion in combustor cpB=1200 J/kg/K,

Fuel heat value: FHV=43 MJ/kg

Flight Mach No

```
M0 = 0.8800
```

Air Mass flow [kg/s]

```
m0 = 100
```

Bypass ratio

```
BPR = 9
```

Turbine inlet temperature [K]

```
Tt4 = 1600
```

Fan pressure ratio

```
FPR = 1.6000
```

Compressor pressure ratio

```
CPR = 20
```

Ambient conditions

Static temperature [K]

```
T0 = 217
```

Static pressure [Pa]

```
P0 = 22000
```

TURBOFAN ENGINE CALCULATION

Section 0

Total temperature [K]

$$T_{t0} = T_0\left(1 + \frac{k-1}{2}M_0^2\right)$$

```
Tt0_id = 250.6090
```

Total pressure [Pa]

$$P_{t0} = P_0\left(1 + \frac{k-1}{2}M_0^2\right)^{\frac{k}{k-1}}$$

```
Pt0_id = 3.6417e+04
```

Speed of sound [m/s]

$a_0 = \sqrt{k * R * T_0}$ - like for ideal engine

```
a0 = 295.2805
```

Flight speed [m/s]

$V_0 = M_0 * a_0$ - like for ideal engine

```
V0_id = 259.8469
```

Section 2 Inlet exit

Total temperature [K]

$T_{t2} = T_{t0}$ - like for ideal engine

```
Tt2_id = 250.6090
```

Total pressure [Pa]

$P_{t2} = P_{t0}$

```
Pt2_id = 3.6417e+04
```

Section 25/13 - Fan outlet

Total temperature [K]

$$T_{t25} = T_{t13} = T_{t2} * \mathrm{FPR}^{\frac{k-1}{k}}$$

```
Tt25_id = 286.6267
Tt13_id = 286.6267
```

Total pressure [Pa]

$$P_{t25} = P_{t13} = P_{t2} * \mathrm{FPR}$$

```
Pt25_id = 5.8267e+04
Pt13_id = 5.8267e+04
```

FAN

Fan work [J/kg]

$$W_F = c_p * (T_{t25} - T_{t2})$$

```
WF_id = 3.6198e+04
```

Fan power [W]

$$P_F = m_0 * W_F$$

```
PC_id = 3.6198e+06
```

STREAM SPLIT

Internal duct mass flow [kg/s]

$$m_{25} = m_0 * \frac{1}{1 + \mathrm{BPR}}$$

```
m25 = 10
```

External duct mass flow [kg/s]

$$m_{13} = m_0 * \frac{\mathrm{BPR}}{1 + \mathrm{BPR}}$$

```
m13 = 90
```

INTERNAL DUCT / CORE ENGINE

Section 3 - Compressor outlet / Burner inlet

Total temperature [K]

$$T_{\mathrm{t3}} = T_{\mathrm{t25}} * \mathrm{CPR}^{\frac{k-1}{k}}$$

```
Tt3_id = 674.5893
```

Total pressure [Pa]

$$P_{\mathrm{t3}} = P_{\mathrm{t25}} * \mathrm{CPR}$$

```
Pt3_id = 1.1653e+06
```

COMPRESSOR

Compressor work [J/kg]

$$W_C = c_p * (T_{\mathrm{t3}} - T_{\mathrm{t25}})$$

```
WC_id = 3.8990e+05
```

Compressor power [W]

$$P_C = m_{25} * W_C$$

```
PC_id = 3.8990e+06
```

Section 4 Burner outlet / Turbine inlet

Total temperature [K]

$$T_{\mathrm{t4}}$$

```
Tt4_id = 1600
```

Total pressure [Pa]

$$P_{\mathrm{t4}} = P_{\mathrm{t3}}$$

```
Pt4_id = 1.1653e+06
```

BURNER

Fuel-air ratio

$$f_B = \frac{m_f}{m_{25}} = c_{\mathrm{pB}} * \frac{T_{\mathrm{t4}} - T_{\mathrm{t3}}}{\mathrm{FHV} * \eta_B}$$

```
fB_id = 0.0258
```

Fuel mass flow   [kg/s]

$$m_{\mathrm{fB}} = m_{25} * f_B$$

```
mfB_id = 0.2583
```

Section 45   High Pressure Turbine outlet

Total temperature   [K]

$$T_{\mathrm{t45}} = T_{\mathrm{t4}} - \frac{W_C}{(1 + f_B) * c_{\mathrm{pt}}}$$

```
Tt45_id = 1.2751e+03
```

Total pressure [Pa]

$$P_{\mathrm{t45}} = P_{\mathrm{t4}} \left(\frac{T_{\mathrm{t45}}}{T_{\mathrm{t4}}} \right)^{\frac{\mathrm{kt}}{\mathrm{kt}-1}}$$

```
Pt45_id = 4.6689e+05
```

HPT Pressure ratio

$$H\mathrm{PT\ PR} = \frac{P_{\mathrm{t4}}}{P_{\mathrm{t45}}}$$

```
HPT_PR_ID = 2.4959
```

Section 5   Low Pressure Turbine outlet

Total temperature   [K]

$$T_{\mathrm{t5}} = T_{\mathrm{t45}} - \frac{(1 + \mathrm{BPR}) W_F}{(1 + f_B) * c_{\mathrm{pt}}}$$

```
Tt5_id = 973.5456
```

Total pressure [Pa]

$$P_{\mathrm{t5}} = P_{\mathrm{t45}} \left(\frac{T_{\mathrm{t5}}}{T_{\mathrm{t45}}} \right)^{\frac{\mathrm{kt}}{\mathrm{kt}-1}}$$

```
Pt5_id = 1.5735e+05
```

LPT Pressure ratio

$$\mathrm{LPT\ PR} = \frac{P_{\mathrm{t45}}}{P_{\mathrm{t5}}}$$

```
LPT_PR_ID = 2.9673
```

Section 8 Core Engine Nozzle outlet

Total temperature [K]

$$T_{t9} = T_{t5}$$

```
Tt9_id = 973.5456
```

Total pressure [Pa]

$$P_{t9} = P_{t5}$$

```
Pt9_id = 1.5735e+05
```

Static pressure [Pa]

$$P_9 = P_0$$

```
P9_id = 22000
```

Static temperature [K]

$$T_9 = T_{t9} * \left(\frac{P_9}{P_{t9}}\right)^{\frac{\mathrm{kt}-1}{\mathrm{kt}}}$$

```
T9_id = 597.5213
```

