TURBOJET ENGINE OFF-DESIGN CALCULATION

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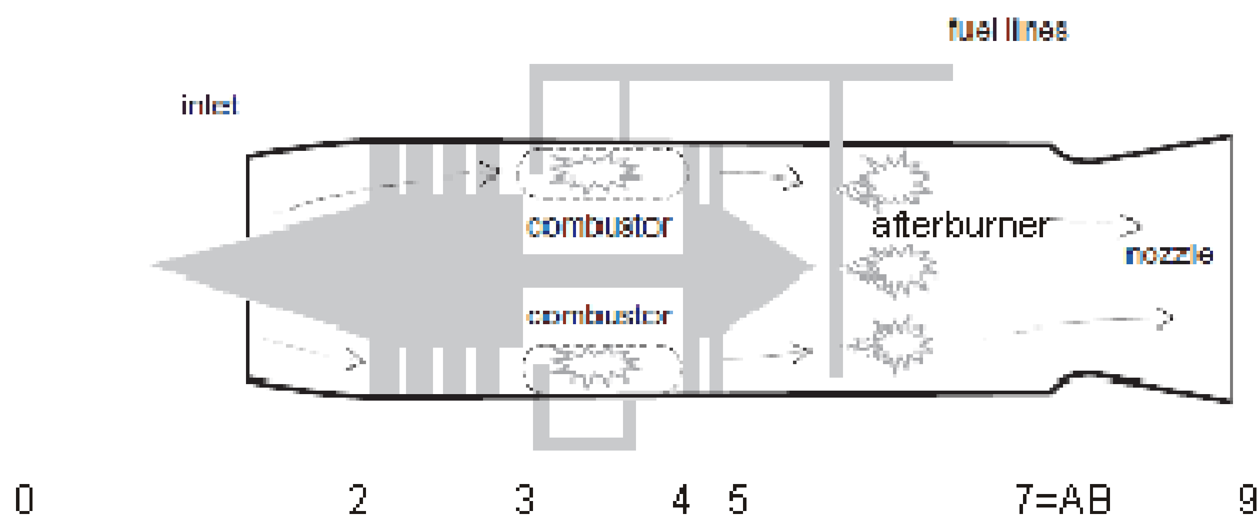
TURBOJET ENGINE with LOSSES

En example of turbojet engin calculation is presented below. Firstly design point (DP) is calculated. OFF-DESIGN calculation is provided next. In the model the engine with convergent nozzle is assumed. Presented example doesn't include AB-ON mode, but it is prepare to extend of AB-ON axmple (could be done in the future)



Given for DP

T0=217 K, P0=22 kPa, M0=0.85, compressor pressure ratio 8, Turbine inlet temperature Tt4=1300 K, mass flow m=10 kg/s. For Afterburner ON: TtAB=1750 K inlet pressure losses coefficient σ_{IN} 0.97, burner pessure losses coefficient σ_B 0.98, nozzle pressure losses coefficient σ_N 0.96, compressor efficeincy η_C 0.83, turbine efficiency η_T 0.9, burner efficiency η_B 0.98, mechanical efficiency η_M= 0.99. For AB ON state afterburner efficiency η_{AB} 0.95, afterburner pressure losses coeficient σ_{AB} 0.96 (for AB OFF assume σ_{AB} 0.975 to include pressure losses caused by injectors and falme holder in AB)

Gas parameters:

Air: k=1.4; cp=1005 J/kg/K, R=287 J/kg/K,

Fumes in turbine and nozzle kt=1.33, cpt=1170 J/kg/K, Rt=290 J/kg/K,

Fumes for Afterburner and nozzle in AB ON mode kAB=1.3, cpAB=1200 J/kg/K, RAB=297 J/kg/K,

For combustion in combustor cpB=1200 J/kg/K,

Fuel heat value: FHV=43 MJ/kg

Flight Mach No

```
M0 = 0.8500
```

Air Mass flow [kg/s]

```
m0 = 10
```

Turbine inlet temperature [K]

```
Tt4 = 1300
```

Compressor pressure ratio

```
CPR = 8
```

Afterburner temperature [K]

```
TtAB = 1750
```

Ambient conditions

Static temperature [K]

```
T0 = 217
```

Static pressure [Pa]

```
P0 = 22000
```

TURBOJET ENGINE DESIGN POINT (DP) CALCULATION

Section 0

Total temperature [K]

$T_{t0} = T_0\left(1 + \frac{k-1}{2} M_0^2\right)$ - like for ideal engine

```
Tt0 = 248.3565
```

Total pressure [Pa]

$P_{t0} = P_0\left(1 + \frac{k-1}{2} M_0^2\right)^{\frac{k}{k-1}}$ - like for ideal engine

```
Pt0 = 3.5284e+04
```

Speed of sound [m/s]

$a_0 = \sqrt{k * R * T_0}$ - like for ideal engine

```
a0 = 295.2805
```

Flight speed [m/s]

$V_0 = M_0 * a_0$ - like for ideal engine

```
V0 = 250.9885
```

Section 2 Compressor inlet

Total temperature [K]

$T_{t2} = T_{t0}$ - like for ideal engine

```
Tt2 = 248.3565
```

Total pressure [Pa]

$P_{t2} = \sigma_{IN} * P_{t0}$

```
Pt2 = 3.4225e+04
```

Section 3 - Compressor outlet / Burner inlet

Total temperature [K]

$$T_{t3} = T_{t2} * \left(1 + \frac{\mathrm{CPR}^{\frac{k-1}{k}} - 1}{\eta_C}\right)$$

```
Tt3 = 491.1616
```

Total pressure [Pa]

$P_{t3} = P_{t2} * \mathrm{CPR}$

```
Pt3 = 2.7380e+05
```

COMPRESSOR

Compressor work [J/kg]

$W_C = c_p * (T_{t3} - T_{t2})$

```
WC = 2.4402e+05
```

Compressor power [W]

$P_C = m_0 * W_C$

```
PC = 2.4402e+06
```

Section 4 Burner outlet / Turbine inlet

Total temperature   [K]

T_{t4}

```
Tt4 = 1300
```

Total pressure   [Pa]

$P_{t4} = \sigma_B * P_{t3}$

```
Pt4 = 2.6833e+05
```

BURNER

Fuel-air ratio

$$f_B = c_{pB} * \frac{T_{t4} - T_{t3}}{\mathrm{FHV} * \eta_B}$$

```
fB = 0.0230
```

Fuel mass flow   [kg/s]

$m_{fB} = m_0 * f_B$

```
mfB = 0.2303
```

Section 4_min Turbine minimum area for critical flow condition

Static temperature for critical flow

$$T_{4\,\mathrm{MIN}} = \frac{2 * T_{t4}}{k_t + 1}$$

```
T4_min = 1.1159e+03
```

Static pressure

$$P_{4\,\mathrm{MIN}} = 0.98\, P_{t4}\left(\frac{2}{k_t + 1}\right)^{\frac{\mathrm{kt}}{\mathrm{kt}-1}}$$

```
P4_min = 1.4209e+05
```

Dencity

$$\rho_{4\,\mathrm{MIN}} = \frac{P_{4\,\mathrm{MIN}}}{R_t T_{4\,\mathrm{MIN}}}$$

```
ro4_min = 0.4391
```

Gass velocity in minimal area:

$$c_{4\,\mathrm{MIN}} = \sqrt{k_t R_t T_{4\,\mathrm{MIN}}}$$

```
c4_min = 656.0449
```

Turbine minimal area

$$A_{4\,\mathrm{MIN}} = \frac{\dot{m}_4}{\rho_{4\,\mathrm{MIN}}\, c_{4\,\mathrm{MIN}}}$$

```
A4_min = 0.0355
```

Section 5 Turbine outlet / Nozzle inlet

Total temperature [K]

$$T_{t5} = T_{t4} - \frac{W_C}{\eta_M * (1 + f_B) * c_{pt}}$$

```
Tt5 = 1.0941e+03
```

Total pressure [Pa]

$$P_{t5} = P_{t4} \left(\frac{\eta_T + \frac{T_{t5}}{T_{t4}} - 1}{\eta_T} \right)^{\frac{kt}{kt-1}}$$

```
Pt5 = 1.2297e+05
```

