ANALYSIS OF GAS TURBINE ENGINE

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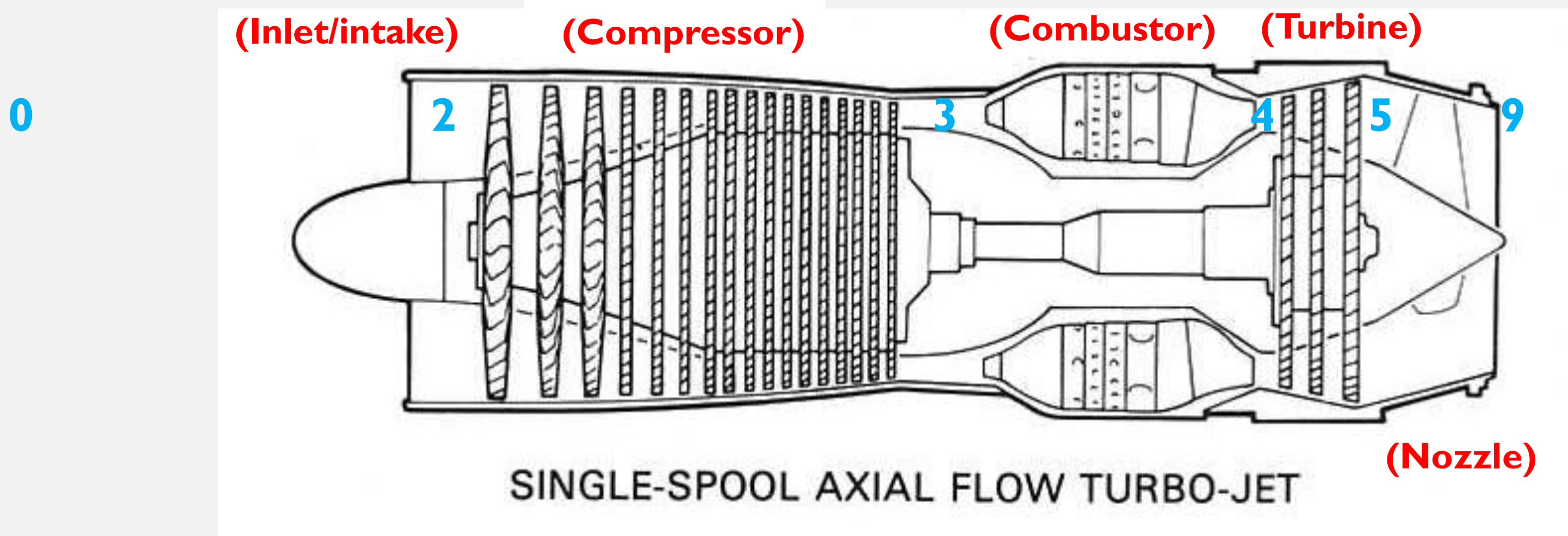
LITERATURE:

- **Jack D. Mattingly, Elements of Propulsion: Gas Turbines and Rockets, AIAA Education Series 2006 (Chapter 7)**
- **Jack D. Mattingly, Elements of Gas Turbine Propulsion, Tata McGraw Hill Education Private Limited, 2013 (Chapter 7)**
- **Gordon C. Oates, Aerothermodynamics of Gas Turbine and Rocket Propulsion, AIAA Education Series, 1997 (Chapter 7)**

SINGLE-SHAFT TURBOJET ENGINE



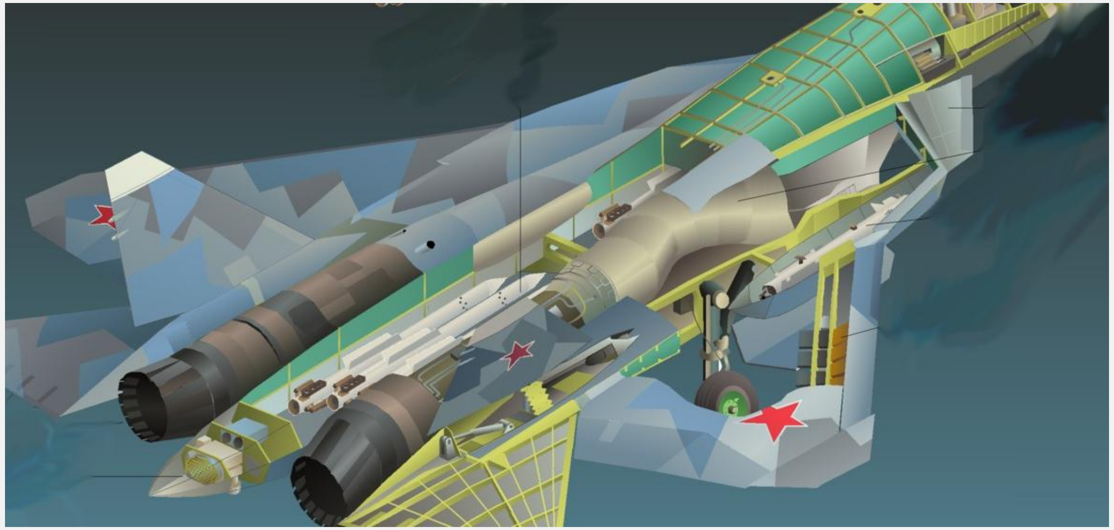
SINGLE-SPOOL AXIAL FLOW TURBO-JET

Engine Components:

- INLET / INTAKE
- COMPRESSOR
- BURNER / COMBUSTOR
- TURBINE
- PROPELING NOZZLE

ENGINE INLET / AIR INTAKE

All **air-breathing jet engines** installed on aircraft must be equipped with an **air inlet** and a **diffuser duct**. Its role is to provide a uniform and stable air supply to ensure efficient engine operation across different flight conditions.



Key functions:

- Capture the required mass flow of air into the engine.
- Decelerate the flow with minimal total pressure loss at high flight speed.
- Ensure that a sufficient mass flow rate of air is drawn into the engine at static engine operation.
- Provide a uniform velocity profile at the compressor face.
- Ensure stable airflow under varying flight conditions.

CLASSIFICATION OF INTAKES

- **Subsonic intakes —** flight Mach number < 1
- **Supersonic intakes —** flight Mach number > 1

SUBSONIC INTAKES

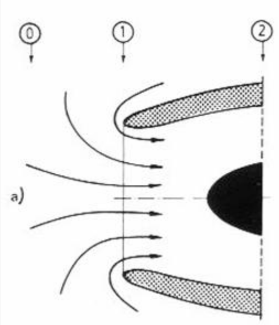
Characteristics:

- **Simple, fixed geometry — no need for variable-geometry elements.**
- **Smooth, aerodynamic shape enabling gradual deceleration without shock waves.**
- **Larger leading-edge radius for low-speed operation to prevent flow separation.**
- **High flow efficiency with low total pressure losses.**
- **Aerodynamic integration with the fuselage to ensure stable compressor inflow across a wide range of angles of attack and speeds.**

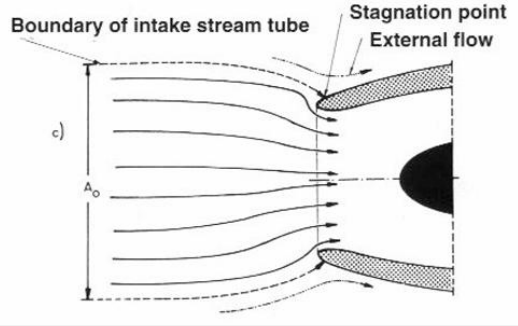
OPERATION OF A SUBSONIC INLET

Operating regimes:

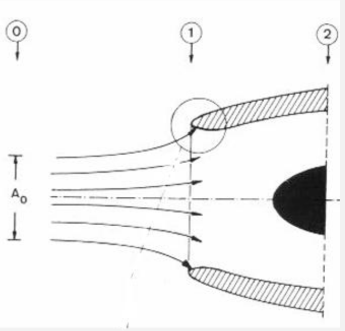
- Static operation

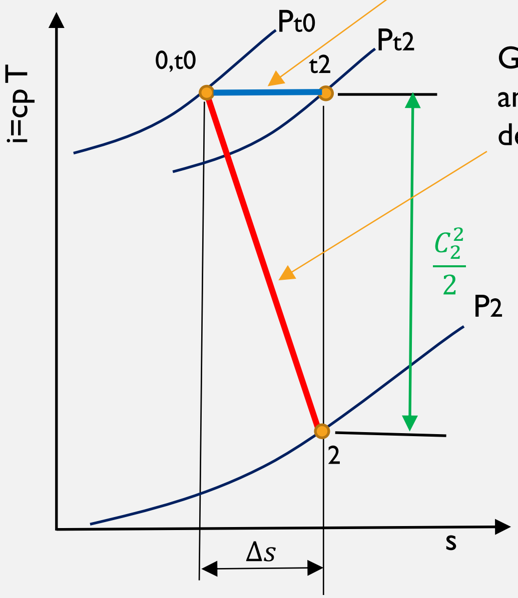


- Flight speed lower than design speed



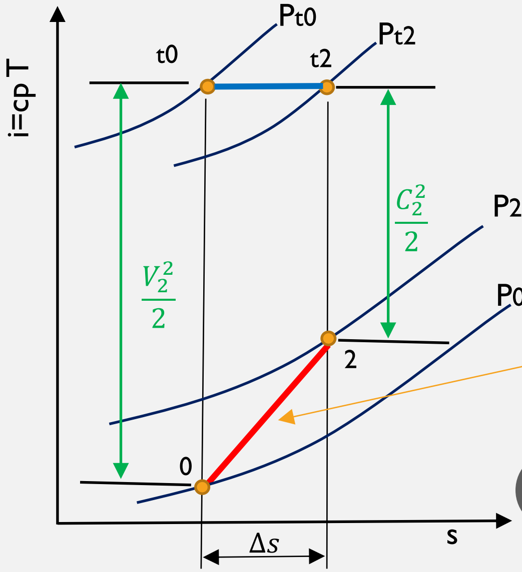
- Flight speed higher than design speed





Const. enthalpy in the inlet

Gas flow acceleration and static pressure decrease



Deceleration of the flow in the inlet and an increase in static pressure

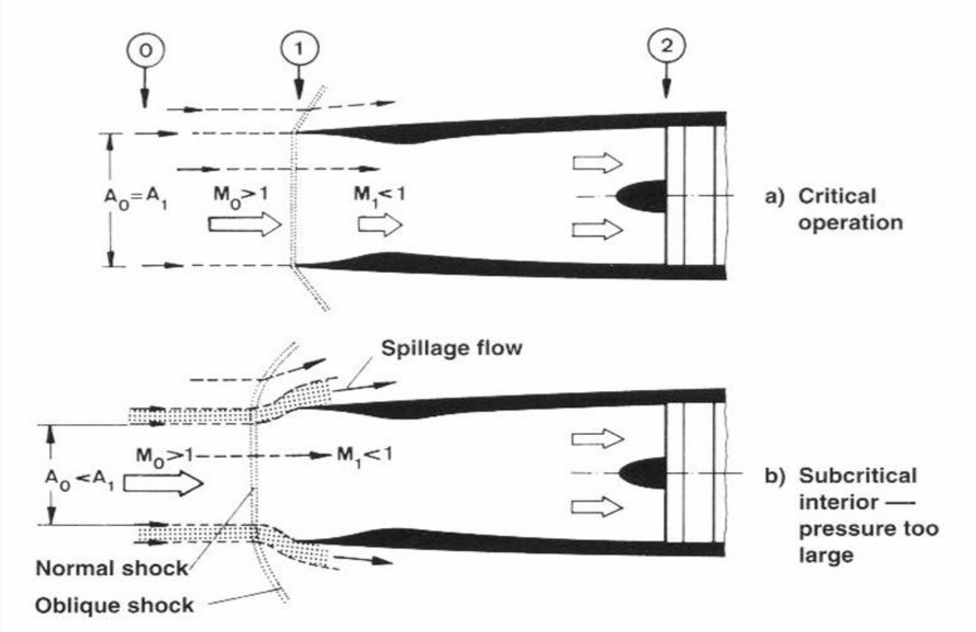
SUPERSONIC INTAKES

Characteristics:

- Complex geometry with variable-position cones, ramps, or wedges to control shock waves.
- Use of shock waves for initial air compression.
- Sharp leading edges to reduce wave drag.
- Ability to maintain proper shock positioning across a wide Mach number range.
- Increased aerodynamic and structural complexity requiring advanced inlet control systems.

SUPERSONIC INTAKES FOR LOW SUPERSONIC SPEEDS

- **Used on aircraft operating at Mach numbers up to approximately 1.4.**
- **Axisymmetric intakes with sharp lips generate a normal shock inside the inlet.**



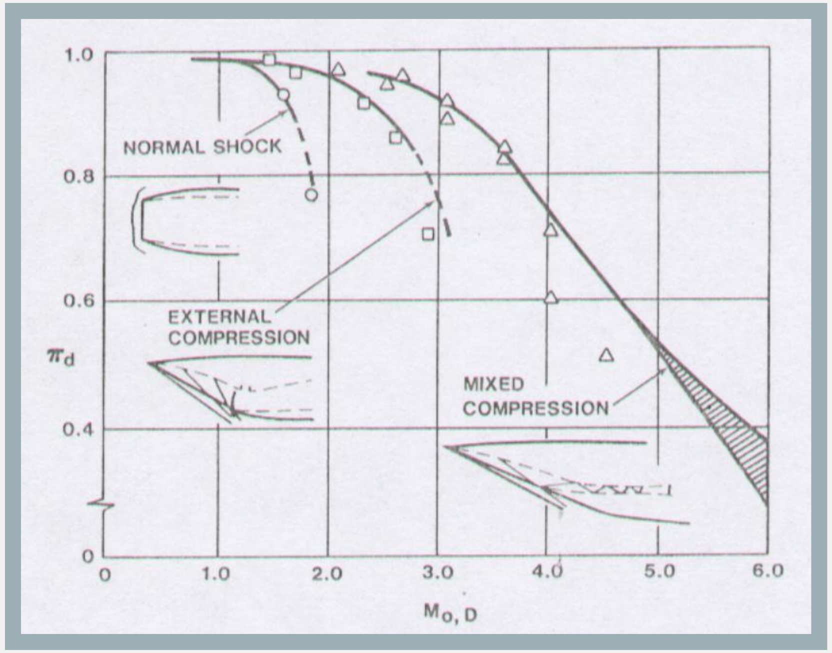
F 100 Super Sabre (Ma=1.3)

MIG 19 (Ma=1.2)

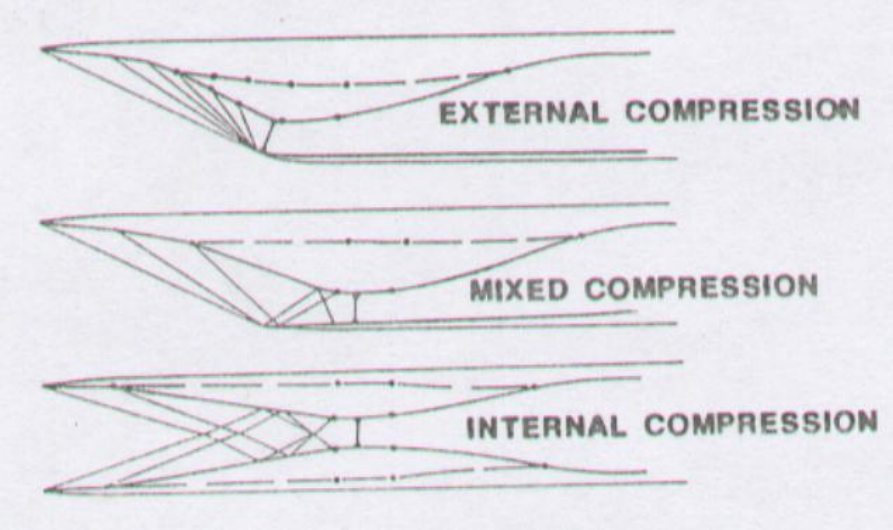
SUPERSONIC INTAKES FOR HIGHER MACH NUMBERS

These intakes generate:

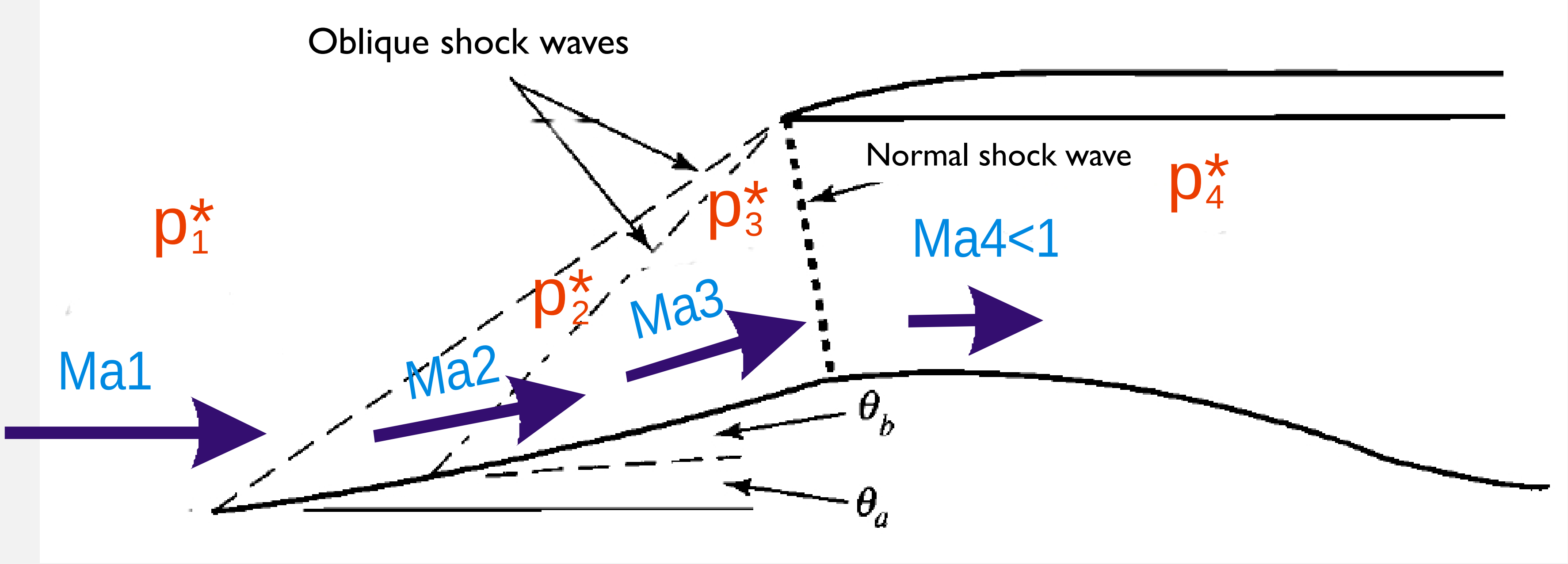
- Oblique shock waves
- A terminal normal shock at the throat
- Subsonic flow downstream of the shock system



Types of supersonic intakes:



PRINCIPLE OF OPERATION OF AN EXTERNAL-COMPRESSION SUPERSONIC INLET



Isentropic pressure rise caused by flight speed

$$p_1^* = p_H\left(1+\frac{k-1}{2}Ma_1^{\,2}\right)^{\frac{k}{k-1}}$$

$$Ma_1 > Ma_2 > Ma_3 > 1 > Ma_4$$

$$p_1^* > p_2^* > p_3^* > p_4^*$$

$$p_1 < p_2 < p_3 < p_4$$

- Deceleration of the incoming flow through a system of shock waves
- Total pressure losses across shocks
- Increase in static pressure

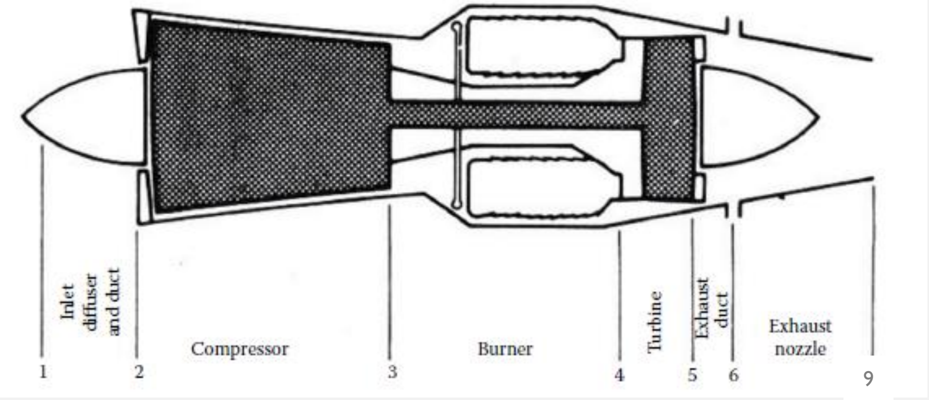
Detailed information in: Ahmed F. El-Sayed, Aircraft Propulsion and Gas Turbine Engines, chapter 9.7.1

INLET WORK

Engine work in static conditions $V_0=0$ → $P_{t0} = P_0, \;\; T_{t0} = T_0$

Ram pressure recovery for flight condition $(M_0 > 0)$

$$P_{t0} = P_0\left(1 + \frac{k-1}{2}{M_0}^2\right)^{k/(k-1)} \qquad T_{t0} = T_0\left(1 + \frac{k-1}{2}{M_0}^2\right)$$



INLET pressure losses → $P_{t2} = \pi_D P_{t0}$
No thermal losses $T_{t2} = T_{t0}$

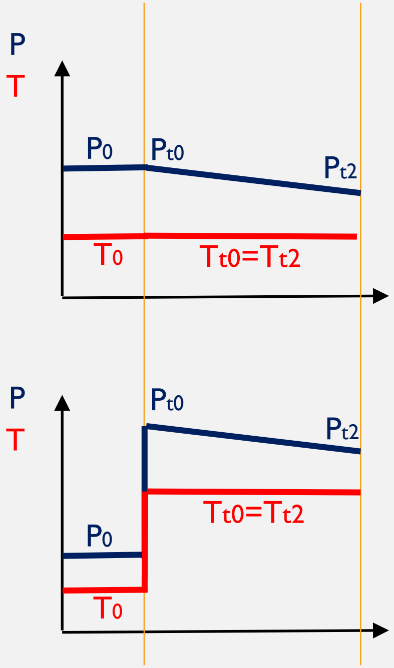
Typical range:

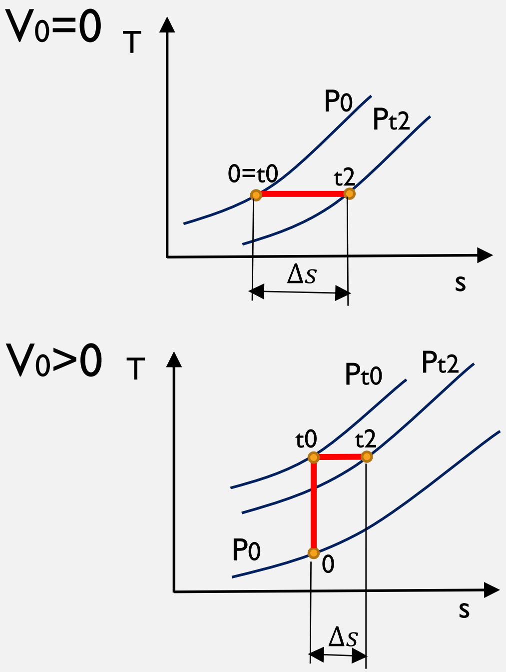
$\pi_D = 0{,}95 - 0{,}99, \quad \pi_D = P_{t2} \,/\, P_{t0}$

π_D is lower for high supoersonic speed

Pressure losses are visible in entropy grow:

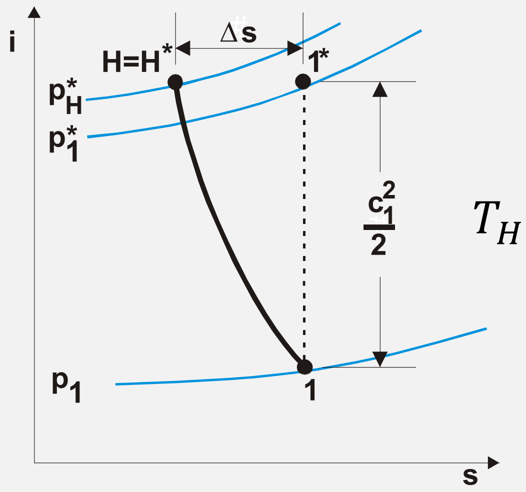
$$\Delta s = -R ln(P_{t2}/P_{t0}) = R ln(1/\pi_D)$$





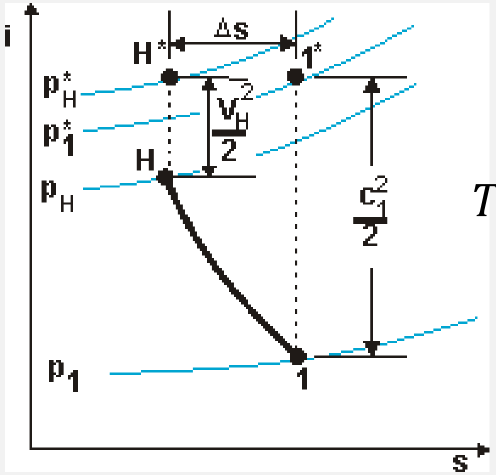
INTAKE ICING PROBLEMS FOR LOW FLIFGT SPEED

$V_0 = 0$

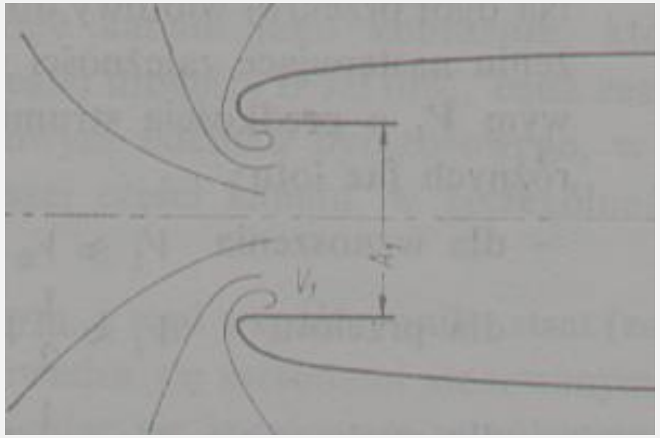


$$T_H = T_1 + \frac{c_1^2}{2\,c_p}$$

$V_0 > 0$ and $V_0 < C_1$



$$T_H + \frac{V_0^2}{2\,c_p} = T_1 + \frac{c_1^2}{2\,c_p}$$



EXAMPLE CALCULATION

Ambient temperature: 8°C (281 K) Air velocity at intake 180 m/s.

Calculate static temperature in the intake for:

a) Static condition,
b) Flight speed 150 km./h

a)

$$T_1 \;= T_H - \frac{c_1^2}{2\,c_p} = 281 - \frac{180^2}{2\,*1000} = 265\,K = -8\,C$$

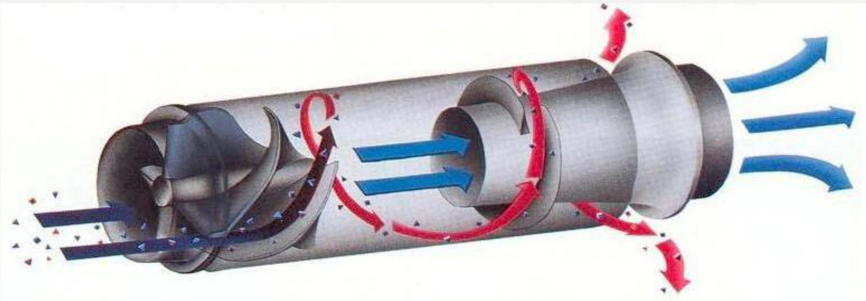
b)

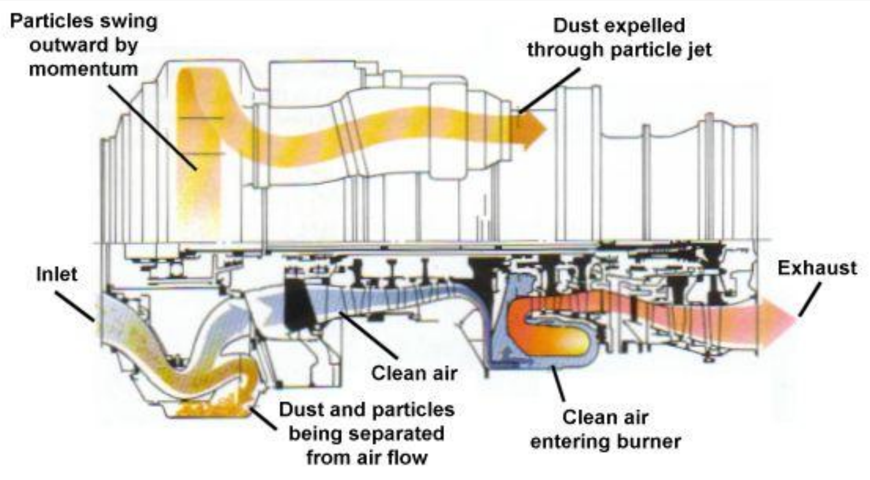
$$V_0 = 150\frac{km}{h} = 41{,}67\,m/s$$

$$T_1 \;= T_H + \frac{V_0^2}{2\,c_p} - \frac{c_1^2}{2\,c_p} = 281 + \frac{41{,}67^2}{2\,*1000} - \frac{180^2}{2\,*1000} = 266\,K = -7\,C$$

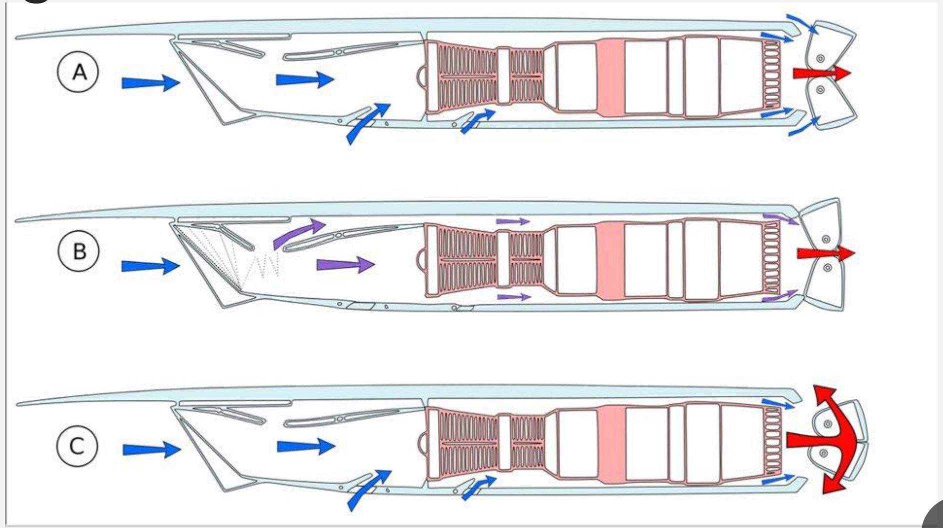
ADDITIONAL FUNCTIONS OF THE INTAKE

- Protection of the engine from dust and debris

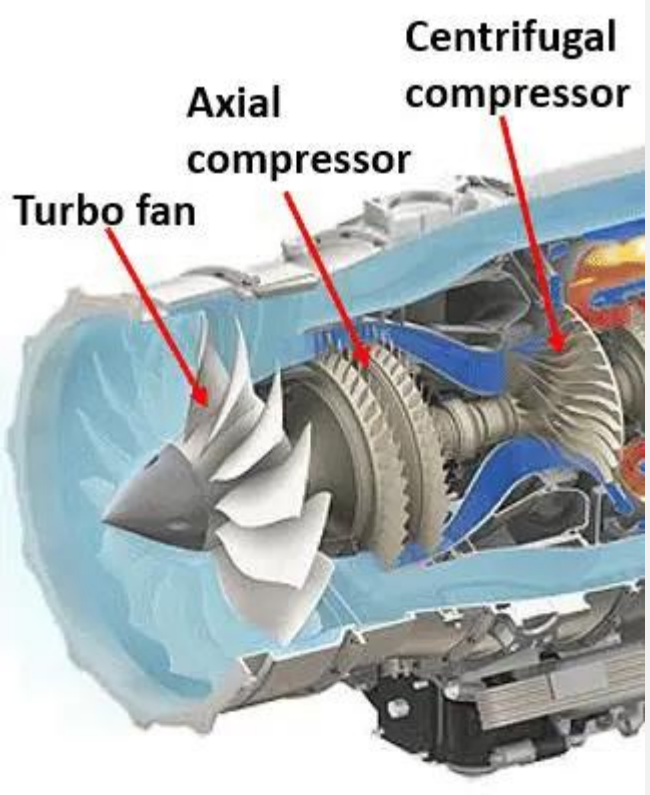




- Stabilization of engine operation under various flight conditions
  a) Adaptation for takeoff conditions
  b) Adaptation for high-speed flight
  c) Operation with thrust reverser deployed