Jet stream Mach No

$$M_9 = \sqrt{\left(\frac{T_{t9}}{T_9} - 1\right) * \frac{2}{\mathrm{kt} - 1}}$$

```
M9_id = 1.9529
```

Speed of sound [m/s]

$$a_9 = \sqrt{\mathrm{kt} * \mathrm{Rt} * T_9}$$

```
a9_id = 480.0666
```

Jet speed [m/s]

$$V_9 = M_9 * a_9$$

Section 19 External Engine Nozzle outlet

Total temperature [K]

$$T_{t19} = T_{t13}$$

Total pressure [Pa]

$$P_{t19} = P_{t13}$$

```
Pt19_id = 5.8267e+04
```

Static pressure [Pa]

$$P_{19} = P_0$$

```
P19_id = 22000
```

Static temperature [K]

$$T_{19} = T_{\mathrm{t}19} * \left(\frac{P_{19}}{P_{\mathrm{t}19}}\right)^{\frac{k-1}{k}}$$

```
T19_id = 217.0000
```

Jet stream Mach No

$$M_{19} = \sqrt{\left(\frac{T_{\mathrm{t}19}}{T_{19}} - 1\right) * \frac{2}{k-1}}$$

```
M19_id = 1.2666
```

Speed of sound [m/s]

$$a_{19} = \sqrt{k * R * T_{19}}$$

```
a19_id = 295.2805
```

Jet speed [m/s]

$$V_{19} = M_{19} * a_{19}$$

```
V19_id = 374.0053
```

TURBOFAN ENGINE PERFORMANCE CALCULATION

Thrust [N]

$$T = m_{25} * (1 + f_B) * V_9 + m_{25} * \mathrm{BPR} * V_{19} - m_{25} * (1 + \mathrm{BPR}) * V_0$$

```
T_id = 1.7293e+04
```

Specific thrust [Ns/kg]

$$\mathrm{ST} = \frac{T}{m_0} = \frac{(1 + f_B) * V_9 + \mathrm{BPR} * V_{19} - (1 + \mathrm{BPR}) * V_0}{1 + \mathrm{BPR}}$$

```
ST_id = 172.9334
```

Specific fuel consumption [kg/N/s]

$$\mathrm{SFC} = \frac{m_{\mathrm{fB}}}{T} = \frac{f_B}{(1 + \mathrm{BPR}) * \mathrm{ST}}$$

```
SFC_id = 1.4934e-05
```

Specific fuel consumption [kg/N/h]

$$\mathrm{SFC} = \mathrm{SFC} * 3600$$

```
SFC_id = 0.0538
```

Thermal efficiency

$$\eta_{\mathrm{th}} = \frac{(1+f_B) * V_9{}^2 + \mathrm{BPR} * V_{19}{}^2 - (1+\mathrm{BPR})V_0{}^2}{2 * f_B * \mathrm{FHV}}$$

```
etha_th_id = 0.6688
```

Propulsive efficiency

$$\eta_p = \frac{2 * V_0 * \mathrm{ST} * (1+\mathrm{BPR})}{(1+f_B) * V_9{}^2 + \mathrm{BPR} * V_{19}{}^2 - (1+\mathrm{BPR})V_0{}^2}$$

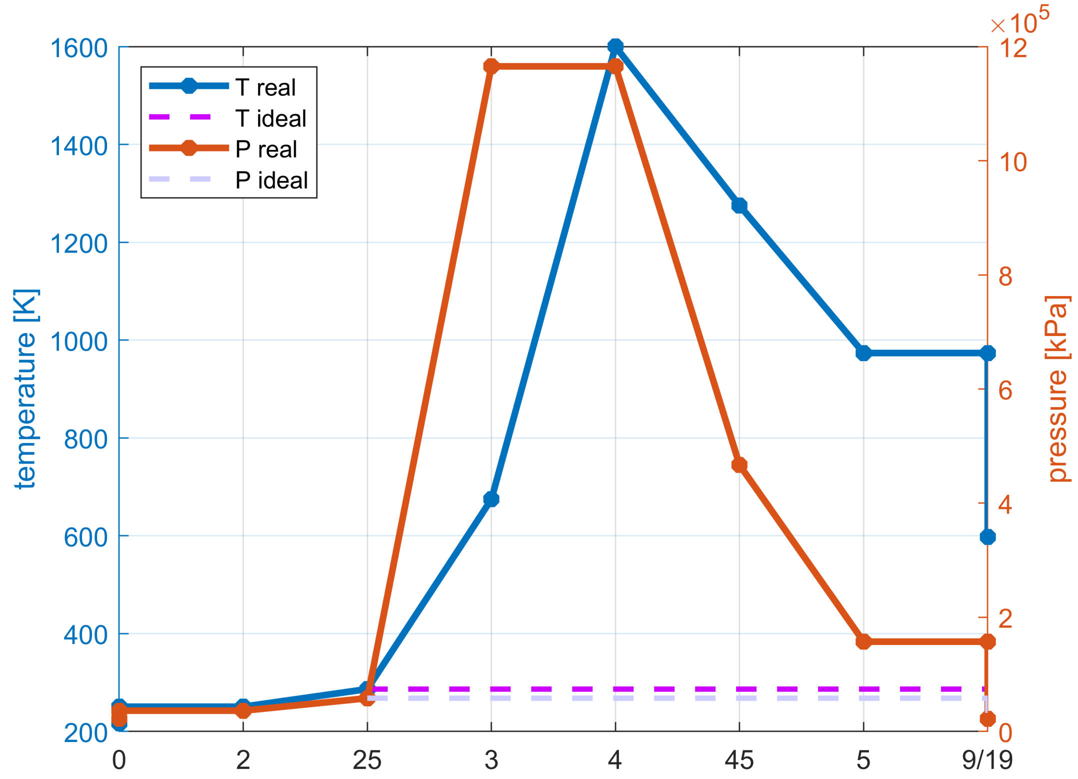
```
etha_p_id = 0.6050
```

Overall efficiency

$$\eta_o = \frac{V_0 * \mathrm{ST} * (1+\mathrm{BPR})}{f_B * \mathrm{FHV}} = \eta_{\mathrm{th}} * \eta_p$$

```
etha_o_id = 0.4047
```