Turbine Pressure Ratio

$$\mathrm{TPR} = \frac{P_{t4}}{P_{t5}}$$

```
TPR = 2.1820
```

Section 7 Engine Nozzle intlet

Total temperature [K]

$$T_{t7} = T_{t5}$$

```
Tt7 = 1.0941e+03
```

Total pressure [Pa]

$$P_{t7} = P_{t5} * \sigma_{AN}$$

```
Pt7 = 1.1990e+05
```

Section 9 Engine Nozzle outlet

Total temperature [K]

$$T_{t9} = T_{t7}$$

```
Tt9 = 1.0941e+03
```

Total pressure [Pa]

$$P_{t9} = P_{t7} * \sigma_N$$

```
Pt9 = 1.1510e+05
```

Section 9 - incomplete expansion in the nozzle

Critical pressure ratio

$$B_{cr} = \left(\frac{k_t + 1}{2}\right)^{\frac{kt}{kt-1}}$$

```
Bcr = 1.8506
```

Static pressure [Pa]

$$P_{9\,IE} = \frac{P_{t9}}{B_{cr}}$$

```
P9_IE = 6.2198e+04
```

Static temperature [K]

$$T_{9\,IE} = T_{t9} * \left(\frac{P_{9\,IE}}{P_{t9}}\right)^{\frac{kt-1}{kt}}$$

```
T9_IE = 939.1185
```

Mach No. in section 9

$$M_{9\,IE} = \sqrt{\left(\frac{T_{t9}}{T_{9IE}} - 1\right) * \frac{2}{k_t - 1}}$$

```
M9_IE = 1
```

Speed of sound [m/s]

$$a_{9\,IE} = \sqrt{k_t * R_t * T_{9\,IE}}$$

```
a9_IE = 601.8455
```

Jet speed in section 9 [m/s]

$$V_{9\,IE} = M_{9\,IE} * a_{9\,IE}$$

```
V9_IE = 601.8455
```

Gas density in section 9 [kg/m^3]

$$\rho_{9\,IE} = \frac{P_{9\,IE}}{R_t * T_{9\,IE}}$$

```
RO9_IE = 0.2284
```

Nozzle minimal area

$$A_{9\,\mathrm{MIN}} = \frac{\dot{m}_9}{\rho_{9\,\mathrm{IE}} V_{9\,\mathrm{IE}}}$$

```
A9_min = 0.0744
```

Section 9e

Jet speed after decompression to ambient pressure [m/s]

$$V_{9\,e} = V_{9\,\mathrm{IE}} + \frac{P_{9\,\mathrm{IE}} - P_0}{\rho_{9\,\mathrm{IE}} * V_{9\,\mathrm{IE}}}$$

```
V9e = 894.3022
```

Static temperature   [K]

$$T_{9\mathrm{e}} = T_{\mathrm{t}9} - \frac{{V_{9e}}^2}{2 * c_{\mathrm{pt}}}$$

```
T9e = 752.2883
```

Staticl pressure   [Pa]

$$P_{9\,e} = P_0$$

```
P9e = 22000
```

PERFORMANCE OF TURBOJET ENGINE WITH INCOMPLTE EXPANSION IN THE NOZZLE

Thrust   [N]

$$T_{\mathrm{DP}} = m_0 * (1 + f_B) * V_{9e} - m_0 * V_0$$

```
T_DP = 6.6391e+03
```

Specific thrust   [Ns/kg]

$$\mathrm{ST}_{\mathrm{DP}} = \frac{T_{\mathrm{DP}}}{m_0} = (1 + f_B) * V_{9e} - V_0$$

```
ST_DP = 663.9121
```

Specific fuel consumption   [kg/N/s]

$$\mathrm{SFC}_{\mathrm{DP}} = \frac{m_{\mathrm{fB}}}{T_{\mathrm{DP}}}$$

```
SFC_DP = 3.4693e-05
```

Specific fuel consumption   [kg/N/h]

$$\mathrm{SFC}_{\mathrm{DP}} = \mathrm{SFC}_{\mathrm{DP}} * 3600$$

```
SFC_DP = 0.1249
```

Thermal efficiency

$$\eta_{\text{th DP}} = \frac{(1+f_B)*V_{9\,e}{}^2 - V_0{}^2}{2*f_B*\text{FHV}}$$

```
etha_th_DP = 0.3813
```

Propulsive efficiency

$$\eta_{p\,\text{DP}} = \frac{2*V_0*\text{ST}_{\text{DP}}}{(1+f_B)*V_{9\,e}{}^2 - V_0{}^2}$$

```
etha_p_DP = 0.4413
```

Overall efficiency

$$\eta_{o\,\text{DP}} = \frac{V_0*\text{ST}_{\text{DP}}}{f_B*\text{FHV}} = \eta_{\text{th DP}}*\eta_{p\,\text{DP}}$$

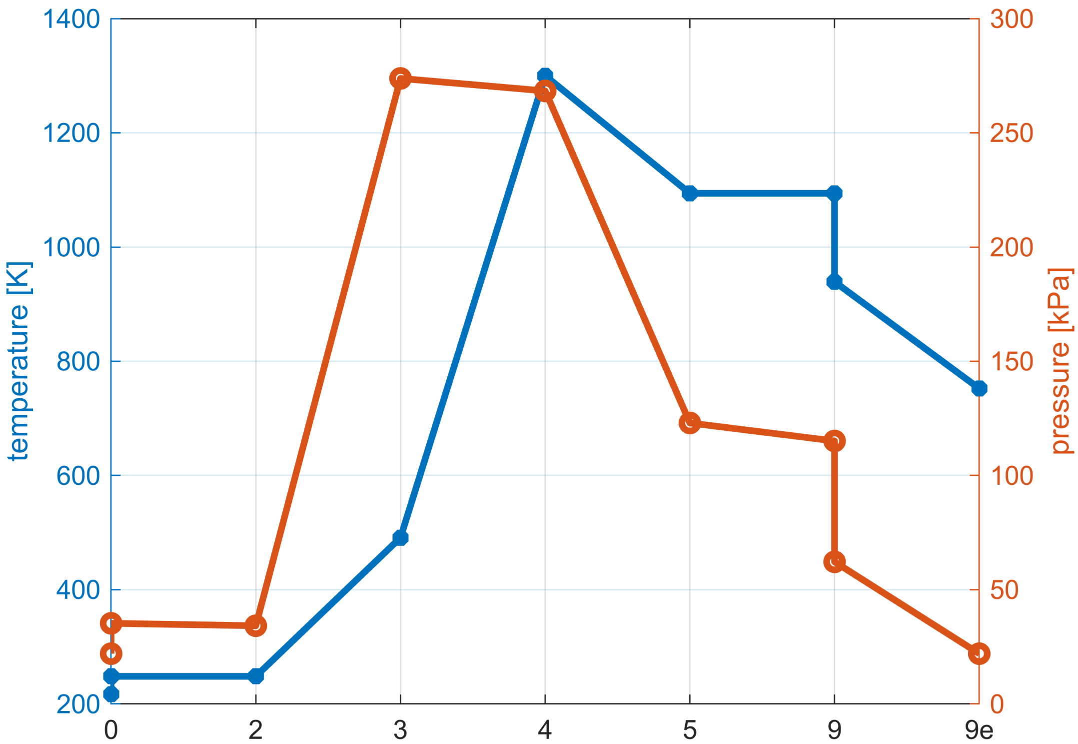
```
etha_o_DP = 0.1682
```

RESULTS PRESENATION

```
Tabela = 13×3 table
```

	Parameter	Unit	CHOCKED NOZZLE
1	'Tt9'	'K'	1.0941e+03
2	'Pt9'	'kPa'	115.1039
3	'T9'	'K'	939.1185
4	'V9'	'm/s'	601.8455
5	'P9'	'kPa'	62.1980
6	'T9e'	'K'	752.2883
7	'V9e'	'm/s'	894.3022
8	'Thrust'	'kN'	6.6391
9	'Specific Thrust'	'N*s/kg'	663.9121
10	'Specific fuel consump'	'kg/N/h'	0.1249
11	'therm. efficiency'	'-'	0.3813
12	'prop. efficiency'	'-'	0.4413
13	'overall efficiency'	'-'	0.1682

Temperature, pressure vs engine sections plot



Temperature - entropy plot

Entropy growth is calculated from equations:

Inlet entropy grow [J/kg/K] :

$$\Delta s_{\mathrm{IN}} = -R * \ln(\sigma_{\mathrm{IN}})$$

```
dS_IN = 8.7418
```