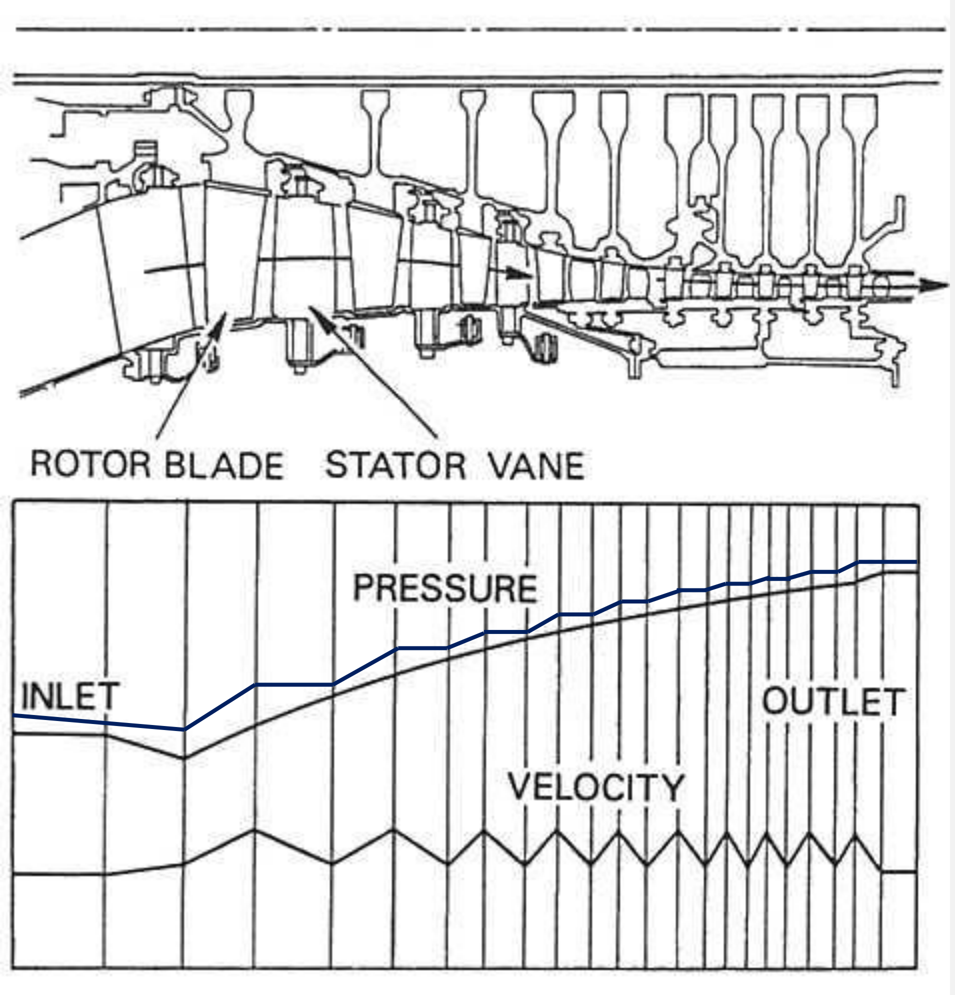


COMPRESSORS AND FANS



- The role of the compressor and fan is to compress the incoming air while consuming as little power as possible — this defines compression efficiency.
- A fan compresses both the bypass and core streams and is an axial-flow machine.
- A compressor is located in the core engine. It may be axial, centrifugal (radial), or mixed (axial-centrifugal).

PARAMETER VARIATION IN A MULTISTAGE COMPRESSOR



- Total pressure increases across the rotor and slightly decreases across the stator due to losses.
- Static pressure increases in both rotor and stator.
- Absolute velocity increases in the rotor and decreases in the stator.

AXIAL COMPRESSOR STAGE

Work of a stage

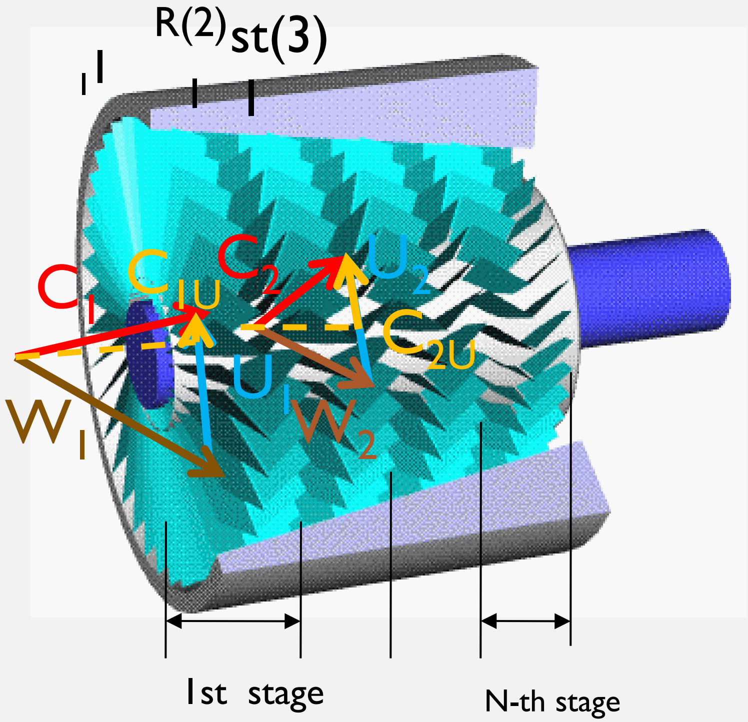
$$W_{st} = c_p(T_{t3} - T_{t1})$$

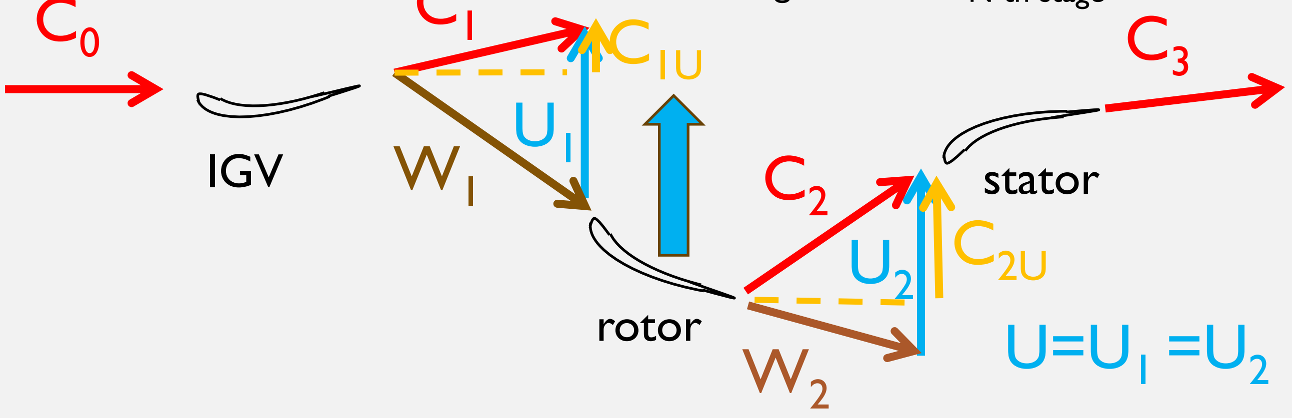
$$= U(C_{2U} - C_{1U})$$

Power of a stage:

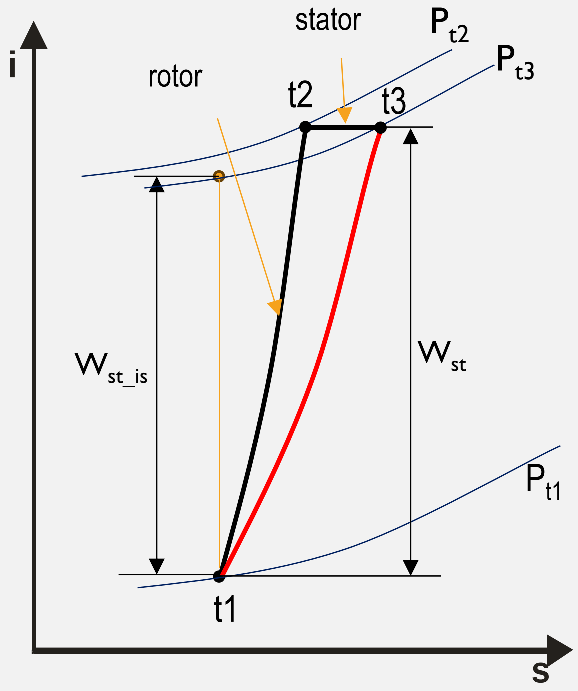
$$P_{st} = \dot{m}\, W_{st}$$

$$= \dot{m}\, c_p\, (T_{t3} - T_{t1})$$





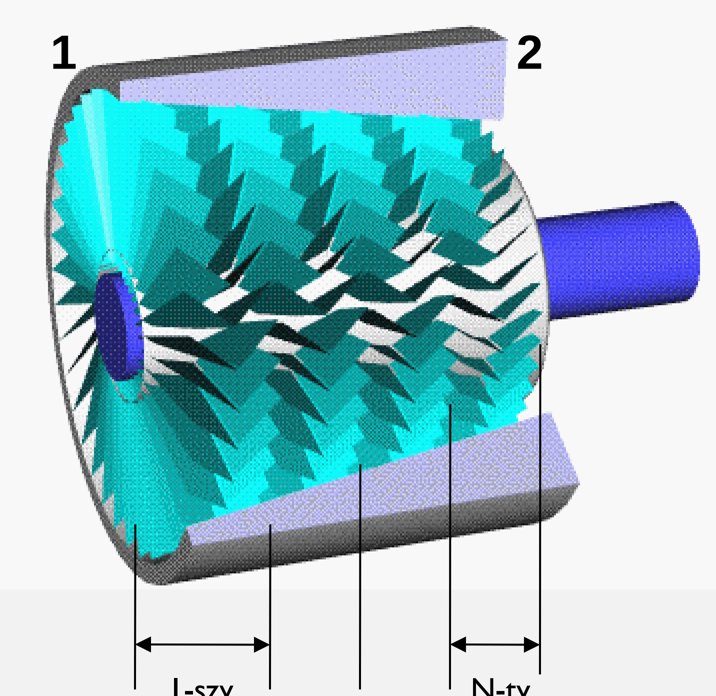
Compressor stage



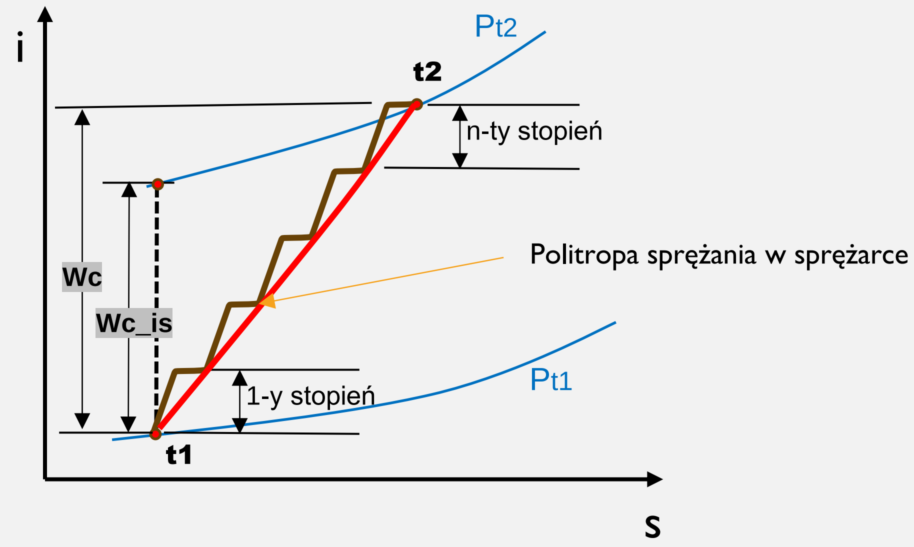
Stage efficiency:

$$\eta_{st} = \frac{W_{st\_is}}{W_{st}} = \frac{i_{t\_st\_is} - i_{t1}}{i_{t\_st} - i_{t1}}$$

MULTISTAGE COMPRESSOR



Charakterystyka sprężarki wielostopniowej



Compressor efficiency:

$$\eta_C = \frac{W_{C\_is}}{W_C} = \frac{i_{t2\_is} - i_{t1}}{i_{t2} - i_{t1}}$$

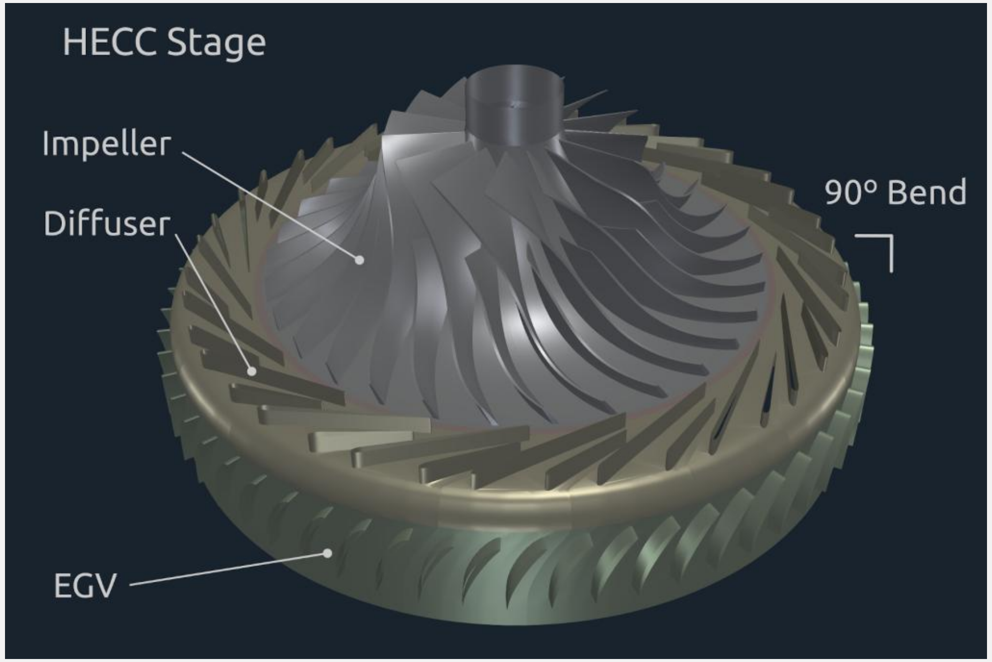
Compressor work:

$$W_C = \sum_{i=1}^{n} W_{st} = c_p (T_{t2} - T_{t1})$$

Compressor pressure ratio:

$$\pi_C = \prod_{i=1}^{n} \pi_{st}$$

CENTRIFUGAL (RADIAL) COMPRESSOR



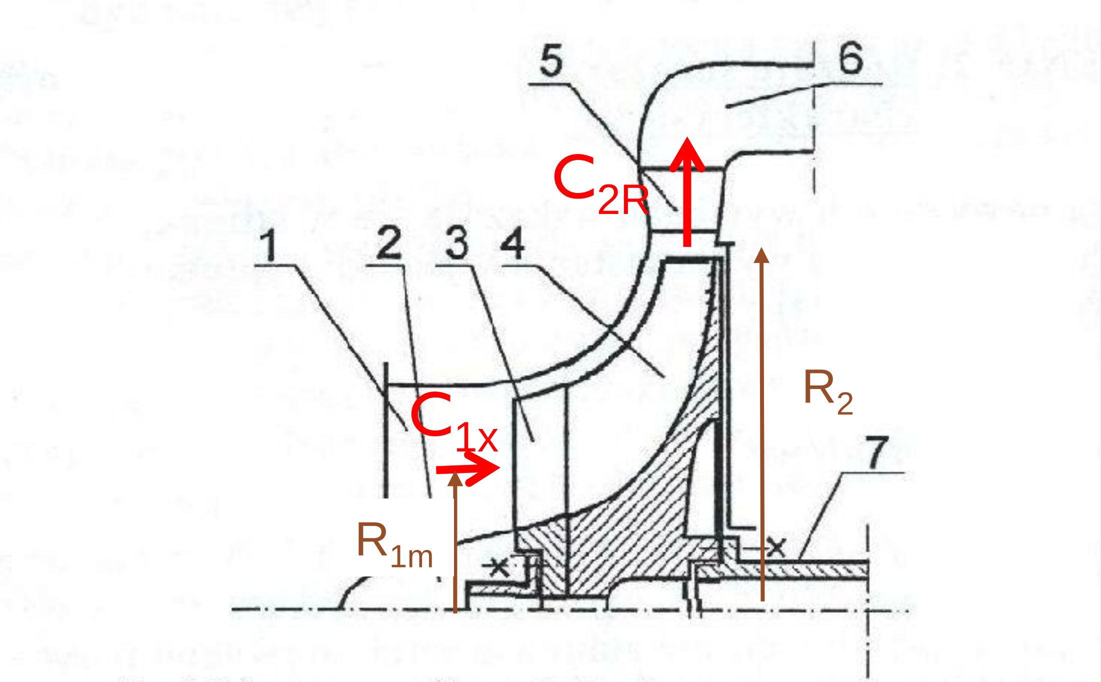
Advantages:

- High pressure ratio in a single stage
- Suitable for small mass-flow engines
- Lower rotational speeds than axial compressors