Temperature, pressure vs engine sections plot



```
Tabela = 12×3 table
```

	Section	T ideal [K]	P ideal [kPa]
1	'0'	217	22

	Section	T ideal [K]	P ideal [kPa]
2	't0'	251	36.4168
3	't2'	251	36.4168
4	't25'	287	58.2669
5	't3'	675	1.1653e+03
6	't4'	1600	1.1653e+03
7	't45'	1275	466.8916
8	't5'	974	157.3471
9	't9'	974	157.3471
10	'9'	598	22
11	't19'	287	58.2669
12	'19'	217	22

Performance of ideal turbofan engine

```
Tabela = 9×3 table
```

	Parameter	Unit	Ideal turbofan
1	'Thrust'	'kN'	17.2933
2	'Specific Thrust'	'N*s/kg'	172.9334
3	'Fuel consumption'	'kg/s'	0.2583
4	'Specific fuel consump'	'kg/N/h'	0.0538
5	'therm. efficiency'	'-'	0.6688
6	'prop. efficiency'	'-'	0.6050
7	'overall efficiency'	'-'	0.4047
8	'V9'	'm/s'	937.5422
9	'V19_id'	'm/s'	374.0053

CONCLUSIONS

In the ideal turbofan

- Temperature and pressure rise in the fan and compressor and drop in turbines is isentropic
- Specific thrust is lower than in the turbojet engine (see turbojet engine calculation)
- Higher temperature after compressor causes lower fuel consumption of the real engine - TIT (turbine inlet temperature) is the same in both cases
- Lower total temperature in the nozzle inlet and higher static temperature in the nozzle outlet causes lower outlet flow velocity and by this way lower thrust and specific thrust of real jet engine
- Specific fuel consumption is higher due to lwer thrust and thermal and overall efficiences are lower in real engine

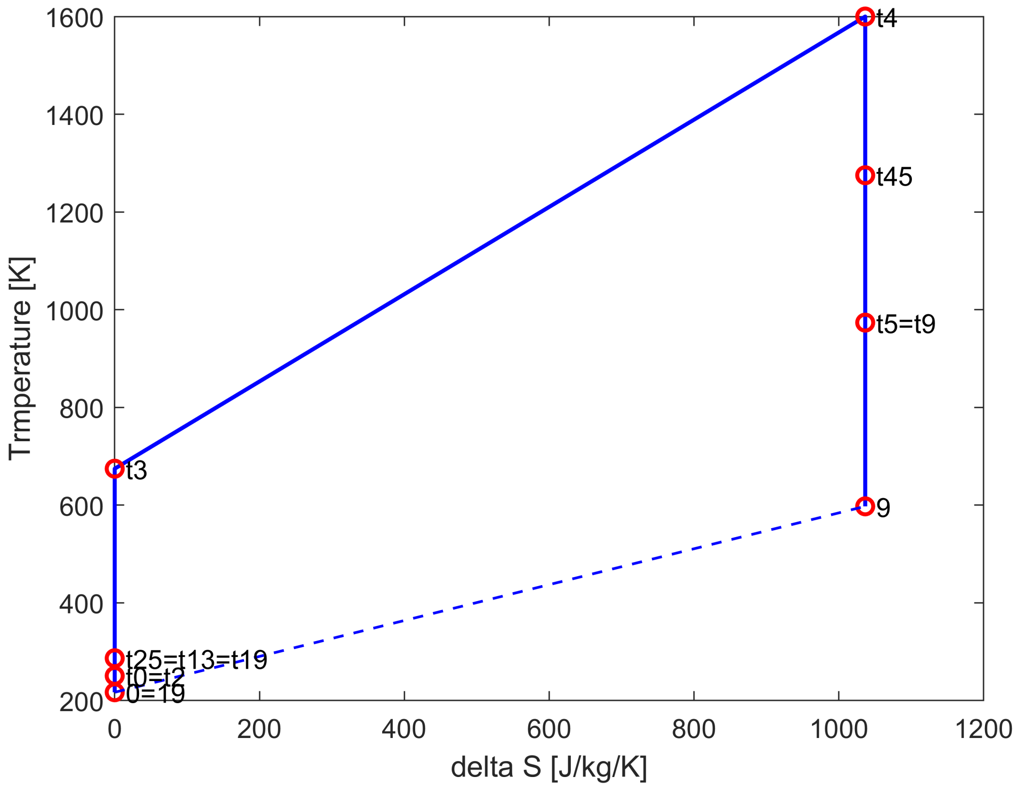
Temperature - entropy plot

Entropy growth is calculated from equations:

Combustor entropy grow   [J/kg/K] :