Compressor entropy grow [J/kg/K] :

$$\Delta s_C = c_p * \ln\frac{T_{\mathrm{t3}}}{T_{\mathrm{t2}}} - R * \ln(\mathrm{CPR})$$

```
dS_C = 88.5178
```

Combustor entropy grow [J/kg/K] :

$$\Delta s_B = c_{\mathrm{pB}} * \ln\frac{T_{\mathrm{t4}}}{T_{\mathrm{t3}}} - R_t * \ln(\sigma_B)$$

```
dS_B = 1.1739e+03
```

Turbine entropy grow [J/kg/K] :

$$\Delta s_T = c_{\mathrm{pt}} * \ln\frac{T_{\mathrm{t5}}}{T_{\mathrm{t4}}} - R_t * \ln\left(\frac{P_{\mathrm{t5}}}{P_{\mathrm{t4}}}\right)$$

```
dS_T = 24.4935
```

Afterburner entropy grow for AB OFF [J/kg/K] :

$$\Delta s_{AB} = -Rt * \ln(\sigma_{AB})$$

```
dS_AB = 7.3422
```

Nozzle entropy grow [J/kg/K] :

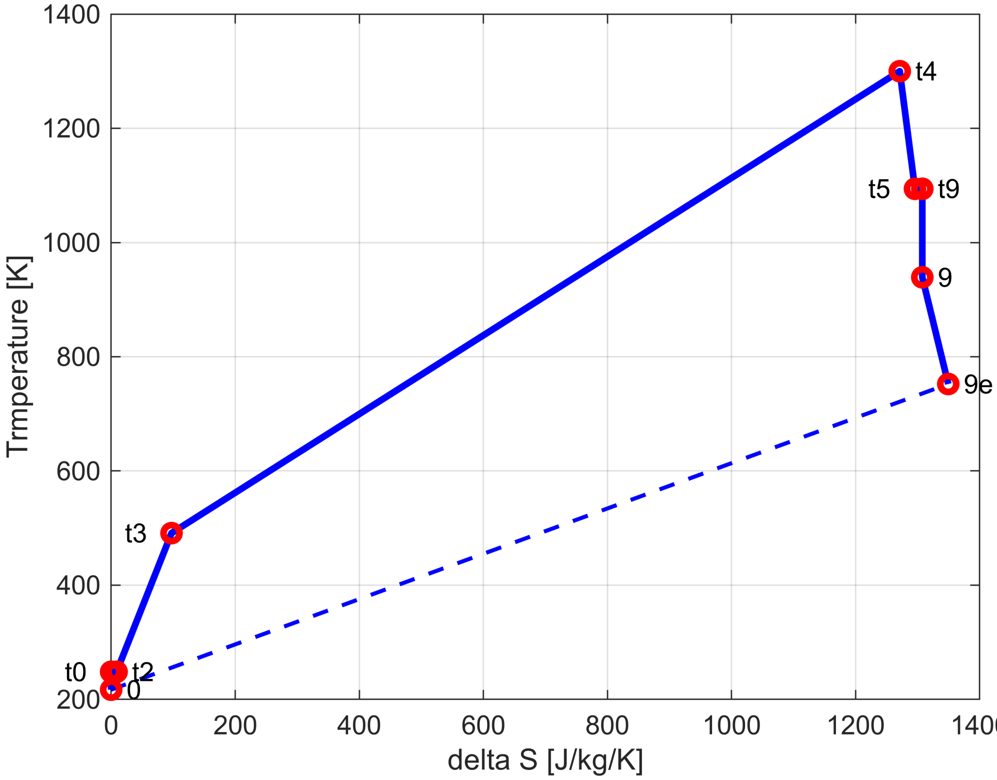
$$\Delta s_N = -Rt * \ln(\sigma_N)$$

```
dS_N = 11.8384
```

Additional entropy growth 9-9e [J/kg/K]

$$\Delta s_{9-9e} = c_{pt} * \ln\frac{T_{9e}}{T_{9\,IE}} - R_t * \ln\frac{P_{9e}}{P_{9\,IE}}$$

```
deltaS_9_9e = 41.8596
```



ENGINE PERFORMANCE CALCULATION FOR VARIOUS PLA SETING (OFF DESIGN)

Example of engine performance calculation for specified PLA - instead of PLA the n/n_DP is specified. In calculation it will be used equation for Tt4 to n relation:

$$\frac{n}{n_{DP}} = \sqrt{\frac{T_{t4}}{T_{t0}}\frac{T_{t0\,DP}}{T_{t4\,DP}}}$$

Additionally TPR=TPR_DP is assumed as constant for constant outlet nozzle area. Engine components efficiencies are assumed as equal in DP state.

Example of performance calculation for the same flight condition as for DP, but for n/n_DP=0.95

Due to the same flight condition all parameters to section 2 are unchanged.

The calculation of the turbocomponent is poroved in loop to match fB (fuel air ratio in two next steps). For that reason in first assumption fb=0 is used. For future calculation the key parameters of DP will be recalled by DP ending.

```
fB_1 = 0
```

TIT off-design

$$T_{t4} = T_{t4\,DP}\left(\frac{n}{n_{DP}}\right)^2$$

```
Tt4 = 1.1733e+03
```

Total temperature in section 5 [K]

$$T_{t5} = T_{t4}\left(1 - \eta_T + \eta_T\left(\frac{1}{\mathrm{TPR}}\right)^{\frac{\mathrm{kt}-1}{\mathrm{kt}}}\right)$$

```
Tt5 = 987.4009
```

Turbine Work [J] calculated according mass flow in section 4

$$W_T = c_{pt}(T_{t4} - T_{t5})$$

```
WT = 2.1744e+05
```

Section 3 - Compressor outlet

Total temperature   [K]

$$T_{t3} = T_{t2} + \frac{(1 + f_B)W_T\eta_m}{c_p}$$

```
Tt3 = 462.5545
```

BURNER

Fuel-air ratio

$$f_B = c_{pB} * \frac{T_{t4} - T_{t3}}{\mathrm{FHV} * \eta_B}$$

```
fB = 0.0202
```

Loop calculation starts

```
while abs(fB-fB_1)/fB>1e-5
fB_1=fB;
```

Section 3 - Compressor outlet

Total temperature [K]

$$T_{t3} = T_{t2} + \frac{\eta_m(1+f_B)W_T}{c_p}$$

BURNER

Fuel-air ratio

$$f_B = c_{pB} * \frac{T_{t4} - T_{t3}}{\mathrm{FHV} * \eta_B}$$

End of loop calculation:

```
end
```

fB and fB_1 comparison:

```
fB = 0.0201
fB_1 = 0.0201
```

fB is equal fB_1 therefore loop calculation is finished

Compressor pressure ratio calculation CPR

CPR and total pressure of the compressor outlet [Pa]

$$\mathrm{CPR} = \left(1 + \eta_C\left(\frac{T_{t3}}{T_{t2}} - 1\right)\right)^{\frac{k}{k-1}}$$

```
CPR = 6.8136
```

$$P_{t3} = P_{t2} * \mathrm{CPR}$$

```
Pt3 = 2.3320e+05
```

COMPRESSOR

Section 4 Burner outlet / Turbine inlet

Total pressure [Pa]

$$P_{t4} = \sigma_B * P_{t3}$$

```
Pt4 = 2.2853e+05
```

Mass flow calculation for turbine minimum area

Section 4_min Turbine minimum area for critical flow condition

Static temperature for critical flow

$$T_{4\,\mathrm{MIN}} = \frac{2 * T_{\mathrm{t4}}}{k_t + 1}$$

```
T4_min = 1.0071e+03
```

Static pressure

$$P_{4\,\mathrm{MIN}} = P_{\mathrm{t4}} * 0.98 \left(\frac{2}{k_t + 1}\right)^{\frac{\mathrm{kt}}{\mathrm{kt}-1}}$$

```
P4_min = 1.2102e+05
```

Dencity

$$\rho_{4\,\mathrm{MIN}} = \frac{P_{4\,\mathrm{MIN}}}{R_t T_{4\,\mathrm{MIN}}}$$

```
ro4_min = 0.4144
```

Gass velocity in minimal area:

$$c_{4\,\mathrm{MIN}} = \sqrt{k_t R_t T_{4\,\mathrm{MIN}}}$$

```
c4_min = 623.2426
```