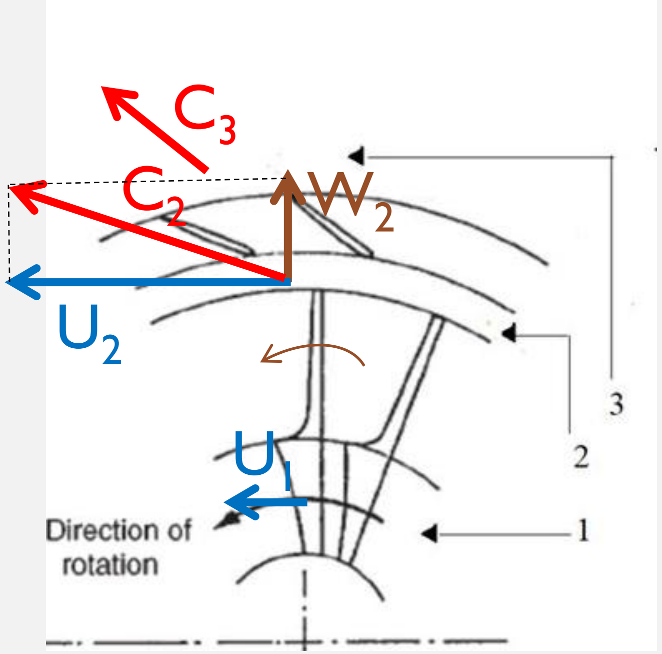
Disadvantages:

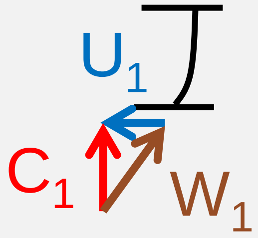
- Limited mass-flow capability
- Larger frontal area compared to axial compressors

SPRĘŻARKCENTRIFUGAL COMPRESSOR STAGE WORKA PROMIENIOWA



Rys.1 Schemat sprężarki promieniowej
1-wlot; 2-owiewka; 3-zabierak; 4-kanały międzyłopatkowe wirnika;
5-dyfuzor; 6-wylot; 7-wał





Velocity diagram includes:

- Absolute velocity C
- Relative velocity W
- Blade speed U

Work per stage:

$$W_{st} = u_2 c_{2u} - u_1 c_{1u} = \omega(R_2 c_{2u} - R_{1m} c_{1u})$$

COMPRESSOR CALCULATIONS (2–3)

COMPRESSOR (2 – 3)
CPR – compressor pressure ratio

$$\pi_C = \frac{P_{t3}}{P_{t2}} = \text{CPR}$$

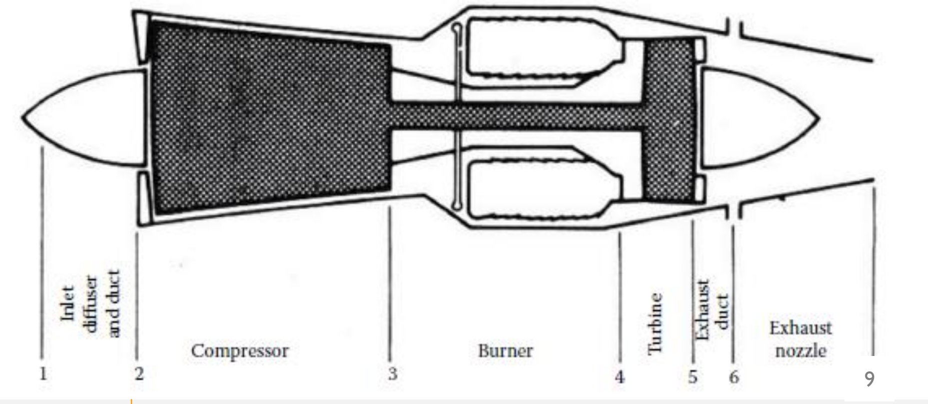
Entropy rise due to losses:
$\Delta s = Cp * ln(T_{t3}/T_{t2}) - Rln(P_{t3}/P_{t2}))$

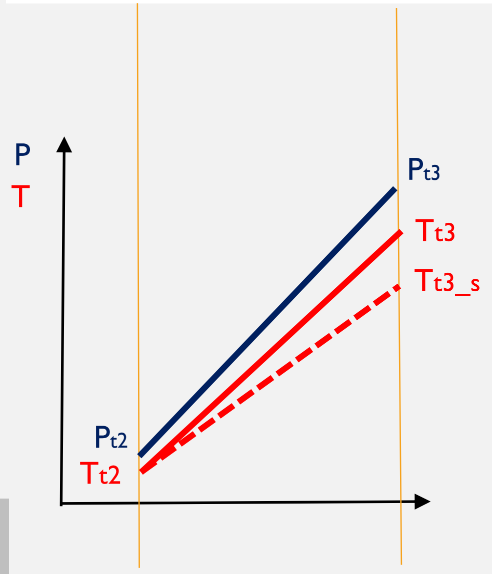
Isentropic efficiency

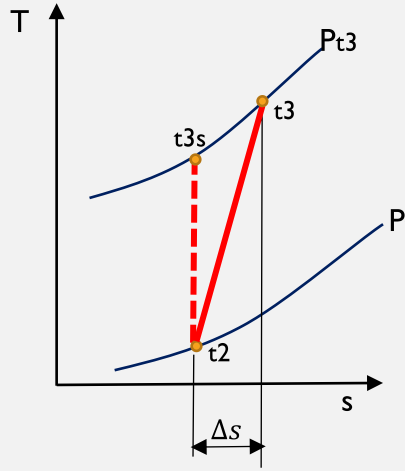
$$\eta_C = \frac{W_{Cs}}{W_C} = \frac{Cp(T_{t3s} - T_{t2})}{Cp(T_{t3} - T_{t2})} = \frac{T_{t3s}/T_{t2} - 1}{T_{t3}/T_{t2} - 1}$$

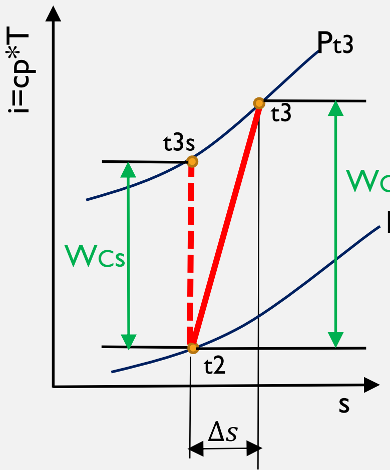
$$= \frac{(P_{t3}/P_{t2})^{\frac{k-1}{k}} - 1}{T_{t3}/T_{t2} - 1} = \frac{(\pi_C)^{\frac{k-1}{k}} - 1}{T_{t3}/T_{t2} - 1}$$

Compressor efficiency describes the relationship between the temperature rise and pressure rise across the compressor.









COMPRESSOR POLYTROPIC EFFICIENCY

$$e_C = \frac{\text{ideal work of compression for a differential pressure change}}{\text{actual work of compression for a differential pressure change}}$$

$$e_C = \frac{di_{ts}}{di_t} = \frac{dT_{ts}}{dT_t} = \frac{dT_{ts}/T_t}{dT_t/T_t}$$

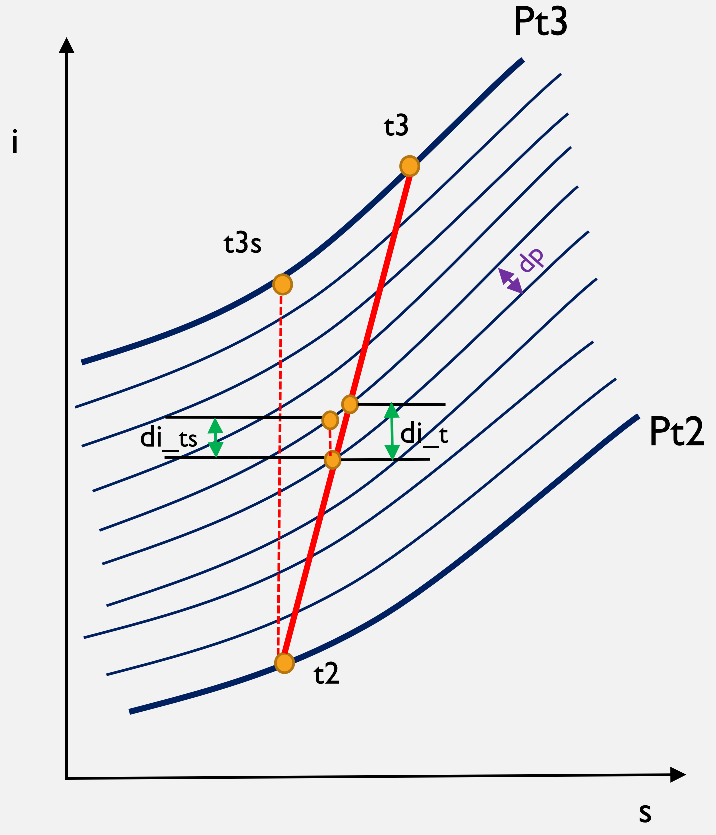
$$dT_{ts}/T_t = \frac{k-1}{k} dP_t/P_t$$

rearangment

$$dT_t/T_t = \frac{k-1}{e_C k} dP_t/P_t \xrightarrow{\text{after differentiation}} \ln(T_{t3}/T_{t2}) = \frac{k-1}{e_C k} ln(P_{t3}/P_{t2})$$

$$e_C = \frac{k-1}{k} ln(P_{t3}/P_{t2})/\ln(T_{t3}/T_{t2})$$

$$T_{t3}/T_{t2} = (P_{t3}/P_{t2})^{\frac{k-1}{e_C\, k}}$$



Polytropic efficiency is treated as a constant for specific compressor. It is value independent of number of stages in the compressor (CPR). Higher polytropic efficiency is for modern compressors

Typical range:

e_C = 0,88 – 0,92

POLYTROPIC VS. ISENTROPIC EFFICIENCY OF COMPRESSOR

Isentropic efficiency

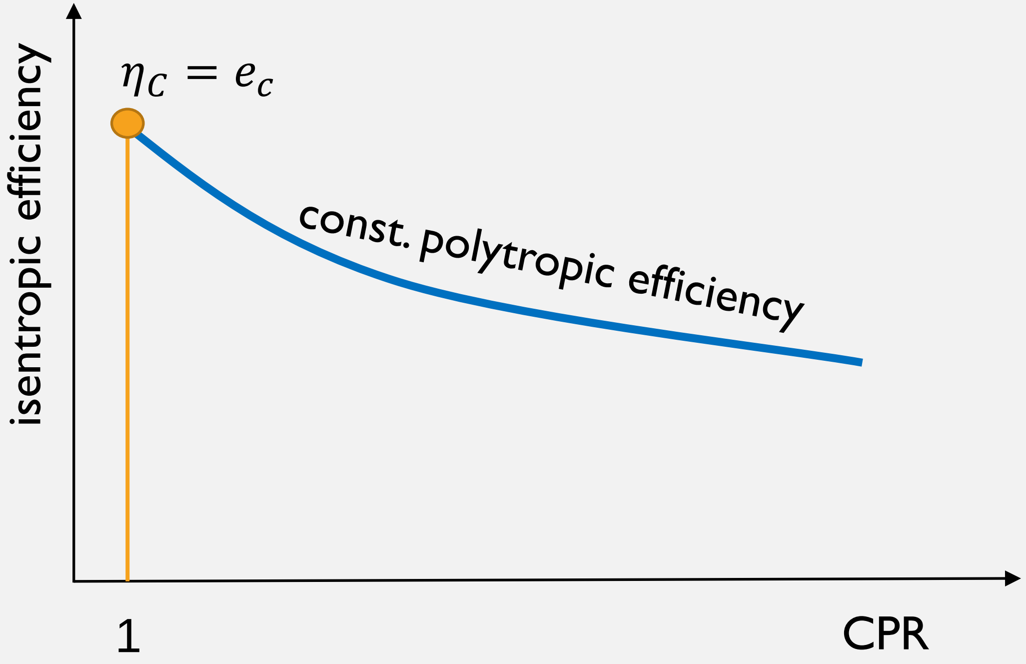
$$\eta_C = \frac{(P_{t3}/P_{t2})^{\frac{k-1}{k}} - 1}{T_{t3}/T_{t2} - 1}$$

Polytropic efficiency relation

$$T_{t3}/T_{t2} = (P_{t3}/P_{t2})^{\frac{k-1}{e_C\,k}}$$

Isentropic vs. polytropic compressor efficiency

$$\eta_C = \frac{(P_{t3}/P_{t2})^{\frac{k-1}{k}} - 1}{(P_{t3}/P_{t2})^{\frac{k-1}{e_C\,k}} - 1}$$



Isentropic efficiency decreases with compressor pressure ratio (CPR) grow for defined polytropic efficiency.

Compressor isentropic efficiency for CPR>1 is lower than polytropic efficiency`

COMPRESSOR WORK AND POWER

Compressor work:

$$W_C = Cp(T_{t3} - T_{t2})$$

for isentropic efficiency →

$$W_C = CpT_{t2}\left(\frac{(\pi_C)^{\frac{k-1}{k}} - 1}{\eta_C}\right)$$

for polytropic efficiency →

$$W_C = CpT_{t2}\left((\pi_C)^{\frac{k-1}{e_C\,k}} - 1\right)$$

Compressor power:

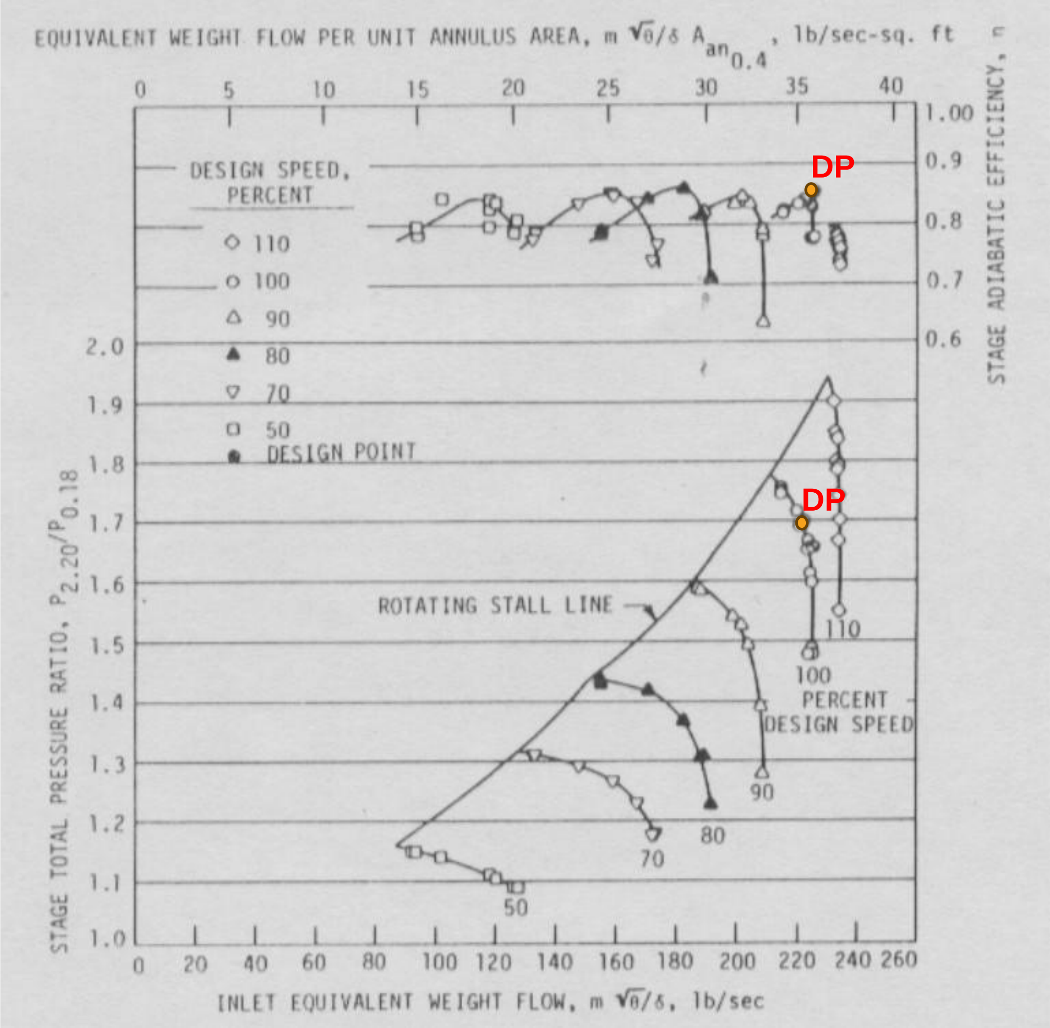
$$P_C = \dot{m}_C W_C = \dot{m}_0 Cp(T_{t3} - T_{t2})$$

The compressor work is associated with the enthalpy rise (temperature rise) across the compressor.