$$\Delta s_B = c_{\mathrm{pB}} * \ln \frac{T_{\mathrm{t4}}}{T_{\mathrm{t3}}}$$

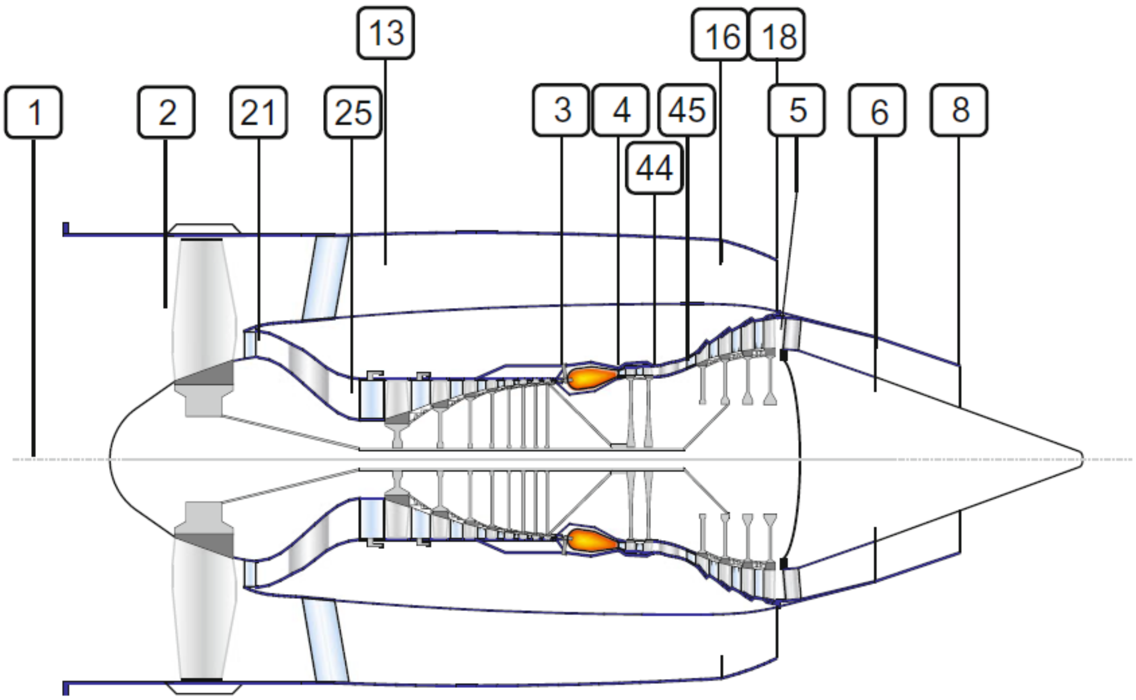
```
dS_B = 1.0364e+03
```



CONCLUSIONS

TURBOFAN ENGINE WITH LOSSES

Calculation of a turbofan engin with losses of the same work parameters like an ideal turbofan. The assumption is that an external and internal engine nozzles expanison is to ambient pressur. The losses coefcients and efficiences are specified below.



Given

Flight conditions: T0=217 K, P0=22 kPa, M0=0.88, bypass ratio 9, fan pressure ratio 1.6, compressor pressure ratio 20, Turbine inlet temperature Tt4=1600 K, mass flow m=100 kg/s.

Gas parameters:

Air: k=1.4; cp=1005 J/kg/K, R=287 J/kg/K,

Fumes in turbine and nozzle kt=1.33, cpt=1170 J/kg/K, Rt=290 J/kg/K,

For combustion in combustor cpB=1200 J/kg/K,

Fuel heat value: FHV=43 MJ/kgGiven

Engine losses coefficients:

inlet pressure losses coefficient σ_{IN} 0.98, burner pessure losses coefficient σ_B 0.98, internal nozzle pressure losses coefficient $\sigma_{N\,\mathrm{int}}$ 0.97, external nozzle pressure losses coefficient $\sigma_{N\,\mathrm{ext}}$ 0.96, fun efficiency η_F 0,91, compressor efficeincy η_C 0.83, high pressure turbine (HPT) efficiency η_{HPT} 0.88, low pressure turbine (LPT) efficiency η_{LPT} 0.9, burner efficiency η_B 0.98, mechanical efficiency low pressure spool $\eta_{M\,\mathrm{LP}}$= 0.995 ,mechanical efficiency high pressure spool $\eta_{M\,\mathrm{HP}}$= 0.99.

Flight Mach No

```
M0 = 0.8800
```

Air Mass flow [kg/s]

```
m0 = 100
```

Bypass ratio

```
BPR = 9
```

Turbine inlet temperature [K]

```
Tt4 = 1600
```

Fan pressure ratio

```
FPR = 1.6000
```

Compressor pressure ratio

```
CPR = 20
```

Ambient conditions

Static temperature [K]

```
T0 = 217
```

Static pressure [Pa]

```
P0 = 22000
```

```
s_N_ext = 0.9600
```

```
e_MHP = 0.9900
```

TURBOJET ENGINE WITH LOSSES

Section 0

Total temperature [K]

$T_{t0} = T_0\left(1 + \frac{k-1}{2}M_0^2\right)$ - like for ideal engine

```
Tt0 = 250.6090
```

Total pressure [Pa]

$P_{t0} = P_0\left(1 + \frac{k-1}{2}{M_0}^2\right)^{\frac{k}{k-1}}$ - like for ideal engine

```
Pt0 = 3.6417e+04
```

Speed of sound [m/s]

$a_0 = \sqrt{k * R * T_0}$ - like for ideal engine

```
a0 = 295.2805
```

Flight speed [m/s]

$V_0 = M_0 * a_0$ - like for ideal engine

```
V0 = 259.8469
```

Section 2 Compressor inlet

Total temperature [K]

$T_{t2} = T_{t0}$ - like for ideal engine

```
Tt2 = 250.6090
```

Total pressure [Pa]

$$P_{t2} = \sigma_{\mathrm{IN}} * P_{t0}$$

```
Pt2 = 3.5688e+04
```

Section 25/13 - Fan outlet

Total temperature [K]

$$T_{t25} = T_{t13} = T_{t2} * \left(1 + \frac{\mathrm{FPR}^{\frac{k-1}{k}} - 1}{\eta_F}\right)$$

```
Tt25 = 290.1889
```

```
Tt13 = 290.1889
```

Total pressure [Pa]

$$P_{t25} = P_{t13} = P_{t2} * \mathrm{FPR}$$

```
Pt25 = 5.7102e+04
```

FAN

Fan work [J/kg]

$$W_F = c_p * (T_{t25} - T_{t2})$$

```
WF = 3.9778e+04
```

Fan power [W]

$$P_F = m_p * W_F$$

```
PF = 3.9778e+06
```

STREAM SPLIT

Internal duct mass flow [kg/s]

$$m_{25} = m_0 * \frac{1}{1 + \mathrm{BPR}}$$

```
m25 = 10
```

External duct mass flow [kg/s]

$$m_{13} = m_0 * \frac{\mathrm{BPR}}{1 + \mathrm{BPR}}$$

```
m13 = 90
```

INTERNAL DUCT / CORE ENGINE

Section 3 - Compressor outlet / Burner inlet

Total temperature   [K]

$$T_{\mathrm{t3}} = T_{\mathrm{t25}} * \left(1 + \frac{\mathrm{CPR}^{\frac{k-1}{k}} - 1}{\eta_C}\right)$$

```
Tt3 = 763.4229
```