Mass flow in section 4

$$\dot{m}_4 = A_{4\,\mathrm{MIN}}\, \rho_{4\,\mathrm{MIN}}\, c_{4\,\mathrm{MIN}}$$

```
m4 = 9.1717
```

Mass flow in section 0

$$\dot{m}_0 = \frac{\dot{m}_4}{1 + f_B}$$

```
m0 = 8.9909
```

Fuel mass flow i

$$\dot{m}_f = f_B\, \dot{m}_0$$

```
mf = 0.1809
```

Calculation of exit nozzle and performance paramethers

Section 7 Engine Nozzle intlet

Total temperature   [K]

$$T_{\mathrm{t7}} = T_{\mathrm{t5}}$$

```
Tt7 = 987.4009
```

Total pressure [Pa]

$$P_{\mathrm{t7}} = \frac{P_{\mathrm{t4}}}{\mathrm{TPR}} * \sigma_{\mathrm{AN}}$$

```
Pt7 = 1.0212e+05
```

Section 9 Engine Nozzle outlet

Total temperature   [K]

$$T_{t9} = T_{t7}$$

```
Tt9 = 987.4009
```

Total pressure [Pa]

$$P_{t9} = P_{t7} * \sigma_N$$

```
Pt9 = 9.8034e+04
```

Section 9 - incomplete expansion in the nozzle

Static pressure [Pa]

$$P_{9\,\mathrm{IE}} = \frac{P_{t9}}{B_{cr}}$$

```
P9_IE = 5.2974e+04
```

Static temperature   [K]

$$T_{9\,\mathrm{IE}} = T_{t9} * \left(\frac{P_{9\,\mathrm{IE}}}{P_{t9}}\right)^{\frac{\mathrm{kt}-1}{\mathrm{kt}}}$$

```
T9_IE = 847.5544
```

Mach No. in section 9

$$M_{9\,\mathrm{IE}} = \sqrt{\left(\frac{T_{t9}}{T_{9\mathrm{IE}}} - 1\right) * \frac{2}{\mathrm{k}_t - 1}}$$

```
M9_IE = 1
```

Speed of sound   [m/s]

$$a_{9\,\mathrm{IE}} = \sqrt{\mathrm{k}_t * R_t * T_{9\,\mathrm{IE}}}$$

```
a9_IE = 571.7532
```

Jet speed in section 9 [m/s]

$$V_{9\,\mathrm{IE}} = M_{9\,\mathrm{IE}} * a_{9\,\mathrm{IE}}$$

```
V9_IE = 571.7532
```

Gas density in section 9 [kg/m^3]

$$\rho_{9\,\mathrm{IE}} = \frac{P_{9\,\mathrm{IE}}}{R_t * T_{9\,\mathrm{IE}}}$$

```
RO9_IE = 0.2155
```

Nozzle minimal area (for model correctness check)

$$A_{9\,\mathrm{MIN}} = \frac{\dot{m}_9}{\rho_{9\,\mathrm{IE}} V_{9\,\mathrm{IE}}}$$

```
A9_min = 0.0744
```

Section 9e

Jet speed after decompression to ambient pressure [m/s]

$$V_{9\,e} = V_{9\,\mathrm{IE}} + \frac{P_{9\,\mathrm{IE}} - P_0}{\rho_{9\,\mathrm{IE}} * V_{9\,\mathrm{IE}}}$$

```
V9e = 823.1099
```

Static temperature [K]

$$T_{9\mathrm{e}} = T_{\mathrm{t}9} - \frac{{V_{9e}}^2}{2 * c_{\mathrm{pt}}}$$

```
T9e = 697.8668
```

Staticl pressure [Pa]

$$P_{9\,e} = P_0$$

```
P9e = 22000
```

PERFORMANCE OF TURBOJET ENGINE WITH INCOMPLTE EXPANSION IN THE NOZZLE

Thrust [N]

$$T = m_0 * (1 + f_B) * V_{9e} - m_0 * V_0$$

```
T = 5.2927e+03
```

Specific thrust [Ns/kg]

$$\mathrm{ST} = \frac{T}{m_0} = (1 + f_B) * V_{9e} - V_0$$

```
ST = 588.6787
```

Specific fuel consumption [kg/N/s]

$$\mathrm{SFC} = \frac{m_{\mathrm{fB}}}{T}$$

```
SFC = 3.4170e-05
```

Specific fuel consumption [kg/N/h]

$$\mathrm{SFC} = \mathrm{SFC} * 3600$$

```
SFC = 0.1230
```

RESULTS PRESENATION

```
Tabela = 8×4 table
```

	Parameter	Unit	DP	0.95*N_DP
1	'm0'	'kg/s'	10	8.9909
2	'CPR'	'-'	8	6.8136
3	'TIT'	'K'	1300	1.1732e+03
4	'fuel mass flow'	'kg/s'	0.2303	0.1809
5	'A9_min'	'm^2'	0.0744	0.0744
6	'Thrust'	'kN'	6.6391	5.2927
7	'Specific Thrust'	'N*s/kg'	663.9121	588.6787
8	'Specific fuel consump'	'kg/N/h'	0.1249	0.1230

Conclusions:

Off design calculation fo N=0.95*N_DP shows:

- decreasing of air mass flow throught the engine
- TIT and CPR decreasing
- Engine thrust and specific thrust are lower
- SFC is lower - it cames from constant efficiency of the engine components. For variable efficiency SFC coul slightly grow in OFF Design calculation for N lower than N_DP

CALCULATION FOR M0=0 and H=0

In analysis N=N_DP is assumed, all components efficiency are equal DP efficiency.

Ambient conditions

Flight Mach No.

```
M0 = 0
```

Static temperature [K]

```
T0 = 288
```

Static pressure [Pa]

```
P0 = 101325
```

Section 0

Total temperature [K]

$T_{t0} = T_0\left(1 + \frac{k-1}{2} M_0^2\right)$ - like for ideal engine

```
Tt0 = 288
```

Total pressure [Pa]

$$P_{t0} = P_0\left(1 + \frac{k-1}{2} M_0^2\right)^{\frac{k}{k-1}}$$ - like for ideal engine

```
Pt0 = 101325
```

Speed of sound [m/s]

$$a_0 = \sqrt{k * R * T_0}$$ - like for ideal engine

```
a0 = 340.1741
```

Flight speed [m/s]

$$V_0 = M_0 * a_0$$ - like for ideal engine

```
V0 = 0
```

Section 2 Compressor inlet

Total temperature [K]

$$T_{t2} = T_{t0}$$ - like for ideal engine

```
Tt2 = 288
```

Total pressure [Pa]

$$P_{t2} = \sigma_{IN} * P_{t0}$$

```
Pt2 = 9.8285e+04
```

Algoritm for off-design calculation

```
fB_1 = 0
```

$$T_{t4} = T_{t4\,DP} * \frac{T_{t0}}{T_{t0\_DP}}\left(\frac{n}{n_{DP}}\right)^2$$, n/n_DP=1

```
Tt4 = 1.5075e+03
```

Total temperature [K]

$$T_{t5} = T_{t4}\left(1 - \eta_T + \eta_T\left(\frac{1}{\mathrm{TPR}}\right)^{\frac{\mathrm{kt}-1}{\mathrm{kt}}}\right)$$

```
Tt5 = 1.2687e+03
```

Turbine Work [J] calculated according mass flow in section 4

$$W_T = c_{pt}(T_{t4} - T_{t5})$$

```
WT = 2.7939e+05
```

Section 3 - Compressor outlet

Total temperature [K]

$$T_{t3} = T_{t2} + \frac{(1 + f_B) W_T \eta_m}{c_p}$$

```
Tt3 = 563.2233
```

BURNER

Fuel-air ratio

$$f_B = c_{pB} * \frac{T_{t4} - T_{t3}}{\mathrm{FHV} * \eta_B}$$

```
fB = 0.0269
```