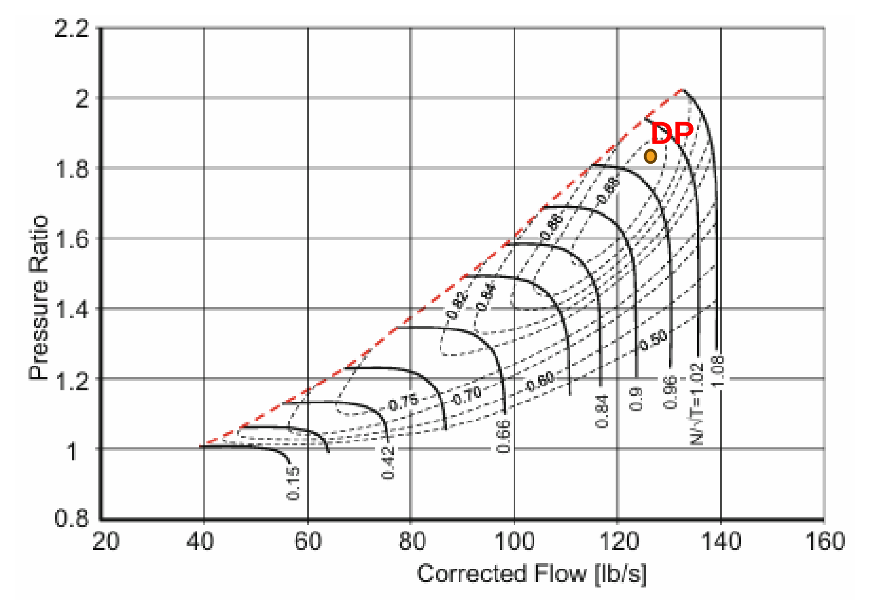
The compressor power depends on the work and, additionally, on the mass flow rate of air passing through the compressor.

Fans in large turbofan engines are characterised by low specific work (a small increase in enthalpy and gas temperature), but due to the large mass flow rate of air, the power required to drive them is high.

COMPRESSOR MAP



m {kg/s} = m {lb/s} × 0.45359237



Compressor map shows relationships between: Pressure ratio Corrected mass flow Corrected rotational speed and Isentropic efficiency

Corrected mass flow:

$$m_{cor} = m\sqrt{T_{t1}/288}\,\frac{1013.25}{P_{t1}}$$

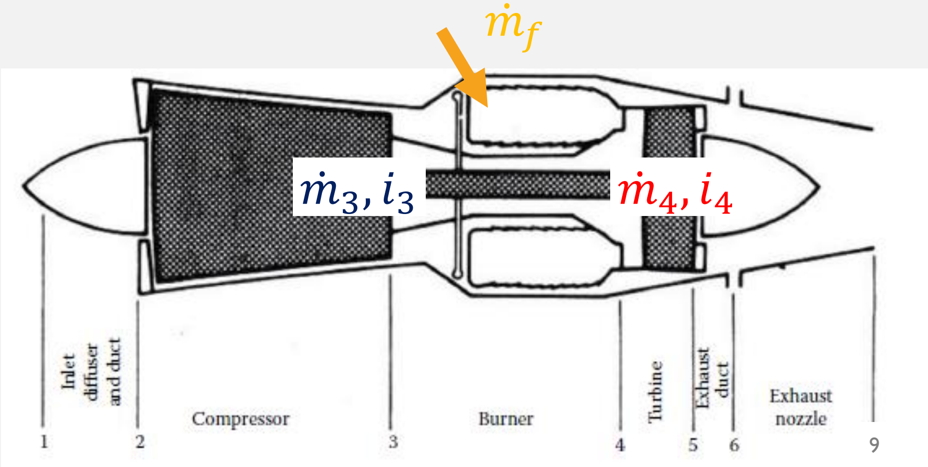
Corrected speed:

$$N_{cor} = N/\sqrt{T_{t1}/288}$$

BURNER/COMBUSTER/COMBUSTION CHAMBER

The combustion chamber in a gas turbine engine is located between the compressor and turbine assemblies.

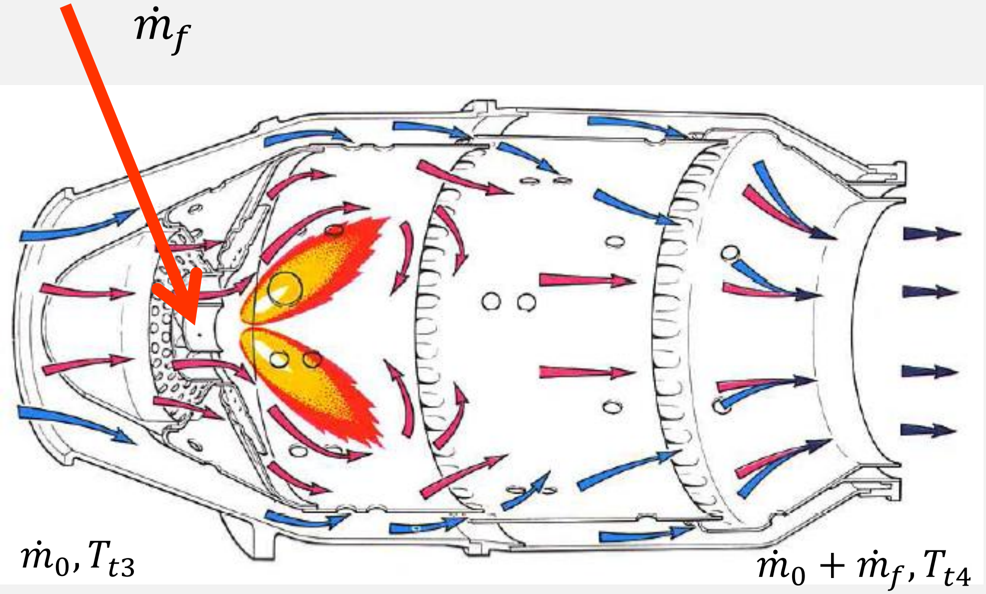
Its role is to efficiently convert the chemical energy of the fuel into thermal energy of the working gas while minimizing flow losses (pressure losses).



ENERGY BALANCE IN THE COMBUSTION CHAMBER

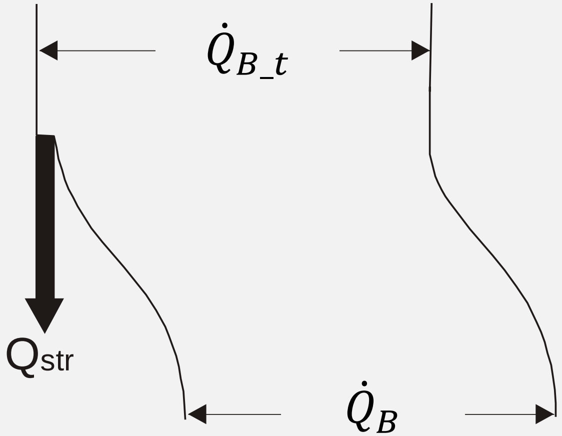
Energy Balance

$$\dot{Q}_B = \Delta\dot{I}_B = \bar{c}_p\left((\dot{m}_0 + \dot{m}_f)T_{t4} - \dot{m}_0 T_{t3}\right) \approx \dot{m}_0 \bar{c}_p (T_{t4} - T_{t3})$$



Thermal efficiency of the combustor defines how effectively the supplied fuel energy increases the gas enthalpy.

$$\eta_B = \frac{\dot{Q}_B}{\dot{Q}_{B\_t}} = \frac{\dot{m}_0\ \bar{c}_p\left(T_{t4} - T_{t3}\right)}{\dot{m}_f FHV} = \frac{\bar{c}_p\left(T_{t4} - T_{t3}\right)}{FHV\ f_{pal}}$$

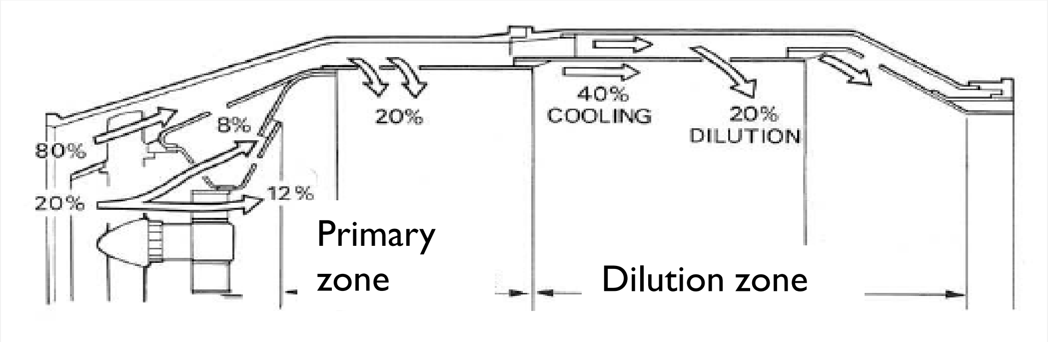


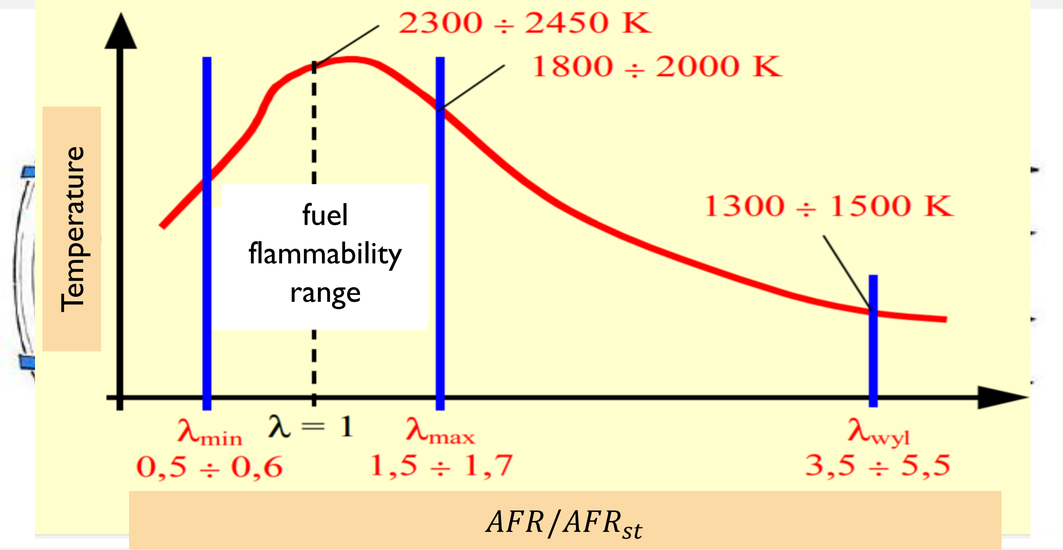
$\dot{Q}_{B\_t}$ - Theoretical heat released by the fuel

$\dot{Q}_B$ - The actual heat transferred to the airflow in the combustor

$f_B = \dot{m}_f / \dot{m}_0$ - Fuel–air ratio determines the relative fuel consumption

ORGANIZATION OF THE COMBUSTION PROCESS





Fuel: **aviation kerosene**

mass fractions

$$C = 0{,}86, \qquad H = 0{,}14$$

Stoichiometric oxygen requirement,

$$O_t = \frac{8}{3} \cdot 0.86 + 8 \cdot 0{,}14 = 3{,}413 \left[\frac{kgO_2}{kg\ fuel}\right]$$

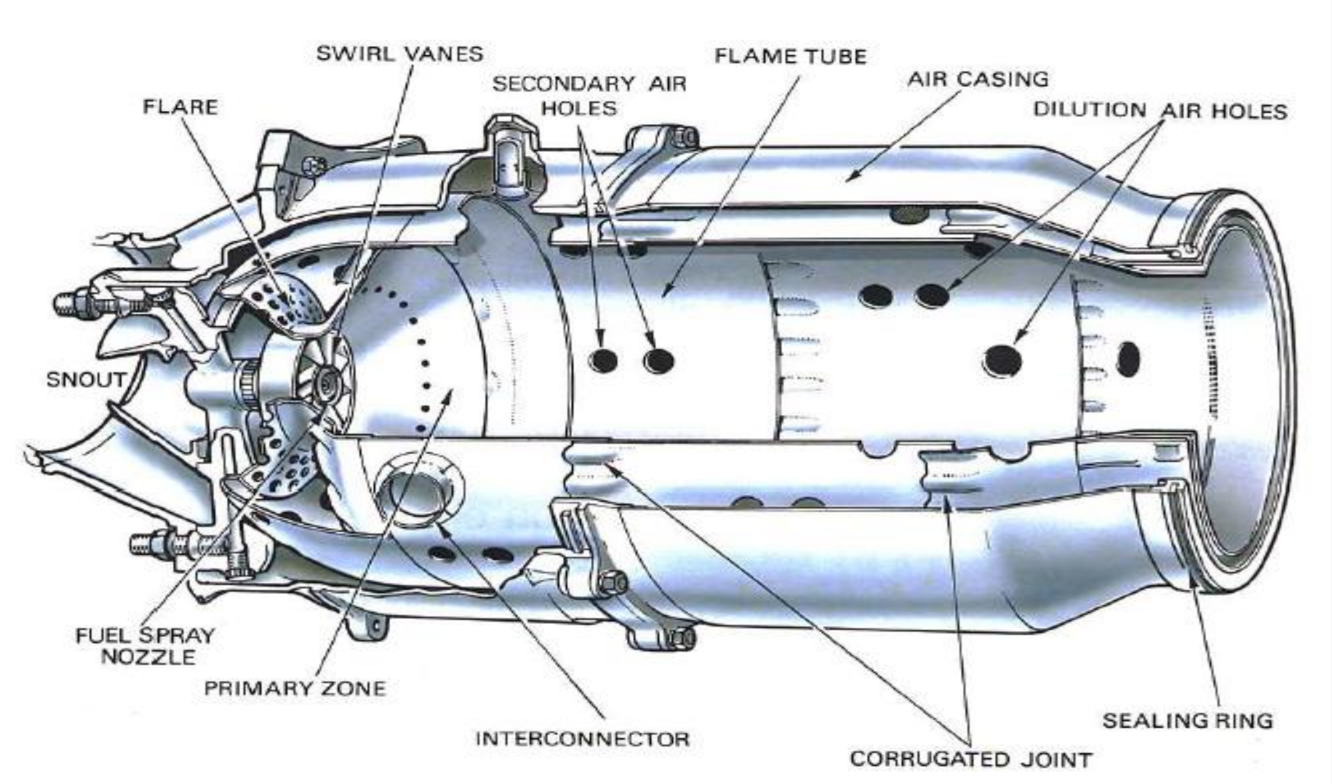
Stoichiometric air–fuel ratio,

$$AFR_{st} = \frac{O_t}{0{,}232} = 14{,}7 \left[\frac{kg\ air}{kg\ fuel}\right]$$

The fuel-to-air mass ratio for a stoichiometric mixture is therefore approximately 0,067, whereas in a single-flow gas turbine engine, fB≈0,02.This means that the amount of air relative to the amount of fuel is about three times greater than required by the combustion balance. Consequently, the exhaust gases from a gas turbine engine contain approximately 30% combustion products, while the remainder consists of unreacted air.

The amount of air supplied to the primary zone is about 1/15 of the total air entering the combustor.