Total pressure   [Pa]

$$P_{\mathrm{t3}} = P_{\mathrm{t25}} * \mathrm{CPR}$$

```
Pt3 = 1.1420e+06
```

COMPRESSOR

Compressor work   [J/kg]

$$W_C = c_p * (T_{\mathrm{t3}} - T_{\mathrm{t25}})$$

```
WC = 4.7560e+05
```

Compressor power   [W]

$$P_C = m_{25} * W_C$$

```
PC = 4.7560e+06
```

Section 4 Burner outlet / Turbine inlet

Total temperature   [K]

$$T_{\mathrm{t4}}$$

```
Tt4 = 1600
```

Total pressure   [Pa]

$$P_{\mathrm{t4}} = \sigma_B * P_{\mathrm{t3}}$$

```
Pt4 = 1.1192e+06
```

BURNER

Fuel-air ratio

$$f_B = c_{\mathrm{pB}} * \frac{T_{\mathrm{t4}} - T_{\mathrm{t3}}}{\mathrm{FHV} * \eta_B}$$

```
fB = 0.0238
```

Fuel mass flow   [kg/s]

$$m_{\mathrm{fB}} = m_{25} * f_B$$

```
mfB = 0.2382
```

Section 45 High Pressure Turbine (HPT) outlet / Low Pressure Turbine (LPT) inlet

Total temperature [K]

$$T_{\mathrm{t45}} = T_{\mathrm{t4}} - \frac{W_C}{\eta_{M\ \mathrm{HP}} * (1 + f_B) * c_{\mathrm{pt}}}$$

```
Tt45 = 1.1990e+03
```

Total pressure [Pa]

$$P_{\mathrm{t45}} = P_{\mathrm{t4}} \left(\frac{\eta_{T\ \mathrm{HP}} + \frac{T_{\mathrm{t45}}}{T_{\mathrm{t4}}} - 1}{\eta_T} \right)^{\frac{\mathrm{kt}}{\mathrm{kt}-1}}$$

```
Pt45 = 2.8981e+05
```

High pressure turbine pressure ratio

$$H\mathrm{PT\ PR} = \frac{P_{\mathrm{t4}}}{P_{\mathrm{t45}}}$$

```
HPT_PR = 3.8618
```

Section 5 LPT outlet / Nozzle inlet

Total temperature [K]

$$T_{\mathrm{t5}} = T_{\mathrm{t45}} - \frac{(1 + \mathrm{BPR}) W_F}{\eta_{M\ \mathrm{LP}} * (1 + f_B) * c_{\mathrm{pt}}}$$

```
Tt5 = 865.2132
```

Total pressure [Pa]

$$P_{\mathrm{t5}} = P_{\mathrm{t45}} \left(\frac{\eta_{\mathrm{LPT}} + \frac{T_{\mathrm{t5}}}{T_{\mathrm{t45}}} - 1}{\eta_{\mathrm{LPT}}} \right)^{\frac{\mathrm{kt}}{\mathrm{kt}-1}}$$

```
Pt5 = 6.5228e+04
```

Low pressure turbine pressure ratio

$$\mathrm{LPT\ PR} = \frac{P_{\mathrm{t45}}}{P_{\mathrm{t5}}}$$

```
LPT_PR = 4.4430
```

Section 9 Core engine nozzle outlet / Internal duct nozzle outlet

Total temperature [K]

$T_{t9} = T_{t5}$

```
Tt9 = 865.2132
```

Total pressure [Pa]

$P_{t9} = P_{t5} * \sigma_{N\,\mathrm{int}}$

```
Pt9 = 6.3271e+04
```

Static pressure [Pa]

$P_9 = P_0$

```
P9 = 22000
```

Static temperature [K]

$$T_9 = T_{t9} * \left(\frac{P_9}{P_{t9}}\right)^{\frac{\mathrm{kt}-1}{\mathrm{kt}}}$$

```
T9 = 665.7165
```

Jet stream Mach No

$$M_9 = \sqrt{\left(\frac{T_{t9}}{T_9} - 1\right) * \frac{2}{\mathrm{kt} - 1}}$$

```
M9 = 1.3477
```

Speed of sound [m/s]

$a_9 = \sqrt{\mathrm{kt} * \mathrm{Rt} * T_9}$

```
a9 = 506.7217
```

Jet speed [m/s]

$V_9 = M_9 * a_9$

```
V9 = 682.8899
```

Section 19 External Engine Nozzle outlet

Total temperature [K]

$T_{t19} = T_{t13}$

Total pressure [Pa]

$P_{t19} = \sigma_{N\,\mathrm{ext}} * P_{t13}$

```
Pt19 = 5.4817e+04
```

Static pressure [Pa]

$P_{19} = P_0$

```
P19 = 22000
```

Static temperature [K]

$$T_{19} = T_{t19} * \left(\frac{P_{19}}{P_{t19}}\right)^{\frac{k-1}{k}}$$

```
T19 = 223.5610
```

Jet stream Mach No

$$M_{19} = \sqrt{\left(\frac{T_{t19}}{T_{19}} - 1\right) * \frac{2}{k-1}}$$

```
M19 = 1.2207
```

Speed of sound [m/s]

$$a_{19} = \sqrt{k * R * T_{19}}$$

```
a19 = 299.7112
```

Jet speed [m/s]

$$V_{19} = M_{19} * a_{19}$$

```
V19 = 365.8624
```

TURBOFAN ENGINE PERFORMANCE CALCULATION

Thrust [N]

$$T = m_{25} * (1 + f_B) * V_9 + m_{25} * \mathrm{BPR} * V_{19} - m_{25} * (1 + \mathrm{BPR}) * V_0$$

```
T = 1.3935e+04
```

Specific thrust [Ns/kg]

$$\mathrm{ST} = \frac{T}{m_0} = \frac{(1 + f_B) * V_9 + \mathrm{BPR} * V_{19} - (1 + \mathrm{BPR}) * V_0}{1 + \mathrm{BPR}}$$

```
ST = 139.3452
```

Specific fuel consumption [kg/N/s]

$$\mathrm{SFC} = \frac{m_{\mathrm{fB}}}{T} = \frac{f_B}{(1 + \mathrm{BPR}) * \mathrm{ST}}$$

```
SFC = 1.7096e-05
```

Specific fuel consumption [kg/N/h]

$$\mathrm{SFC} = \mathrm{SFC} * 3600$$

```
SFC = 0.0615
```

Thermal efficiency

$$\eta_{\mathrm{th}} = \frac{(1+f_B)*V_9{}^2 + \mathrm{BPR}*V_{19}{}^2 - (1+\mathrm{BPR})*V_0{}^2}{2*f_B*\mathrm{FHV}}$$

```
etha_th = 0.4915
```