Loop calculation starts

```
while abs(fB-fB_1)/fB>1e-5
fB_1=fB;
```

Section 3 - Compressor outlet

Total temperature [K]

$$T_{t3} = T_{t2} + \frac{\eta_m (1 + f_B) W_T}{c_p}$$

BURNER

Fuel-air ratio

$$f_B = c_{pB} * \frac{T_{t4} - T_{t3}}{\mathrm{FHV} * \eta_B}$$

End of loop calculation

```
end
```

fB and fB_1 comparison

```
fB = 0.0267
```

```
fB_1 = 0.0267
```

fB is almost equal fB_1 therefore loop calculation is finished

Compressor pressure ratio calculation CPR

CPR and total pressure of the compressor outlet [Pa]

$$\mathrm{CPR} = \left(1 + \eta_C \left(\frac{T_{t3}}{T_{t2}} - 1\right)\right)^{\frac{k}{k-1}}$$

```
CPR = 8.0448
```

$$P_{t3} = P_{t2} * \mathrm{CPR}$$

```
Pt3 = 7.9069e+05
```

COMPRESSOR

Section 4 Burner outlet / Turbine inlet

Total pressure   [Pa]

$$P_{t4} = \sigma_B * P_{t3}$$

```
Pt4 = 7.7487e+05
```

Mass flow calculation for turbine minimum area

Section 4_min Turbine minimum area for critical flow condition

Static temperature for critical flow

$$T_{4\,MIN} = \frac{2 * T_{t4}}{k_t + 1}$$

```
T4_min = 1.2940e+03
```

Static pressure

$$P_{4\,MIN} = P_{t4} * 0.98 \left(\frac{2}{k_t + 1} \right)^{\frac{kt}{kt-1}}$$

```
P4_min = 4.1034e+05
```

Dencity

$$\rho_{4\,MIN} = \frac{P_{4\,MIN}}{R_t T_{4\,MIN}}$$

```
ro4_min = 1.0935
```

Gass velocity in minimal area:

$$c_{4\,MIN} = \sqrt{k_t R_t T_{4\,MIN}}$$

```
c4_min = 706.4672
```

Mass flow in section 4

$$\dot{m}_4 = A_{4\,MIN}\, \rho_{4\,MIN}\, c_{4\,MIN}$$

```
m4 = 27.4344
```

Mass flow in section 0

$$\dot{m}_0 = \frac{\dot{m}_4}{1 + f_B}$$

```
m0 = 26.7215
```

Fuel mass flow i

$$\dot{m}_f = f_B\, \dot{m}_0$$

```
mf = 0.7130
```

Calculation of exit nozzle and performance paramethers

Section 7 Engine Nozzle intlet

Total temperature   [K]

$$T_{t7} = T_{t5}$$

```
Tt7 = 1.2687e+03
```

Total pressure [Pa]

$$P_{t7} = \frac{P_{t4}}{\mathrm{TPR}} * \sigma_{\mathrm{AN}}$$

```
Pt7 = 3.4625e+05
```

Section 9 Engine Nozzle outlet

Total temperature   [K]

$$T_{t9} = T_{t7}$$

```
Tt9 = 1.2687e+03
```

Total pressure [Pa]

$$P_{t9} = P_{t7} * \sigma_N$$

```
Pt9 = 3.3240e+05
```

Section 9 - incomplete expansion in the nozzle

Static pressure [Pa]

$$P_{9\,\mathrm{IE}} = \frac{P_{t9}}{B_{\mathrm{cr}}}$$

```
P9_IE = 1.7961e+05
```

Static temperature   [K]

$$T_{9\,\mathrm{IE}} = T_{t9} * \left(\frac{P_{9\,\mathrm{IE}}}{P_{t9}}\right)^{\frac{\mathrm{kt}-1}{\mathrm{kt}}}$$

```
T9_IE = 1.0890e+03
```

Mach No. in section 9

$$M_{9\,\mathrm{IE}} = \sqrt{\left(\frac{T_{t9}}{T_{9\mathrm{IE}}} - 1\right) * \frac{2}{\mathrm{k}_t - 1}}$$

```
M9_IE = 1
```

Speed of sound   [m/s]

$$a_{9\,\text{IE}} = \sqrt{\mathrm{k}_t * R_t * T_{9\,\text{IE}}}$$

```
a9_IE = 648.1022
```

Jet speed in section 9 [m/s]

$$V_{9\,\text{IE}} = M_{9\,\text{IE}} * a_{9\,\text{IE}}$$

```
V9_IE = 648.1022
```

Gas density in section 9 [kg/m^3]

$$\rho_{9\,\text{IE}} = \frac{P_{9\,\text{IE}}}{R_t * T_{9\,\text{IE}}}$$

```
RO9_IE = 0.5687
```

Nozzle minimal area

$$A_{9\,\text{MIN}} = \frac{\dot{m}_9}{\rho_{9\,\text{IE}}\, V_{9\,\text{IE}}}$$

```
A9_min = 0.0744
```

Section 9e

Jet speed after decompression to ambient pressure [m/s]

$$V_{9\,e} = V_{9\,\text{IE}} + \frac{P_{9\,\text{IE}} - P_0}{\rho_{9\,\text{IE}} * V_{9\,\text{IE}}}$$

```
V9e = 860.5019
```

Static temperature   [K]

$$T_{9\text{e}} = T_{\text{t}9} - \frac{{V_{9e}}^2}{2 * c_{\text{pt}}}$$

```
T9e = 952.2753
```

Staticl pressure   [Pa]

$$P_{9\,e} = P_0$$

```
P9e = 101325
```

PERFORMANCE OF TURBOJET ENGINE WITH THE CHOCKED NOZZLE

Thrust   [N]

$$T = m_0 * (1 + f_B) * V_{9e} - m_0 * V_0$$

```
T = 2.3607e+04
```

Specific thrust   [Ns/kg]

$$\text{ST} = \frac{T}{m_0} = (1 + f_B) * V_{9e} - V_0$$

```
ST = 883.4609
```

Specific fuel consumption   [kg/N/s]

$$\text{SFC} = \frac{m_{\text{fB}}}{T}$$

```
SFC = 3.0200e-05
```

Specific fuel consumption   [kg/N/h]

$$\text{SFC} = \text{SFC} * 3600$$

```
SFC = 0.1087
```

RESULTS PRESENATION

```
Tabela = 8×4 table
```

	Parameter	Unit	DP	0.95*N_DP
1	'm0'	'kg/s'	10	26.7215
2	'CPR'	'-'	8	8.0448
3	'TIT'	'K'	1300	1.5075e+03
4	'fuel mass flow'	'kg/s'	0.2303	0.7130
5	'A9_min'	'm^2'	0.0744	0.0744
6	'Thrust'	'kN'	6.6391	23.6074
7	'Specific Thrust'	'N*s/kg'	663.9121	883.4609
8	'Specific fuel consump'	'kg/N/h'	0.1249	0.1087

Conclusions:

On the ground TIT growed in comparision to DP for N=N_DP

CPR is almost equal CPR_DP

Gas mass flow throught the engine grew as well, as Thrust, and SFC went down.

CALCULATION FOR M0=0 and H=0 and different PLA (n/n_DP) setting

In analysis N=(0,78:0,2:1,02) N_DP is assumed.

Ambient conditions

Flight Mach No.

```
M0 = 0
```

Static temperature [K]

```
T0 = 288
```

Static pressure [Pa]

```
P0 = 101325
```

Section 0

Total temperature [K]

$T_{t0} = T_0\left(1 + \frac{k-1}{2}M_0^2\right)$ - like for ideal engine

```
Tt0 = 288
```

Total pressure [Pa]

$P_{t0} = P_0\left(1 + \frac{k-1}{2}M_0^2\right)^{\frac{k}{k-1}}$ - like for ideal engine

```
Pt0 = 101325
```

Speed of sound [m/s]

$a_0 = \sqrt{k * R * T_0}$ - like for ideal engine

```
a0 = 340.1741
```

Flight speed [m/s]

$V_0 = M_0 * a_0$ - like for ideal engine

```
V0 = 0
```

Section 2 Compressor inlet

Total temperature [K]

$T_{t2} = T_{t0}$ - like for ideal engine

```
Tt2 = 288
```

Total pressure [Pa]

$P_{t2} = \sigma_{IN} * P_{t0}$

```
Pt2 = 9.8285e+04
```

Algoritm for off design calculation

TIT calculation for assumed N/N_DP

```
n_nDP = 1×13
    0.7800    0.8000    0.8200    0.8400    0.8600    0.8800    0.9000    0.9200 ···
```

$T_{t4} = T_{t4\,DP}*\frac{T_{t0}}{T_{t0\_DP}}\left(\frac{n}{n_{DP}}\right)^2$, n/n_DP=1

```
Tt4 = 1×13
```

```
10^3 ×
    0.9172    0.9648    1.0136    1.0637    1.1150    1.1674    1.2211    1.2760 ···
```