COMBUSTION CHAMBER DESIGN



The liner/ flame tube contains openings arranged to:

- Create a proper fuel–air mixture near the injector
- Sustain and complete combustion by adding air downstream
- Cool the combustion products in the dilution zone
- Form a protective cooling film along the liner walls

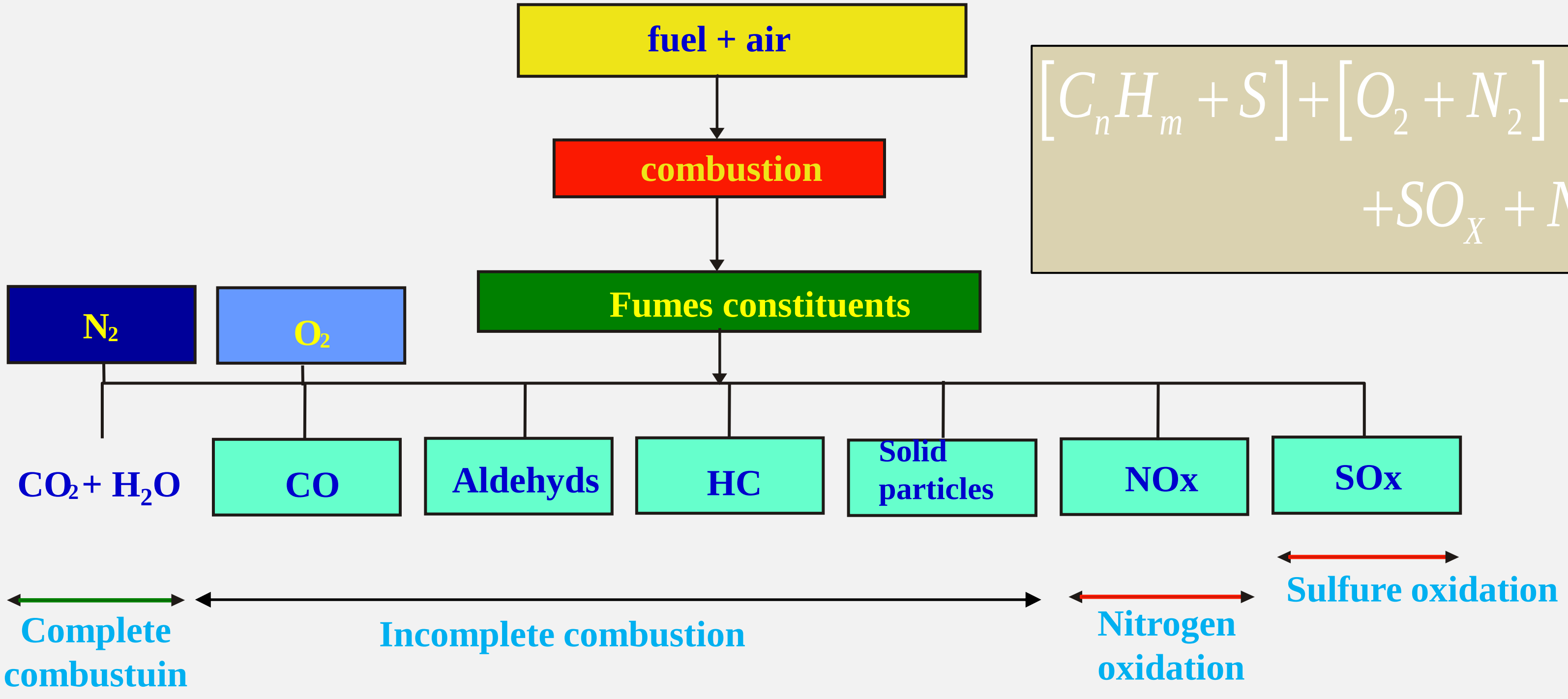
COMBUSTION

Ideal combustion

$$[C_nH_m + S] + [O_2 + N_2] \rightarrow CO_2 + H_2O + N_2 + O_2 + SO_2$$

fuel air → fumes

Real combustion



fuel + air

combustion

Fumes constituents

N_2

O_2

$CO_2 + H_2O$

CO

Aldehyds

HC

Solid particles

NOx

SOx

Complete combustuin

Incomplete combustion

Nitrogen oxidation

Sulfure oxidation

$$[C_nH_m + S] + [O_2 + N_2] \rightarrow CO_2 + H_2O + N_2 + O_2 + \\ + SO_X + NO_X + CO + C_{(sadza)} + UHC$$

FLOW LOSSES IN THE COMBUSTION CHAMBER

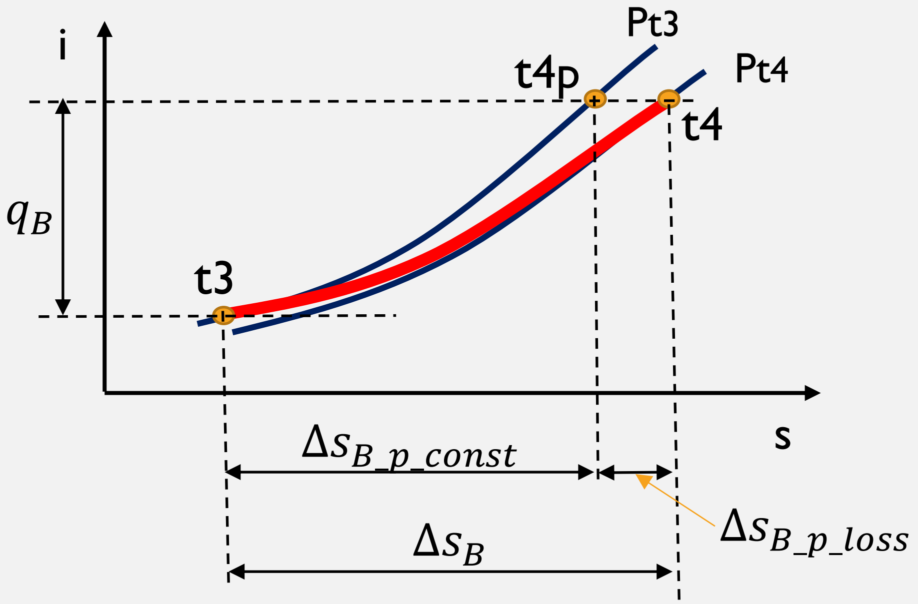
Pressure losses occur due to:

- Aerodynamic friction
- Mixing processes
- Heat addition

$\pi_B = \pi_{B\_M} \cdot \pi_{B\_T}$ - Burner pessure losses coefficient:

π_{B_M} - Mechanical lossese coefficient

π_{B_T} - Thermal lossese coefficient



Pressure losses:

$$\pi_B = \frac{P_{t4}}{P_{t3}}$$

$$q_B = \dot{Q}_B / \dot{m}_0$$

Entropy increase in a burner:

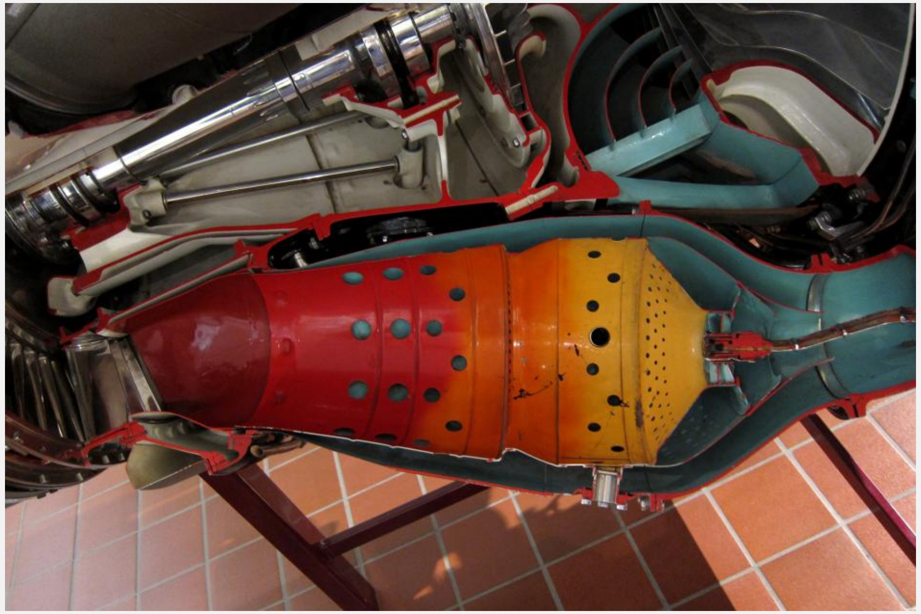
Isobaric entropy rise $\Delta s_{B\_p\_const} = cp_B ln\frac{T_{t4}}{T_{t3}}$

Pressure losses entropy rise $\Delta s_{B\_p\_loss} = -R_t ln\frac{P_{t4}}{P_{t3}}$

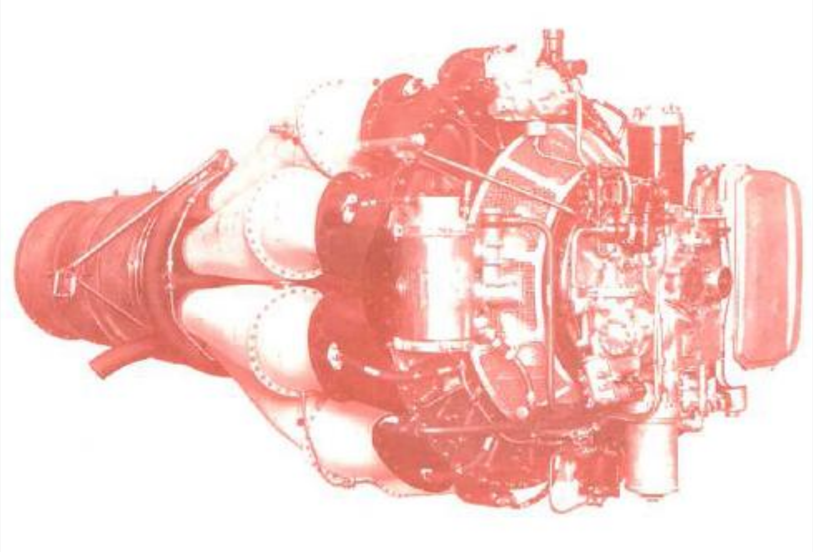
Overall entropy rise $\Delta s_B = cp_B ln\frac{T_{t4}}{T_{t3}} - R_t ln\frac{P_{t4}}{P_{t3}}$

EARLY COMBUSTION CHAMBER DESIGNS

Can-type combustor (individual cans):



Komora spalania z pierwszych konstrukcji silników Whitl’a



Rolls-Royce RB Derwent I

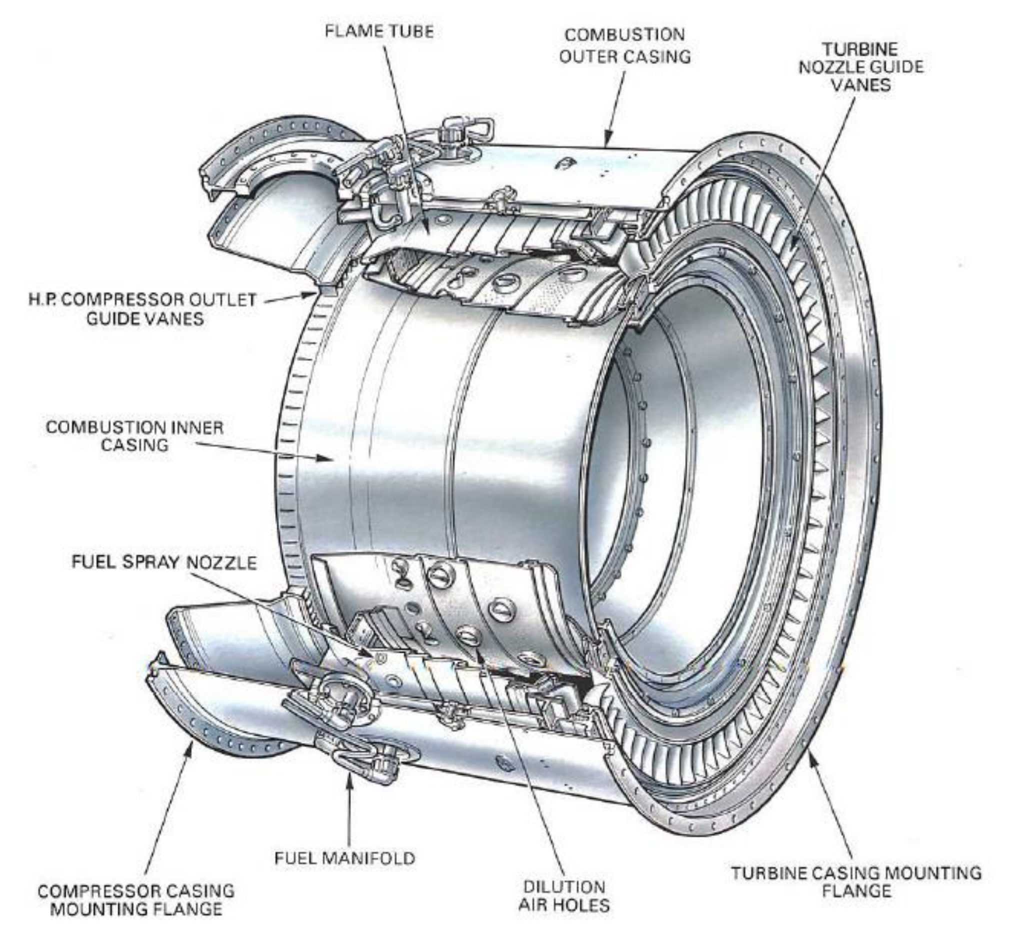
Advantages:

- Easy experimental evaluation
- Individual can replacement

Disadvantages:

- High flow resistance
- Poor circumferential temperature uniformity at combustor exit

ANNULAR COMBUSTION CHAMBER



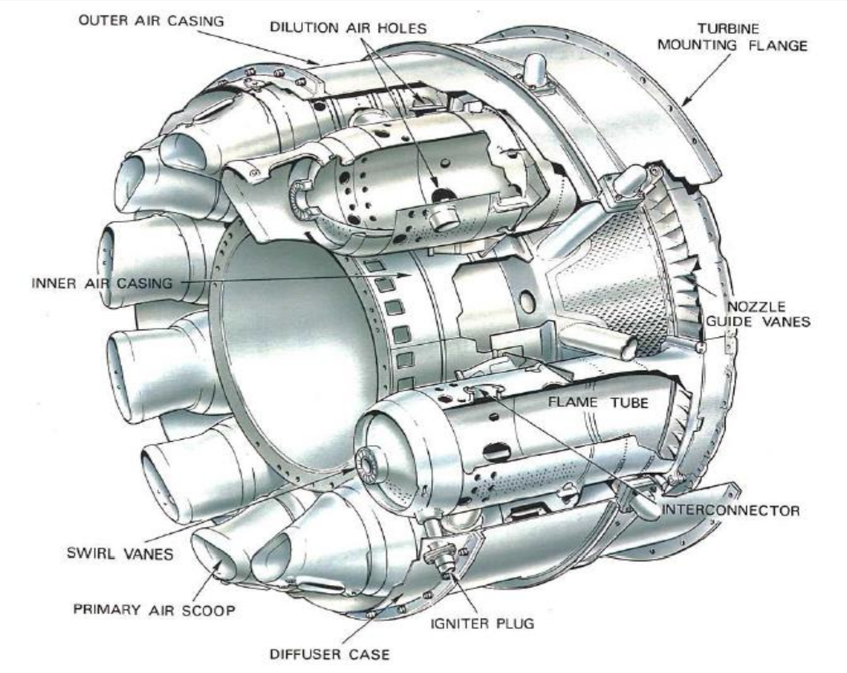
Advantages:

- Compact and lightweight
- Low flow resistance
- Good circumferential temperature uniformity

Disadvantages:

- Testing must be performed on the entire module
- Damage requires replacement of the whole module

CAN-ANNULAR COMBUSTION CHAMBER

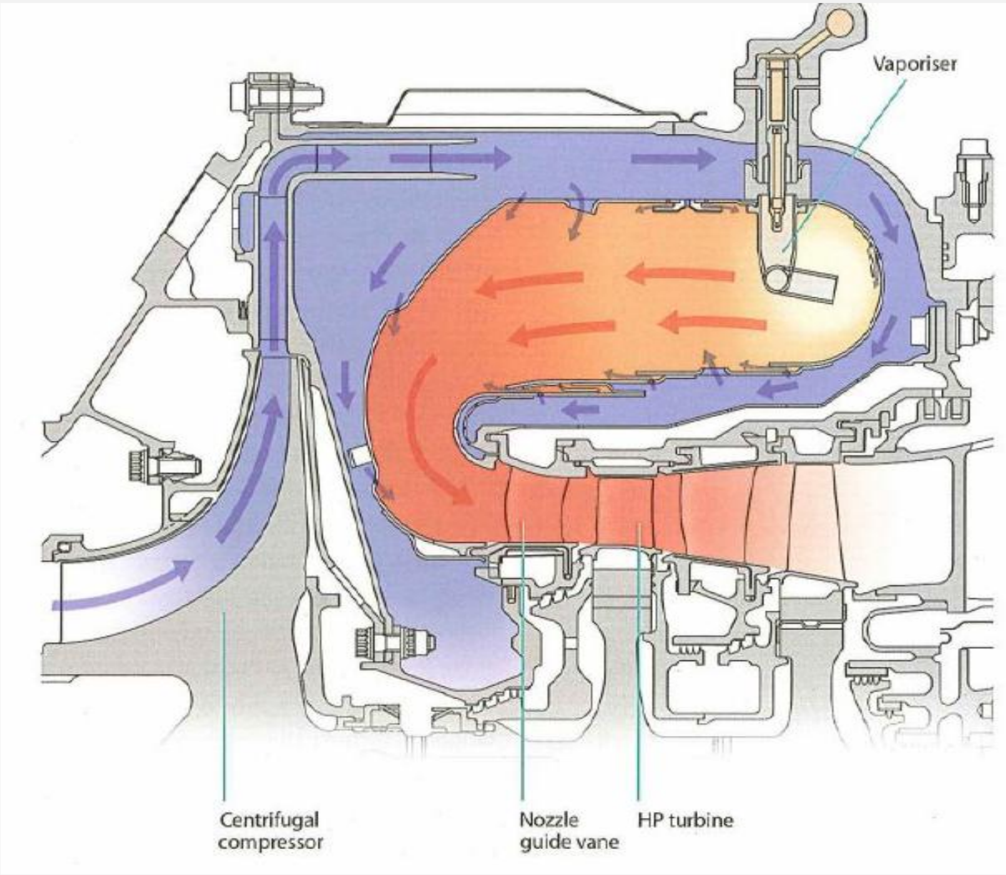


Combustion occurs in individual liners

The outer casing forms a structural load-bearing element

A hybrid solution between can-type and annular combustors

REVERSE-FLOW COMBUSTION CHAMBER



Characteristics:

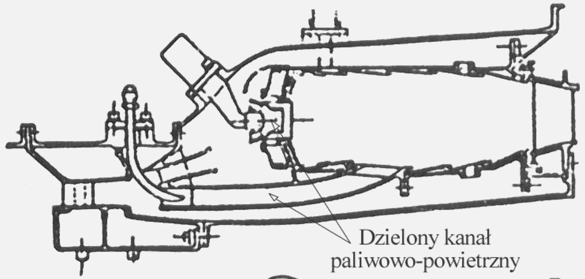
- Shorter axial length at the cost of increased radial dimension
- Improved thermal characteristics due to counter-flow air supply
- Higher flow losses due to multiple flow direction changes

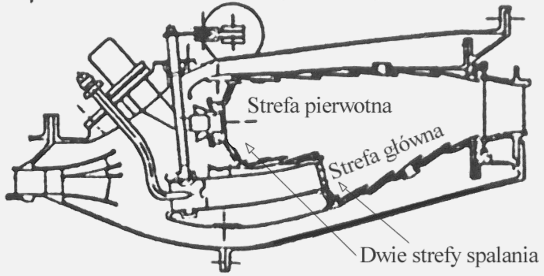
Applications:

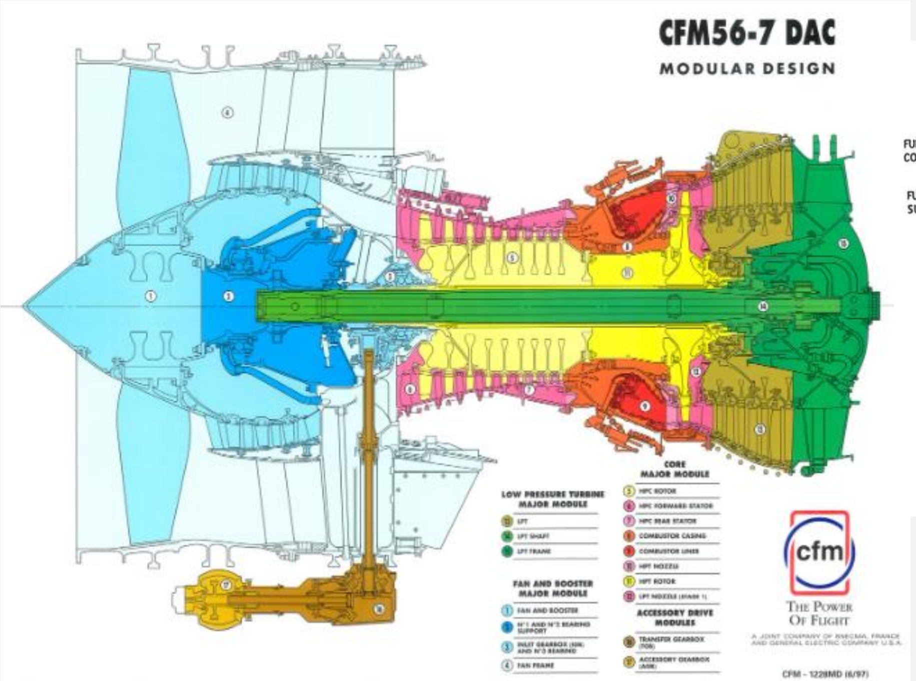
- Small turbofan engines
- Turboshaft and helicopter engines, especially when the last compressor stage is centrifugal

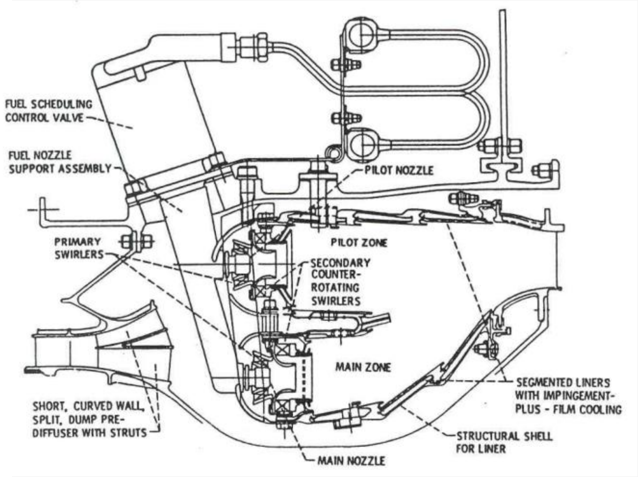
MODERN DUAL ANNULAR COMBUSTOR (DAC)

double-annular combustor arrangements:
- Series configuration
- Parallel configuration



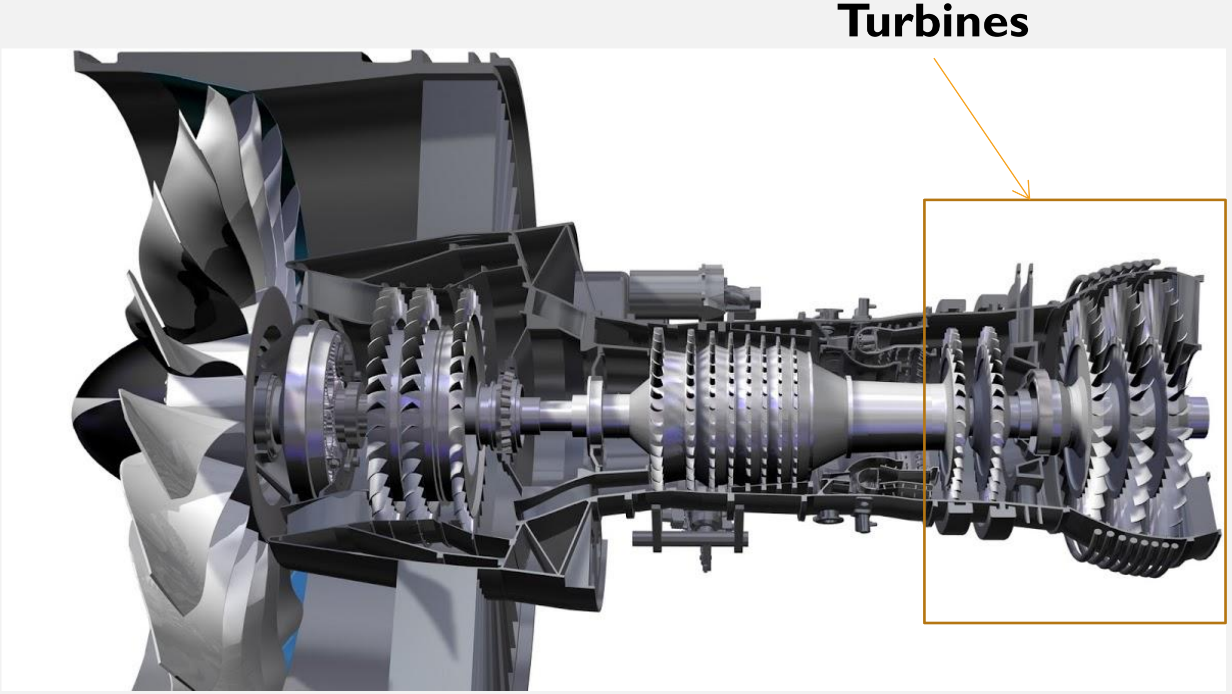






„Double-annular burner” (double-annular combustor, DAC). A dual-annular combustor contains two concentric combustion zones:an outer pilot zone operating at low thrust,an inner main zone activated at higher thrust levels.This design improves combustion stability at low power and reduces NOx emissions at high power compared to single-annular combustors.

TURBINE IN AN AIRCRAFT ENGINE



Turbine rotor blades

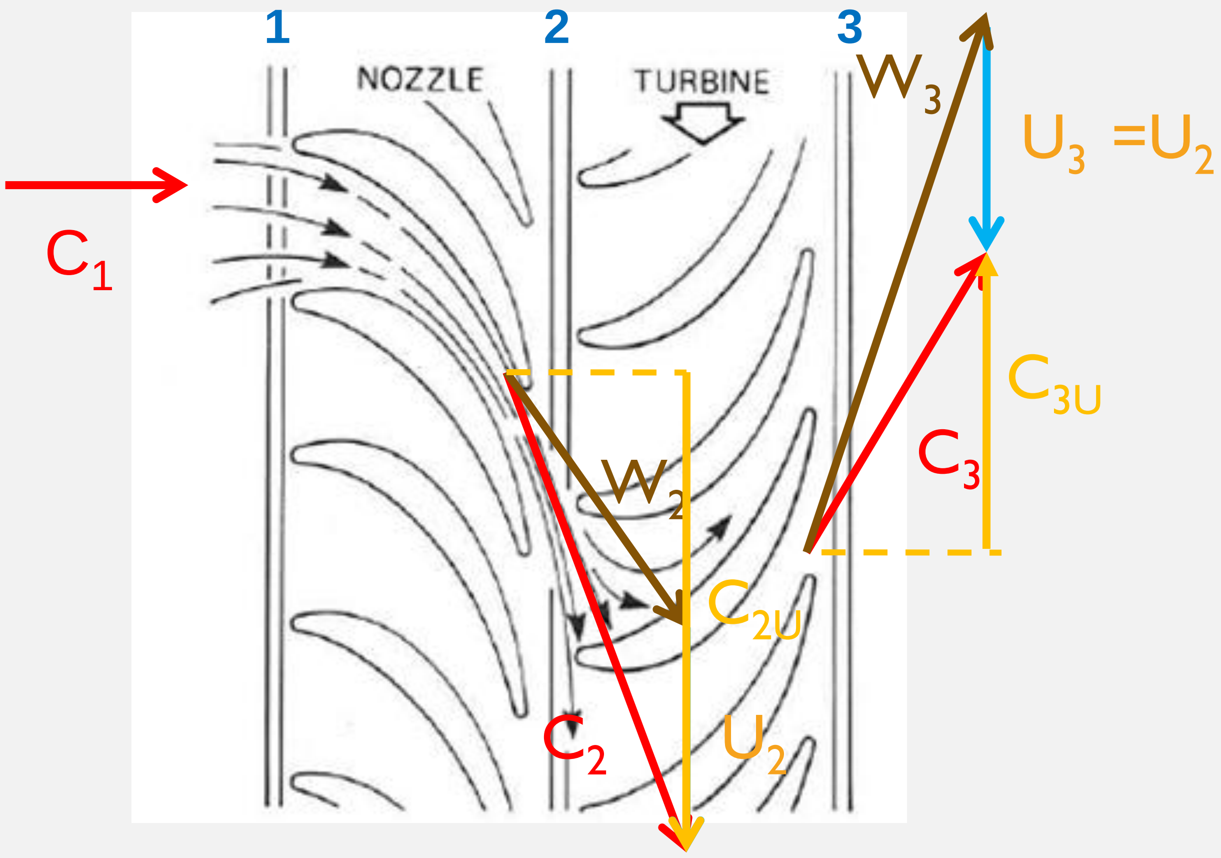
A turbine may consist of one to three turbine modules, each containing one or more stages.
Large high-bypass turbofan engines can extract 30–50 MW of mechanical power.
A single turbine blade can produce power comparable to that of a sports-car engine. Turbine inlet temperatures exceed 1800 K.

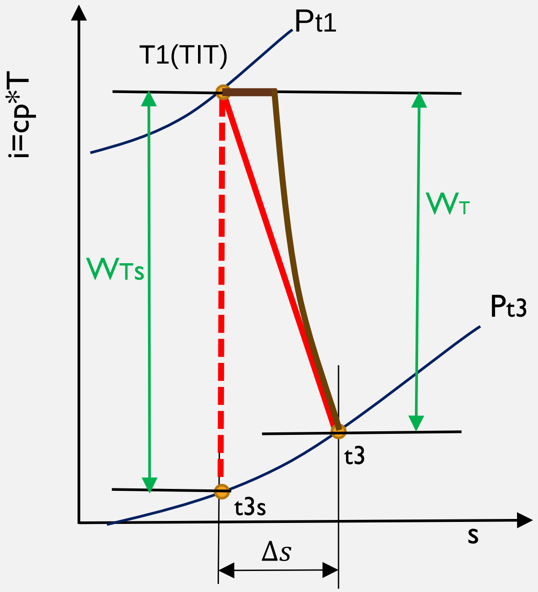
AIRCRAFT ENGINE TURBINES

Turbines convert the thermal and kinetic energy of the exhaust gases into mechanical power used to drive:

- **compressors,**
- **the fan,**
- **engine accessories (oil pumps, fuel pumps, generators)**
- **and, in turboshaft and helicopter engines, the main output shaft producing thrust or lift.**

OPERATION OF A TURBINE STAGE





Turbine stage work

$$W_{st} = u_2(c_{2u} + c_{3u})$$

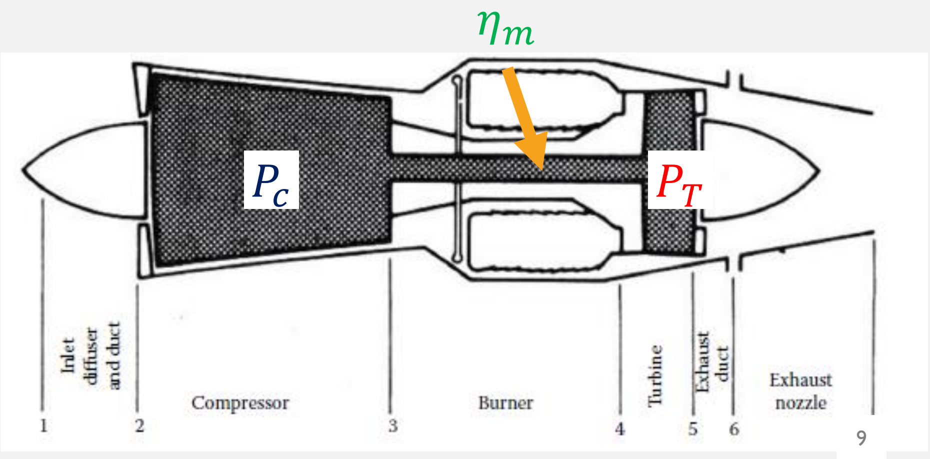
Turbine stage work

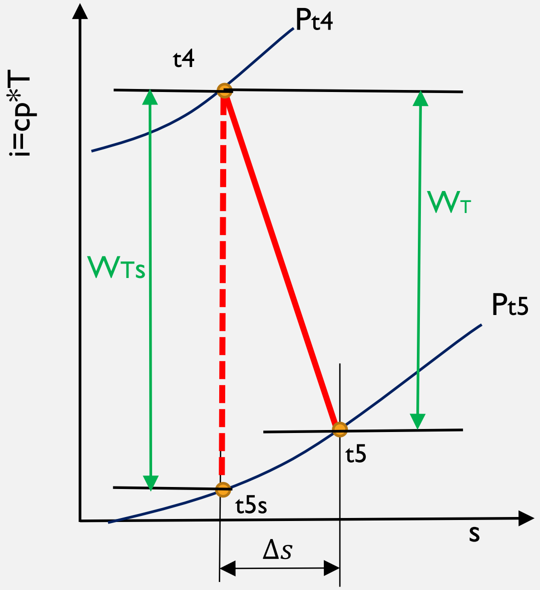
$$W_{st} = c_p(T_{t1} - T_{t3})$$

Turbine efficiency

$$\eta_T = \frac{W_T}{W_{Ts}}$$

TURBINE





Compressor-turbine energy balance:

$$P_T = \dot{m}_4 Cp_T (T_{t4} - T_{t5}) = \frac{1}{\eta_m} P_C$$

Turbine outlet temperature

$$T_{t5} = T_{t4} - \frac{P_C}{\eta_m \dot{m}_4 Cp_T}$$

for: $\dot{m}_2 = \dot{m}_0$ and $\dot{m}_4 = \dot{m}_0 + \dot{m}_f$

$$T_{t5} = T_{t4} - \frac{w_C}{\eta_m (1+f) Cp_T}$$

Turbine pressure ratio (TPR)