Propulsive efficiency

$$\eta_p = \frac{2*V_0*\mathrm{ST}*(1+\mathrm{BPR})}{(1+f_B)*V_9{}^2 + \mathrm{BPR}*V_{19}{}^2 - (1+\mathrm{BPR})*V_0{}^2}$$

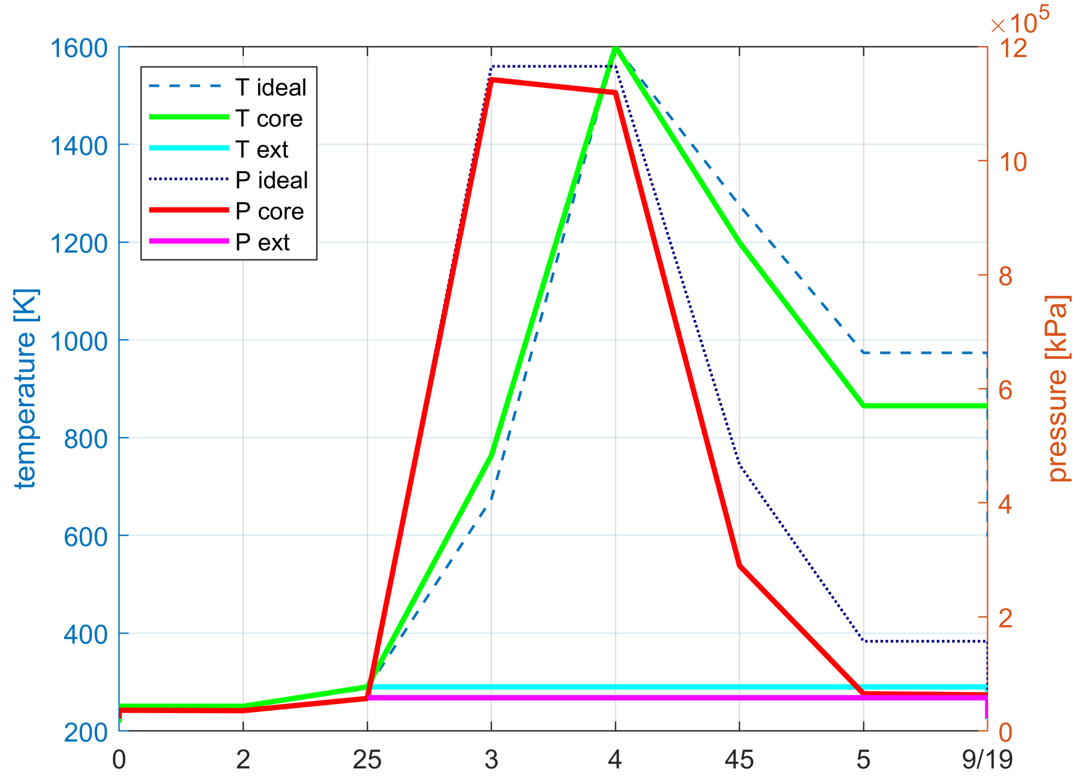
```
etha_p = 0.7192
```

Overall efficiency

$$\eta_o = \frac{V_0*\mathrm{ST}*(1+\mathrm{BPR})}{f_B*\mathrm{FHV}} = \eta_{\mathrm{th}}*\eta_p$$

```
etha_o = 0.3535
```

Temperature, pressure vs engine sections plot



```
Tabela = 12×5 table
```

	Section	T ideal [K]	T real [K]	P ideal [kPa]	P real [kPa]
1	'0'	217	217	22	22
2	't0'	251	251	36.4168	36.4168
3	't2'	251	251	36.4168	35.6885
4	't25'	287	290	58.2669	57.1015

	Section	T ideal [K]	T real [K]	P ideal [kPa]	P real [kPa]
5	't3'	675	763	1.1653e+03	1.1420e+03
6	't4'	1600	1600	1.1653e+03	1.1192e+03
7	't45'	1275	1199	466.8916	289.8122
8	't5'	974	865	157.3471	65.2283
9	't9'	974	865	157.3471	63.2714
10	'9'	598	666	22	22
11	't19'	287	290	58.2669	54.8175
12	'19'	217	224	22	22

Performance comparison of real vs ideal turbofan engine

```
Tabela = 11×4 table
```

	Parameter	Unit	Ideal	real
1	'Thrust'	'kN'	17.2933	13.9345
2	'Specific Thrust'	'N*s/kg'	172.9334	139.3452
3	'Fuel consumption'	'kg/s'	0.2583	0.2382
4	'Specific fuel consump'	'kg/N/h'	0.0538	0.0615
5	'therm. efficiency'	'-'	0.6688	0.4915
6	'prop. efficiency'	'-'	0.6050	0.7192
7	'overall efficiency'	'-'	0.4047	0.3535
8	'V9'	'm/s'	937.5422	682.8899
9	'V19'	'm/s'	374.0053	365.8624
10	'HPT_PR'	'-'	2.4959	3.8618
11	'LPT_PR'	'-'	2.9673	4.4430

CONCLUSIONS

Real to ideal jet engine comparison shows

- Total pressure in engine sections is lower in real engine
- Total temperature after compressor is higher, but after turbine is lower in real engine
- Higher temperature after compressor causes lower fuel consumption of the real engine - TIT (turbine inlet temperature) is the same in both cases
- Lower total temperature in the nozzle inlet and higher static temperature in the nozzle outlet causes lower outlet flow velocity and by this way lower thrust and specific thrust of real jet engine
- Specific fuel consumption is higher due to lwer thrust and thermal and overall efficiences are lower in real engine

Temperature - entropy plot

Entropy growth is calculated from equations:

Inlet entropy grow   [J/kg/K] :

$$\Delta s_{\mathrm{IN}} = -R * \ln(\sigma_{\mathrm{IN}})$$

```
dS_IN = 5.7982
```

Fan entropy grow   [J/kg/K] :

$$\Delta s_F = c_p * \ln \frac{T_{\mathrm{t25}}}{T_{\mathrm{t2}}} - R * \ln(\mathrm{FPR})$$

```
dS_F = 12.4803
```