Total temperature [K]

$$T_{t5} = T_{t4}\left(1 - \eta_T + \eta_T\left(\frac{1}{\mathrm{TPR}}\right)^{\frac{\mathrm{kt}-1}{\mathrm{kt}}}\right)$$

```
Tt5 = 1×13
10^3 ×
    0.7719    0.8120    0.8531    0.8952    0.9383    0.9825    1.0277    1.0738 ···
```

Turbine Work [J] calculated according mass flow in section 4

$$W_T = c_{\mathrm{pt}}(T_{t4} - T_{t5})$$

```
WT = 1×13
10^5 ×
    1.6998    1.7881    1.8786    1.9714    2.0664    2.1636    2.2631    2.3648 ···
```

Section 3 - Compressor outlet

Total temperature [K]

$$T_{t3} = T_{t2} + \frac{(1 + f_B)W_T\eta_m}{c_p}$$

```
Tt3 = 1×13
  460.1475  469.0888  478.2564  487.6504  497.2707  507.1174  517.1905  527.4899 ···
```

BURNER

Loop calculation

```
for i=1:length(Tt4)
    fB_1=0; %first  assumption of fB
```

Fuel-air ratio

$$f_B = c_{\mathrm{pB}} * \frac{T_{t4}(i) - T_{t3}(i)}{\mathrm{FHV} * \eta_B}$$

```
    while abs(fB-fB_1)/fB>1e-5
        fB_1=fB;
```

Section 3 - Compressor outlet

Total temperature [K]

$$T_{t3}(i) = T_{t2} + \frac{\eta_m(1 + f_B)W_T(i)}{c_p}$$

BURNER

Fuel-air ratio

$$f_B = c_{pB} * \frac{T_{t4}(i) - T_{t3}(i)}{\mathrm{FHV} * \eta_B}$$

```
Tt3 = 1×13
  457.6370  466.6416  475.8940  485.3956  495.1479  505.1524  515.4107  525.9244 ⋯

fB = 1×13
    0.0131    0.0142    0.0153    0.0165    0.0176    0.0189    0.0201    0.0214 ⋯
```

End of loop calculation

```
        end % WHILE loop
    end  % FOR loop
```

Compressor pressure ratio calculation CPR

CPR and total pressure of the compressor outlet [Pa]

$$\mathrm{CPR} = \left(1 + \eta_C\left(\frac{T_{t3}}{T_{t2}} - 1\right)\right)^{\frac{k}{k-1}}$$

```
CPR = 1×13
    4.0273    4.2784    4.5478    4.8369    5.1470    5.4797    5.8364    6.2190 ⋯
```

$$P_{t3} = P_{t2} * \mathrm{CPR}$$

```
Pt3 = 1×13
10^5 ×
    3.9582    4.2050    4.4698    4.7540    5.0588    5.3857    5.7363    6.1123 ⋯
```

COMPRESSOR

Section 4 Burner outlet / Turbine inlet

Total pressure   [Pa]

$$P_{t4} = \sigma_B * P_{t3}$$

```
Pt4 = 1×13
10^5 ×
    3.8791    4.1209    4.3804    4.6589    4.9576    5.2780    5.6216    5.9901 ⋯
```

Mass flow calculation for turbine minimum area

Section 4_min Turbine minimum area for critical flow condition

Static temperature for critical flow

$$T_{4\,\mathrm{MIN}} = \frac{2 * T_{t4}}{k_t + 1}$$

```
T4_min = 1×13
10^3 ×
```

```
    0.7873    0.8282    0.8701    0.9130    0.9570    1.0021    1.0481    1.0952 ···
```

Static pressure

$$P_{4\,\text{MIN}} = P_{t4} * 0.98\left(\frac{2}{k_t+1}\right)^{\frac{\text{kt}}{\text{kt}-1}}$$

```
P4_min = 1×13
10^5 ×
    2.0542    2.1823    2.3197    2.4671    2.6253    2.7950    2.9770    3.1721 ···
```

Dencity

$$\rho_{4\,\text{MIN}} = \frac{P_{4\,\text{MIN}}}{R_t T_{4\,\text{MIN}}}$$

```
ro4_min = 1×13
    0.8997    0.9086    0.9193    0.9318    0.9459    0.9618    0.9794    0.9987 ···
```

Gass velocity in minimal area:

$$c_{4\,\text{MIN}} = \sqrt{k_t R_t T_{4\,\text{MIN}}}$$

```
c4_min = 1×13
  551.0444  565.1738  579.3031  593.4325  607.5618  621.6911  635.8205  649.9498 ···
```

Mass flow in section 4

$$\dot{m}_4 = A_{4\,\text{MIN}}\,\rho_{4\,\text{MIN}}\,c_{4\,\text{MIN}}$$

```
m4 = 1×13
   17.6075   18.2376   18.9133   19.6367   20.4098   21.2350   22.1148   23.0520 ···
```

Mass flow in section 0

$$\dot{m}_0 = \frac{\dot{m}_4}{1+f_B}$$

```
m0 = 1×13
   17.3801   17.9825   18.6281   19.3185   20.0558   20.8419   21.6792   22.5700 ···
```

Fuel mass flow i

$$\dot{m}_f = f_B\,\dot{m}_0$$

```
mf = 1×13
    0.2274    0.2551    0.2853    0.3181    0.3540    0.3931    0.4356    0.4821 ···
```

Calculation of exit nozzle and performance paramethers

Section 7 Engine Nozzle intlet

Total temperature [K]

$$T_{t7} = T_{t5}$$

```
Tt7 = 1×13
10^3 ×
```

```
    0.7719    0.8120    0.8531    0.8952    0.9383    0.9825    1.0277    1.0738 ···
```

Total pressure [Pa]

$$P_{t7} = \frac{P_{t4}}{\mathrm{TPR}} * \sigma_{\mathrm{AN}}$$

```
Pt7 = 1×13
10^5 ×
    1.7333    1.8414    1.9574    2.0818    2.2153    2.3584    2.5120    2.6766 ···
```

Section 9 Engine Nozzle outlet

Total temperature [K]

$$T_{t9} = T_{t7}$$

```
Tt9 = 1×13
10^3 ×
    0.7719    0.8120    0.8531    0.8952    0.9383    0.9825    1.0277    1.0738 ···
```

Total pressure [Pa]

$$P_{t9} = P_{t7} * \sigma_N$$

```
Pt9 = 1×13
10^5 ×
    1.6640    1.7677    1.8791    1.9985    2.1266    2.2641    2.4115    2.5695 ···
```

Section 9 - incomplete expansion in the nozzle

Static pressure [Pa]

$$P_{9\,\mathrm{IE}} = \frac{P_{t9}}{B_{cr}}$$

```
P9_IE = 1×13
10^5 ×
    0.8992    0.9552    1.0154    1.0799    1.1492    1.2234    1.3031    1.3885 ···
```

Searching for P9_IE<P0 - in this case nozzle isn't chocked and applied model isn't appropriate. In this case NaN will be insert instead of data. The data calculated previously on the found position should be disregarded.

```
[i,j]=find(P9_IE<P0)
P9_IE(i,j)=NaN;
```

```
i = 1×2
     1     1
j = 1×2
     1     2

P9_IE = 1×13
10^5 ×
       NaN       NaN    1.0154    1.0799    1.1492    1.2234    1.3031    1.3885 ···
```

Static temperature [K]

$$T_{9\,\mathrm{IE}} = T_{\mathrm{t}9} * \left(\frac{P_{9\,\mathrm{IE}}}{P_{\mathrm{t}9}}\right)^{\frac{\mathrm{kt}-1}{\mathrm{kt}}}$$

```
T9_IE = 1×13
10^3 ×
       NaN       NaN    0.7323    0.7684    0.8054    0.8433    0.8821    0.9217 ⋯
```

Mach No. in section 9

$$M_{9\,\mathrm{IE}} = \sqrt{\left(\frac{T_{\mathrm{t}9}}{T_{9\mathrm{IE}}} - 1\right) * \frac{2}{\mathrm{k}_t - 1}}$$

```
M9_IE = 1×13
       NaN       NaN    1.0000    1.0000    1.0000    1.0000    1.0000    1.0000 ⋯
```

Speed of sound [m/s]

$$a_{9\,\mathrm{IE}} = \sqrt{\mathrm{k}_t * R_t * T_{9\,\mathrm{IE}}}$$

```
a9_IE = 1×13
       NaN       NaN  531.4438  544.4058  557.3679  570.3299  583.2920  596.2540 ⋯
```

Jet speed in section 9 [m/s]

$$V_{9\,\mathrm{IE}} = M_{9\,\mathrm{IE}} * a_{9\,\mathrm{IE}}$$

```
V9_IE = 1×13
       NaN       NaN  531.4438  544.4058  557.3679  570.3299  583.2920  596.2540 ⋯
```

Gas density in section 9 [kg/m^3]

$$\rho_{9\,\mathrm{IE}} = \frac{P_{9\,\mathrm{IE}}}{R_t * T_{9\,\mathrm{IE}}}$$

```
RO9_IE = 1×13
       NaN       NaN    0.4782    0.4846    0.4920    0.5002    0.5094    0.5194 ⋯
```

Nozzle minimal area

$$A_{9\,\mathrm{MIN}} = \frac{\dot{m}_9}{\rho_{9\,\mathrm{IE}} V_{9\,\mathrm{IE}}}$$

```
A9_min = 1×13
       NaN       NaN    0.0744    0.0744    0.0744    0.0744    0.0744    0.0744 ⋯
```

Section 9e

Jet speed after decompression to ambient pressure [m/s]

$$V_{9\,e} = V_{9\,\mathrm{IE}} + \frac{P_{9\,\mathrm{IE}} - P_0}{\rho_{9\,\mathrm{IE}} * V_{9\,\mathrm{IE}}}$$

```
V9e = 1×13
       NaN       NaN  532.2812  569.6768  606.9323  643.9999  680.8378  717.4102 ⋯
```

Static temperature [K]

$$T_{9e} = T_{t9} - \frac{{V_{9e}}^2}{2 * c_{pt}}$$

```
T9e = 1×13
       NaN       NaN  732.0041  756.5149  780.9182  805.2536  829.5632  853.8908 ⋯
```

Staticl pressure [Pa]

$$P_{9e} = P_0$$

```
P9e = 101325
```

PERFORMANCE OF TURBOJET ENGINE WITH INCOMPLTE EXPANSION IN THE NOZZLE

Thrust [N]

$$T = m_0 * (1 + f_B) * V_{9e} - m_0 * V_0$$

```
T = 1×13
10^4 ×
       NaN       NaN    1.0067    1.1187    1.2387    1.3675    1.5057    1.6538 ⋯
```

Specific thrust [Ns/kg]

$$\mathrm{ST} = \frac{T}{m_0} = (1 + f_B) * V_{9e} - V_0$$

```
ST = 1×13
       NaN       NaN  540.4323  579.0583  617.6446  656.1450  694.5193  732.7329 ⋯
```

Specific fuel consumption [kg/N/s]

$$\mathrm{SFC} = \frac{m_{fB}}{T}$$

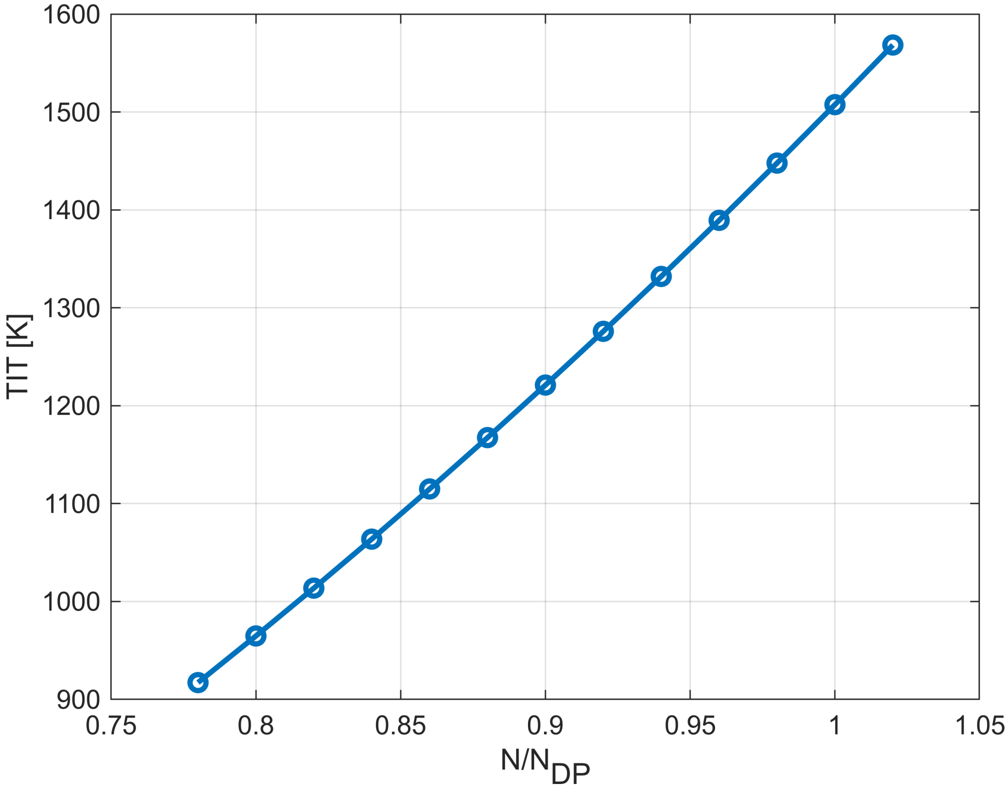
```
SFC = 1×13
10^-4 ×
       NaN       NaN    0.2834    0.2844    0.2858    0.2874    0.2893    0.2915 ⋯
```

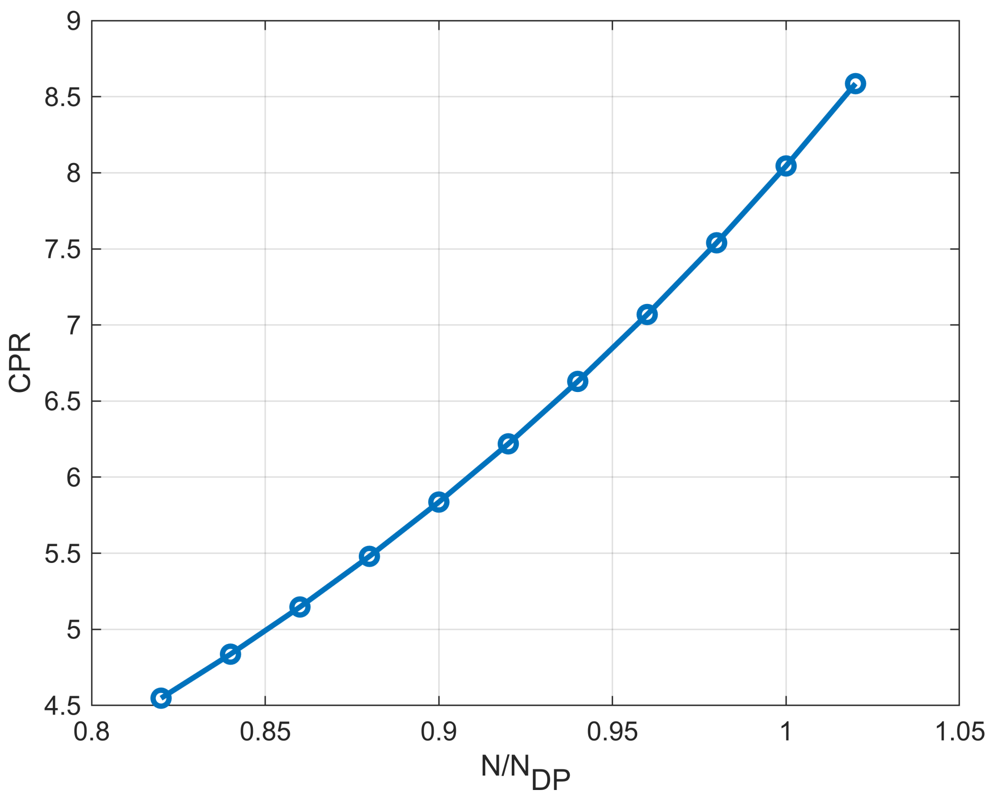
Specific fuel consumption [kg/N/h]