Compressor entropy grow   [J/kg/K] :

$$\Delta s_C = c_p * \ln \frac{T_{\mathrm{t3}}}{T_{\mathrm{t25}}} - R * \ln(\mathrm{CPR})$$

```
dS_C = 112.3414
```

Combustor entropy grow   [J/kg/K] :

$$\Delta s_B = c_{\mathrm{pB}} * \ln \frac{T_{\mathrm{t4}}}{T_{\mathrm{t3}}} - R_t * \ln(\sigma_B)$$

```
dS_B = 893.7950
```

HPT entropy grow   [J/kg/K] :

$$\Delta s_{\mathrm{HPT}} = c_{\mathrm{pt}} * \ln \frac{T_{\mathrm{t45}}}{T_{\mathrm{t4}}} - R_t * \ln\left(\frac{P_{\mathrm{t45}}}{P_{\mathrm{t4}}}\right)$$

```
dS_HPT = 54.2168
```

LPT entropy grow   [J/kg/K] :

$$\Delta s_{\mathrm{LPT}} = c_{\mathrm{pt}} * \ln \frac{T_{\mathrm{t5}}}{T_{\mathrm{t4}}5} - R_t * \ln\left(\frac{P_{\mathrm{t5}}}{P_{\mathrm{t45}}}\right)$$

```
dS_LPT = 50.8027
```

Core nozzle entropy grow   [J/kg/K] :

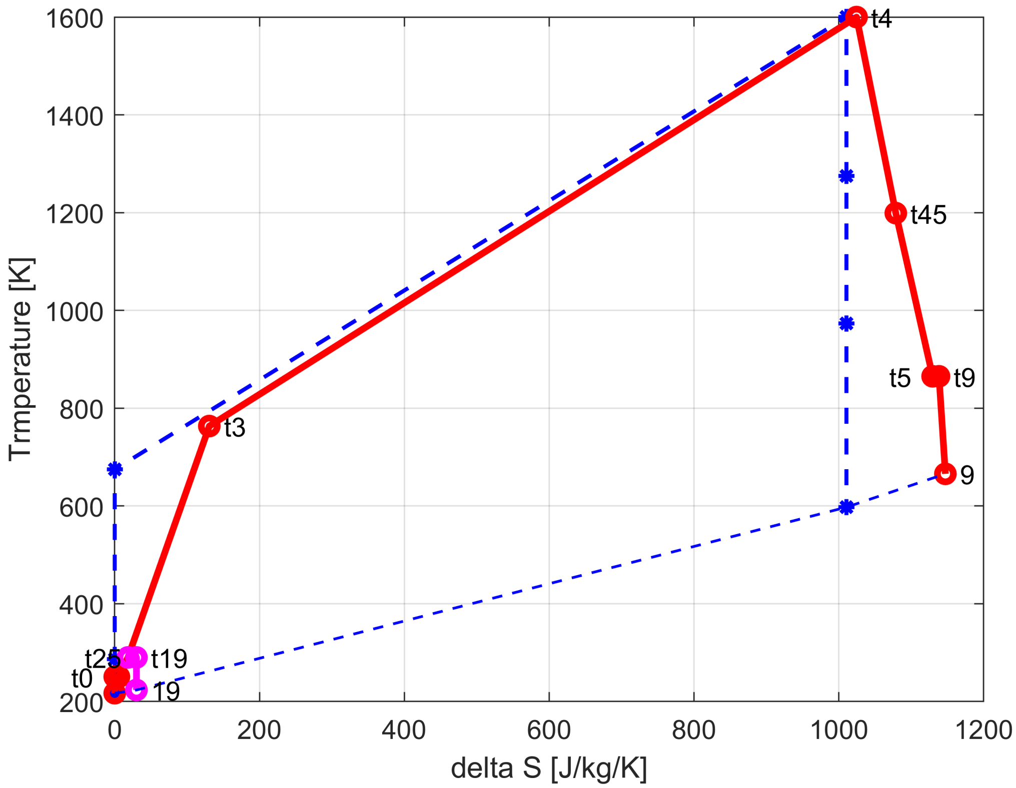
$$\Delta s_{N\ \mathrm{int}} = -\mathrm{Rt} * \ln(\sigma_{N\ \mathrm{int}})$$

```
dS_N_int = 8.8332
```

External nozzle entropy grow   [J/kg/K] :

$$\Delta s_{N\ \mathrm{ext}} = -R * \ln(\sigma_{N\ \mathrm{ext}})$$

```
dS_N_ext = 11.7159
```



TURBOFAN ENGINE WITH CONVERGENT NOZZLES

Maximum possible gas expansion in convergent nozzle is to critical pressure - speed of jet stream is equal speed of sound (M=1). Engine calculation for other components than nozzle looks like for the engine with full expansion in the nozzle. Difference starts from nozzle paramethers calculation therefore the example presented below refers to the real engine calculation presented above.

Example of turbofan engine calculation with convergent nozzles of internal and external duct for flight condition and engine work parameters like in example above.

All paramethers calculated to total section t19 and t9 are like above. Difference starts from static parameters calculation in both sections.

Internal nozzle calculatios:

Total temperature in section 9

```
Tt9 = 865.2132
```

Total pressure in section 9

```
Pt9 = 6.3271e+04
```

Total to critical presure paramether calculation:

$$\beta = \left(\frac{k_t + 1}{2}\right)^{\frac{k_t}{k_t - 1}}$$

```
beta = 1.8506
```

Total pressure in internal nozzle to static ambient pressure calculation:

$$\frac{P_{\mathrm{t9}}}{P_0} =$$

```
ans = 2.8760
```

Pt9/P0 is higher than β then full expansion in the convergent nozzle isn't possible. The critical pressure is available in the nozzle only (IE indext - incomplete expansion):

$$P_{9\,\mathrm{IE}} = \frac{P_{\mathrm{t9}}}{\beta} \quad \text{[Pa]}$$

```
P9_IE = 3.4190e+04
```

Static pressure in the nozzle exit

$$T_{9\,\mathrm{IE}} = \frac{2 * T_{\mathrm{t9}}}{k_t + 1} \quad \text{[K]}$$

```
T9_IE = 742.6723
```

Secrion 9 density calculation:

$$\rho_{9\,\mathrm{IE}} = \frac{P_{9\,\mathrm{IE}}}{R_t * T_{9\,\mathrm{IE}}}$$

```
rho_9_IE = 0.1587
```