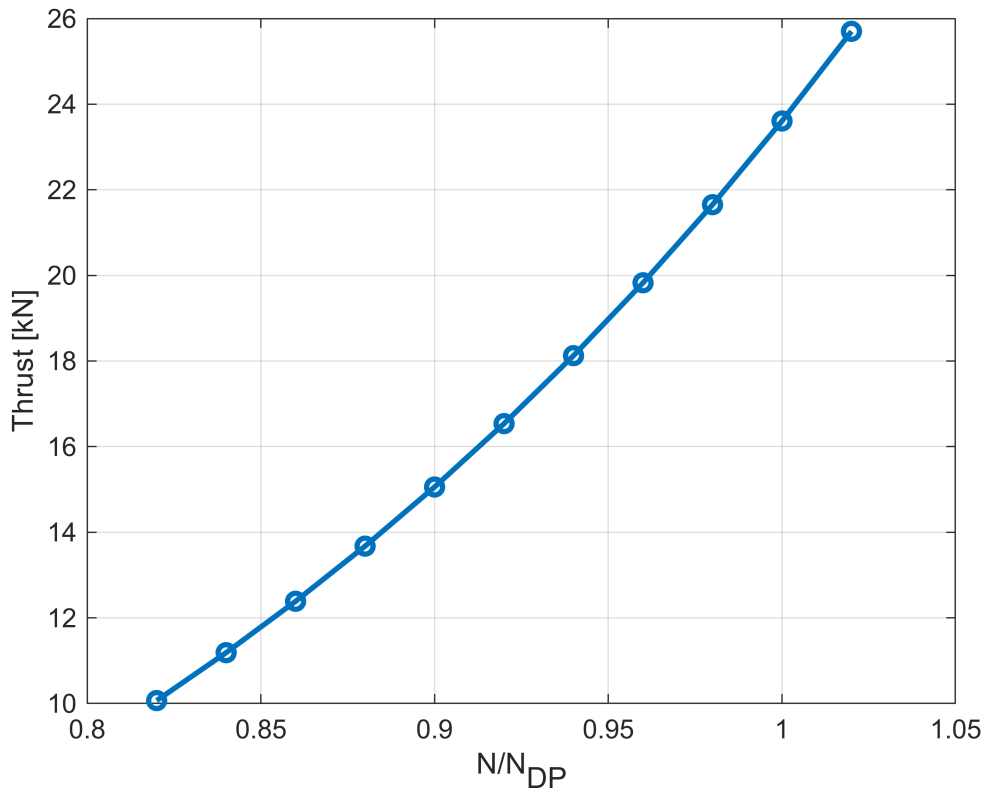
$$\mathrm{SFC} = \mathrm{SFC} * 3600$$

```
SFC = 1×13
       NaN       NaN    0.1020    0.1024    0.1029    0.1035    0.1042    0.1049 ⋯
```

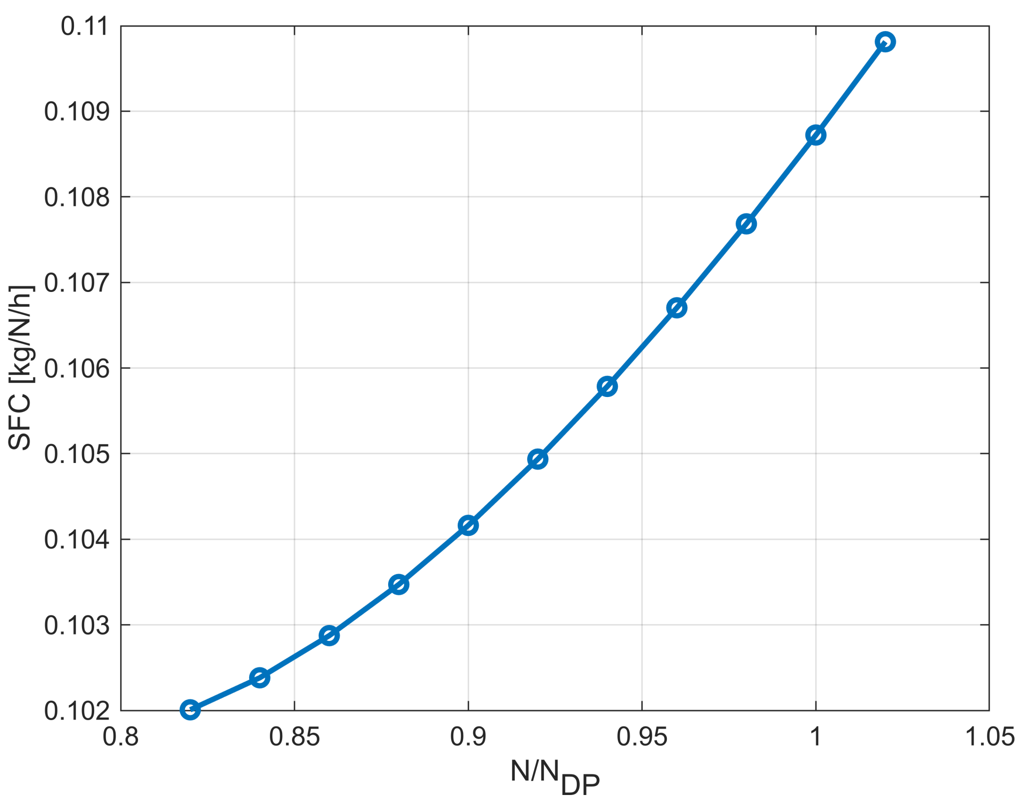
Results presentation


1600
1500
1400
1300
1200
1100
1000
900
TIT [K]
0.75
0.8
0.85
0.9
0.95
1
1.05
N/N_DP

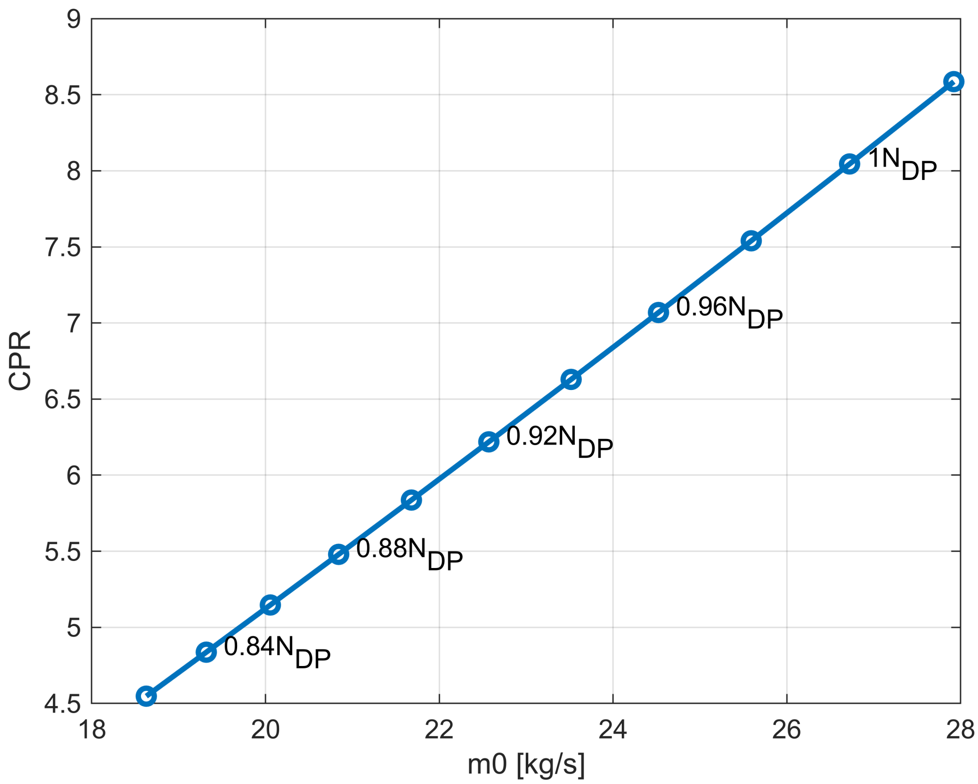

9
8.5
8
7.5
7
6.5
6
5.5
5
4.5
CPR
0.8
0.85
0.9
0.95
1
1.05
N/N_DP



Thrust [kN]
26
24
22
20
18
16
14
12
10
0.8
0.85
0.9
0.95
1
1.05
N/N_DP



SFC [kg/N/h]
0.11
0.109
0.108
0.107
0.106
0.105
0.104
0.103
0.102
0.8
0.85
0.9
0.95
1
1.05
N/N_DP



All presented figures are consistant with the theory instead of SFC plot. For proper SPC vs N evaluation variable component's efficiencies should be used.