Jet speed in the propelling nozzle exit

$$V_{9\,\mathrm{IE}} = a_9 = \sqrt{k_t * R_t * T_{9\,\mathrm{IE}}} \quad \text{[m/s]}$$

```
V9_IE = 535.2090
```

Speed of gas streem after expansion to ambient pressure outside the nozzle:

$$V_{9e} = V_{9\,\mathrm{IE}} + \frac{(P_{9\,\mathrm{IE}} - P_0)}{\rho_{9\,\mathrm{IE}} * V_{9\,\mathrm{IE}}}$$

```
V9e = 678.6810
```

Static temperaure in 9e

$$P_{9e} = P_0$$

```
P9e = 22000
```

Static temperature in section 9e

$$T_{9e} = T_{\mathrm{t9}} - \frac{V_{9e}^2}{2 * \mathrm{cp}_t}$$

```
T9e = 668.3722
```

External nozzle calculatios:

Total temperature in section 19

```
Tt19 = 290.1889
```

Total pressure in section 19

```
Pt19 = 5.4817e+04
```

Total to critical presure paramether calculation in external nozzle (air):

$$\beta = \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}$$

```
beta = 1.8929
```

Total pressure in internal nozzle to static ambient pressure calculation:

$$\frac{P_{\mathrm{t19}}}{P_0} =$$

```
ans = 2.4917
```

Pt19/P0 is higher than β then full expansion in the convergent nozzle isn't possible. The critical pressure is available in the external nozzle only (IE indext - incomplete expansion):

$$P_{19\,\mathrm{IE}} = \frac{P_{\mathrm{t19}}}{\beta} \quad \text{[Pa]}$$

```
P19_IE = 2.8959e+04
```

Static pressure in the nozzle exit

$$T_{19\,\mathrm{IE}} = \frac{2 * T_{\mathrm{t19}}}{k+1} \quad \text{[K]}$$

```
T19_IE = 241.8240
```

Secrion 9 density calculation:

$$\rho_{19\,\mathrm{IE}} = \frac{P_{19\,\mathrm{IE}}}{R * T_{19\,\mathrm{IE}}}$$

```
rho_19_IE = 0.4173
```

Jet speed in the propelling nozzle exit

$$V_{19\,\mathrm{IE}} = a_{19} = \sqrt{k * R * T_{19\,\mathrm{IE}}} \quad \text{[m/s]}$$

```
V19_IE = 311.7129
```

Speed of gas streem after expansion to ambient pressure outside the nozzle:

$$V_{19e} = V_{19\,\mathrm{IE}} + \frac{(P_{19\,\mathrm{IE}} - P_0)}{\rho_{19\,\mathrm{IE}} * V_{19\,\mathrm{IE}}}$$

```
V19e = 365.2177
```

Static temperaure in 9e

$$P_{19e} = P_0$$

```
P19e = 22000
```

Static temperature in section 9e

$$T_{19e} = T_{t19} - \frac{V_{19}^2}{2 * \mathrm{cp}}$$

```
T19e = 223.8287
```

Performance parameters calculation for incomplete expansion in propelling nozzles

Thrust [N]

$$T = m_{25} * (1 + f_B) * V_{9e} + m_{25} * \mathrm{BPR} * V_{19e} - m_{25} * (1 + \mathrm{BPR}) * V_0$$

```
T_IE = 1.3833e+04
```

Specific thrust [Ns/kg]

$$\mathrm{ST} = \frac{T}{m_0} = \frac{(1 + f_B) * V_{9e} + \mathrm{BPR} * V_{19e} - (1 + \mathrm{BPR}) * V_0}{1 + \mathrm{BPR}}$$

```
ST_IE = 138.3340
```

Specific fuel consumption [kg/N/s]

$$\mathrm{SFC} = \frac{m_{\mathrm{fB}}}{T} = \frac{f_B}{(1 + \mathrm{BPR}) * \mathrm{ST}}$$

```
SFC_IE = 1.7221e-05
```

Specific fuel consumption [kg/N/h]

$$\mathrm{SFC} = \mathrm{SFC} * 3600$$

```
SFC_IE = 0.0620
```

Thermal efficiency

$$\eta_{\mathrm{th}} = \frac{(1 + f_B) * {V_{9e}}^2 + \mathrm{BPR} * {V_{19e}}^2 - (1 + \mathrm{BPR}) * {V_0}^2}{2 * f_B * \mathrm{FHV}}$$

```
etha_th_IE = 0.4866
```

Propulsive efficiency

$$\eta_p = \frac{2 * V_0 * \mathrm{ST} * (1 + \mathrm{BPR})}{(1 + f_B) * {V_{9e}}^2 + \mathrm{BPR} * {V_{19e}}^2 - (1 + \mathrm{BPR}) * {V_0}^2}$$

```
etha_p_IE = 0.7212
```

Overall efficiency

$$\eta_o = \frac{V_0 * \mathrm{ST} * (1+\mathrm{BPR})}{f_B * \mathrm{FHV}} = \eta_{\mathrm{th}} * \eta_p$$

```
etha_o_IE = 0.3509
```

Performance comparisson of full expansion turbofan and turbofan with convergent nozzle

```
Tabela = 11×4 table
```

	Parameter	Unit	CONV Nozzles	FULL EXPANS
1	'Thrust'	'kN'	13.8334	13.9345
2	'Specific Thrust'	'N*s/kg'	138.3340	139.3452
3	'Fuel consumption'	'kg/s'	0.2382	0.2382
4	'Specific fuel consump'	'kg/N/h'	0.0620	0.0615
5	'therm. efficiency'	'-'	0.4866	0.4915
6	'prop. efficiency'	'-'	0.7212	0.7192
7	'overall efficiency'	'-'	0.3509	0.3535
8	'V9'	'm/s'	535.2090	682.8899
9	'V9e'	'm/s'	678.6810	682.8899
10	'V19'	'm/s'	311.7129	365.8624
11	'V19e'	'm/s'	365.2177	365.8624

CONCLUSIONS:

Incomplete expansion in the engine propelling nozzle causes:

- Lower thrust and specific thrust than it is in full decompression mode
- Higher specific fuel consumption
- Additional entropy increas caused by jet decompression outside the nozzle