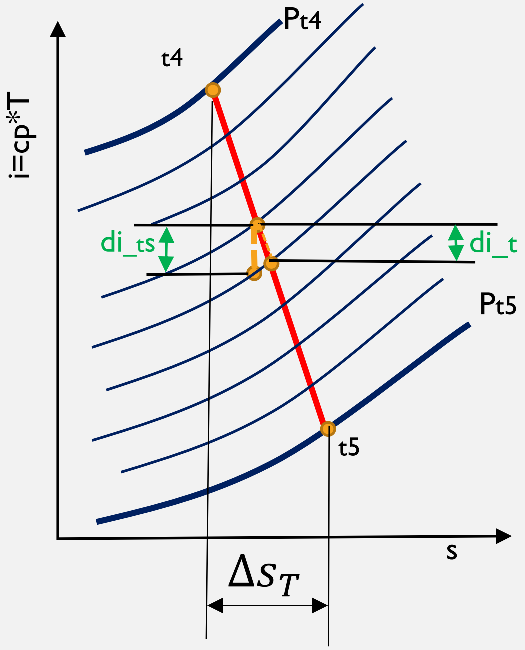
$$\pi_T = P_{t4}/P_{t5} > 1$$

$$\frac{1}{\pi_T} = \left(1 - \frac{1 - T_{t5}/T_{t4}}{\eta_T}\right)^{\frac{k_t}{k_t - 1}}$$

Turbine isentropic efficiency

$$\eta_T = \frac{W_T}{W_{Ts}} = \frac{Cp_t (T_{t4} - T_{t5})}{Cp_t (T_{t4} - T_{t5s})} = \frac{1 - T_{t5}/T_{t4}}{1 - T_{t5s}/T_{t4}} = \frac{1 - T_{t5}/T_{t4}}{1 - (P_{t5}/P_{t4})^{\frac{k_t - 1}{k_t}}}$$

TURBINE POLYTROPIC EFFICIENCY



$$e_T = \frac{\text{actual turbine work for a differential pressure change}}{\text{ideal work of turbine for a differential pressure change}}$$

$$e_T = \frac{di_t}{di_{ts}} = \frac{dT_t}{dT_{ts}} = \frac{dT_t/T_t}{dT_{ts}/T_t}$$

$$dT_{ts}/T_t = \frac{k_t - 1}{k_t} dP_t/P_t$$

rearangment

$$dT_t/T_t = e_T \frac{k_t - 1}{k_t} dP_t/P_t$$

after differentiation

$$\ln(T_{t5}/T_{t4}) = e_T \frac{k_t - 1}{k_t} ln(P_{t5}/P_{t4})$$

$$e_T = \frac{k_t}{k_t - 1} ln\,(T_{t5}/T_{t4})/ln(P_{t5}/P_{t4})$$

$$T_{t5}/T_{t4} = (P_{t5}/P_{t4})^{\frac{e_T(k_t-1)}{k_t}}$$

Turbine entropy grow:

$$\Delta s_T = Cp_t * ln(T_{t5}/T_{t4}) - R_t ln(P_{t5}/P_{t4}))$$

Polytropic efficiency is assumed as a constant for the turbine. It’s value is independent of number of stages in the turbine (TPR). Higher polytropic efficiency is for modern turbine.

Typical range:

e_T = 0,85 – 0,9

POLYTROPIC VS. ISENTROPIC EFFICIENCY OF TURBINE

Isentropic efficiency

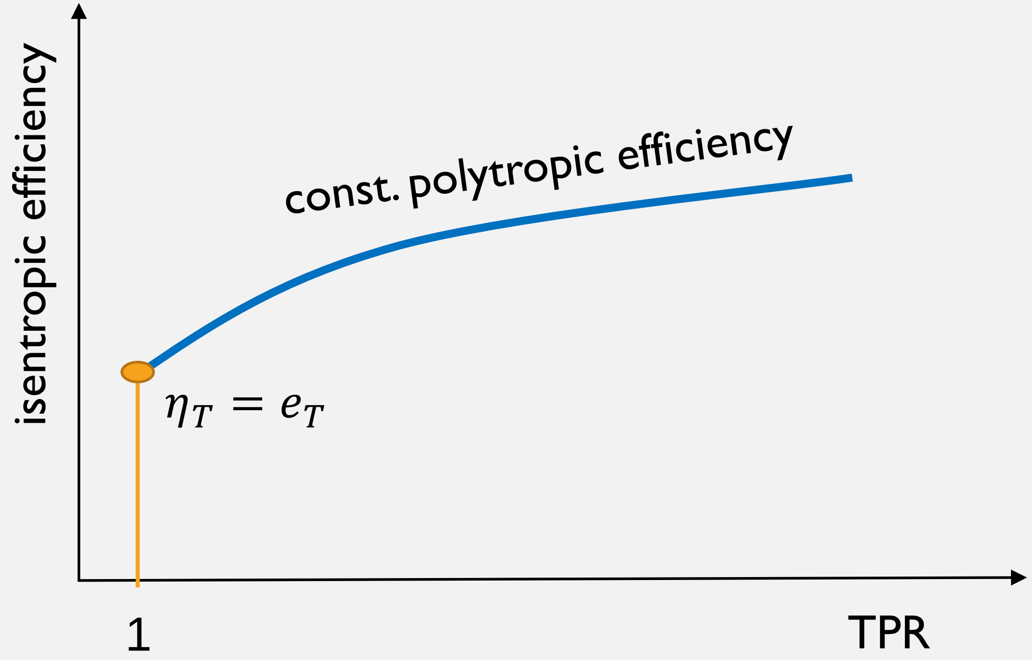
$$\eta_T = \frac{1 - T_{t5}/T_{t4}}{1 - (P_{t5}/P_{t4})^{\frac{k_t - 1}{k_t}}}$$

Polytropic efficiency relation

$$T_{t5}/T_{t4} = (P_{t5}/P_{t4})^{\frac{e_T(k_t - 1)}{k_t}}$$

Isentropic vs. polytropic compressor efficiency

$$\eta_T = \frac{1 - (P_{t5}/P_{t4})^{\frac{e_T(k_t - 1)}{k_t}}}{1 - (P_{t5}/P_{t4})^{\frac{k_t - 1}{k_t}}}$$

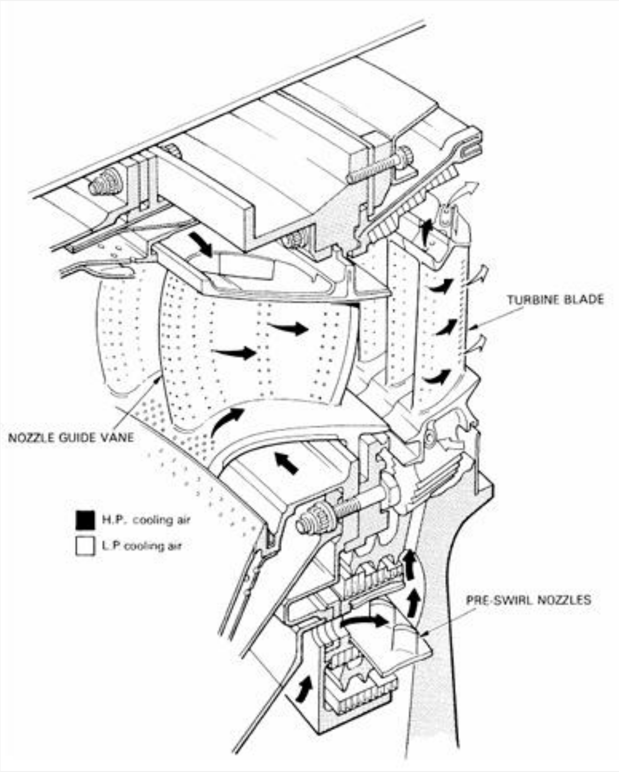


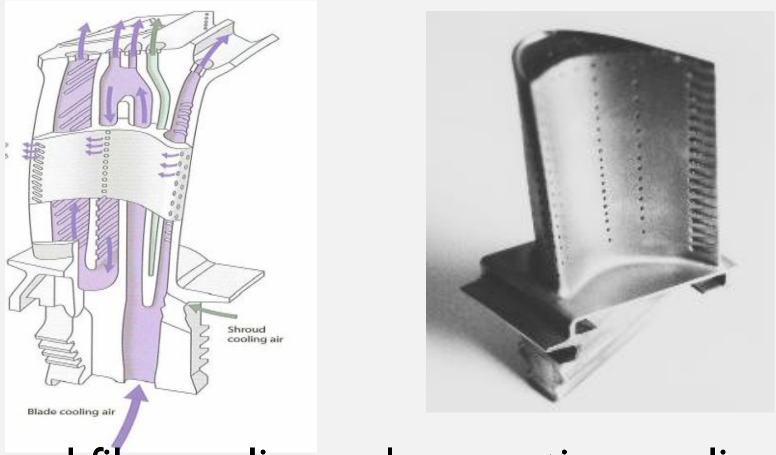
Isentropic efficiency increases with growing of turbine pressure ratio (TPR) for defined polytropic efficiency.

Turbine isentropic efficiency for TPR>1 is higher than polytropic efficiency`

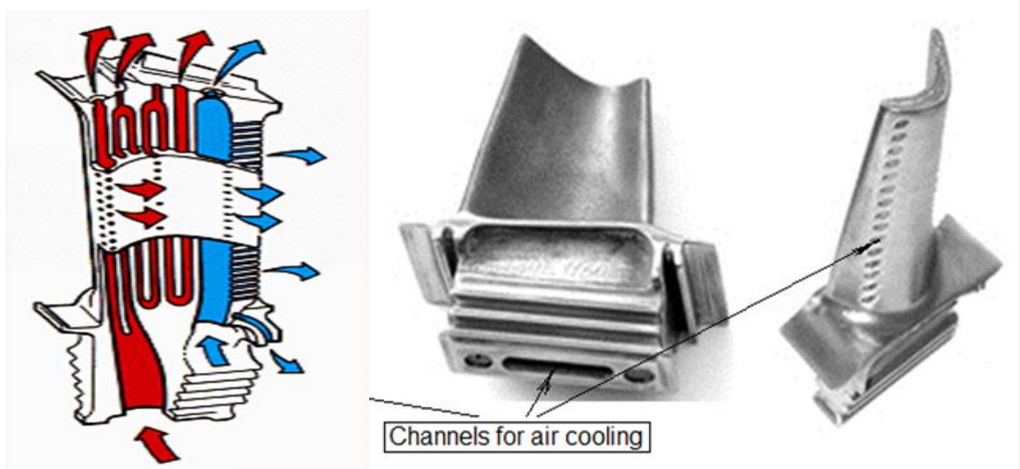
TURBINE COOLING

High-pressure turbine stages operate at extremely high temperatures and require intensive cooling.
Cooling methods include:



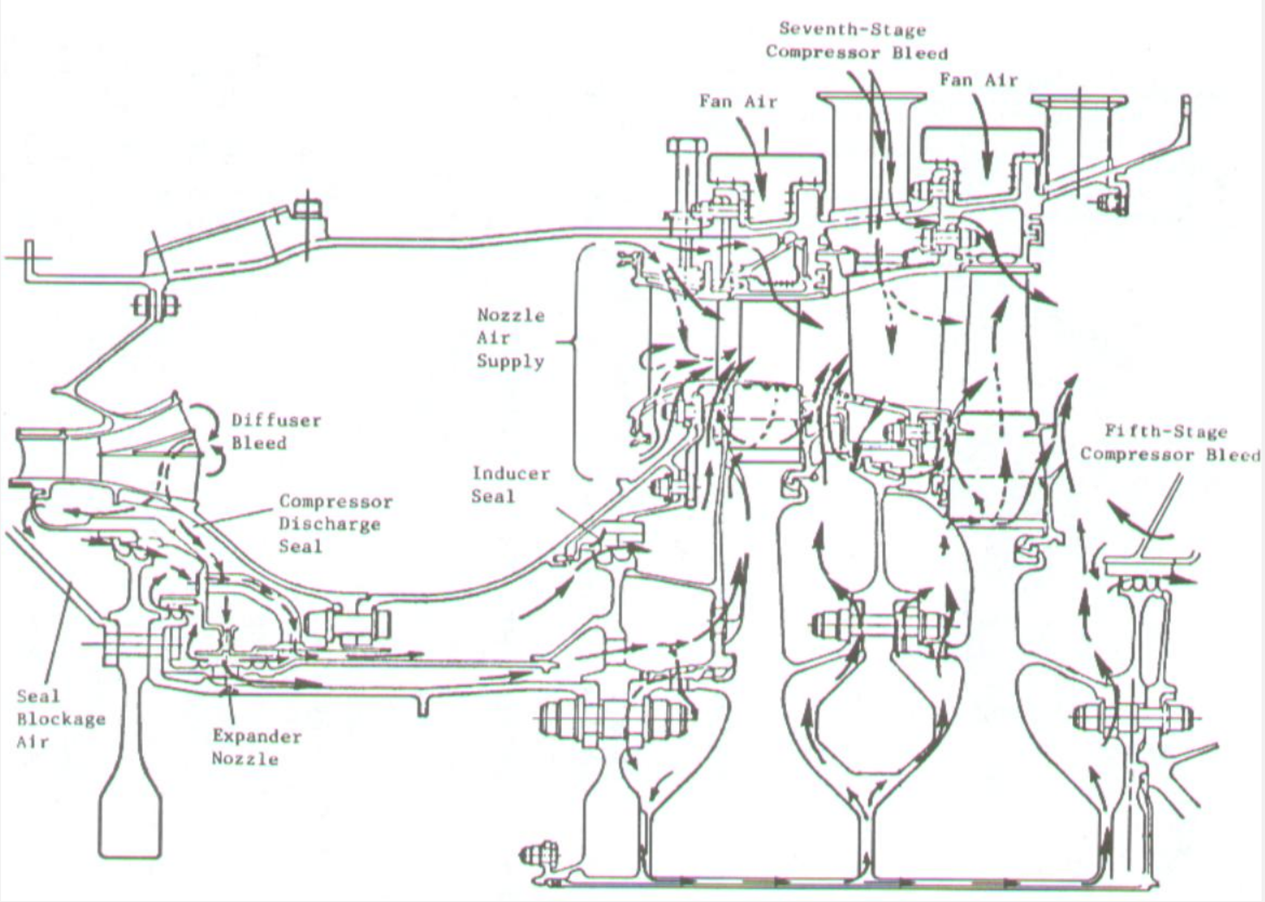


external film cooling and convection cooling,



Internal convective cooling passages

AIR FOR TURBINE COOLING



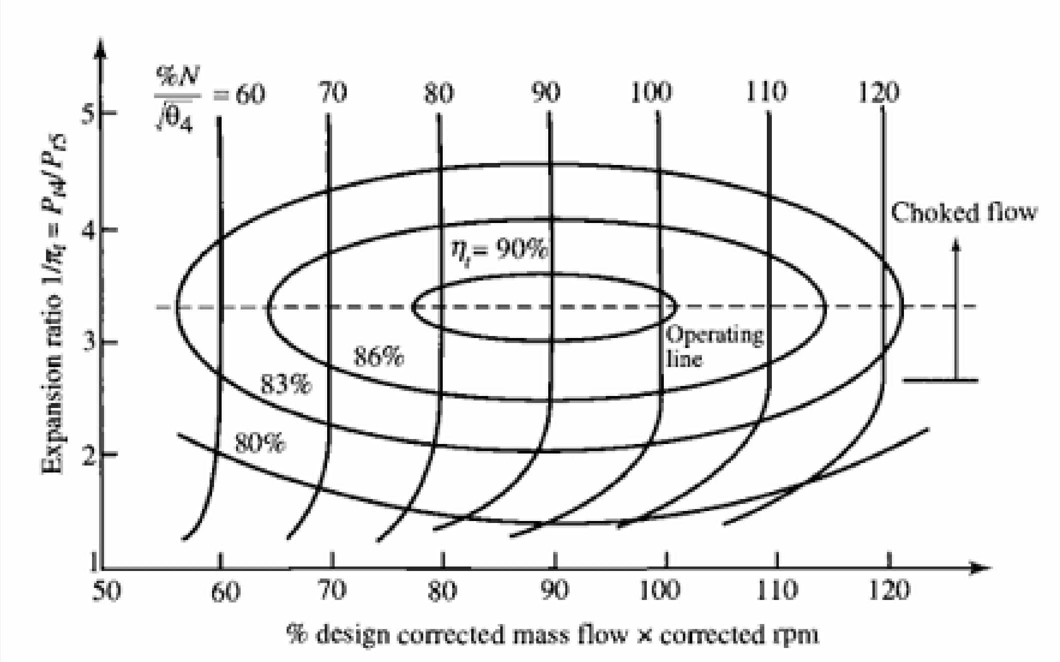
Cooling air must have sufficiently high pressure and temperature. It is extracted from the compressor:

- The first stage of the high-pressure turbine is cooled using air taken directly from the compressor exit.
- Subsequent turbine stages are cooled using air extracted from earlier compressor stages.

Cooling increases flow losses and reduces turbine efficiency.

Schema of turbine cooling of NASA E3 engine (NASA CR-167955) [Je-Chin Han, Sandip Dutta, Srinath V. Ekkad: Gas turbine heat transfer and cooling technology, Taylor & Francis, New York 2001]

TURBINE CHARACTERISTICS

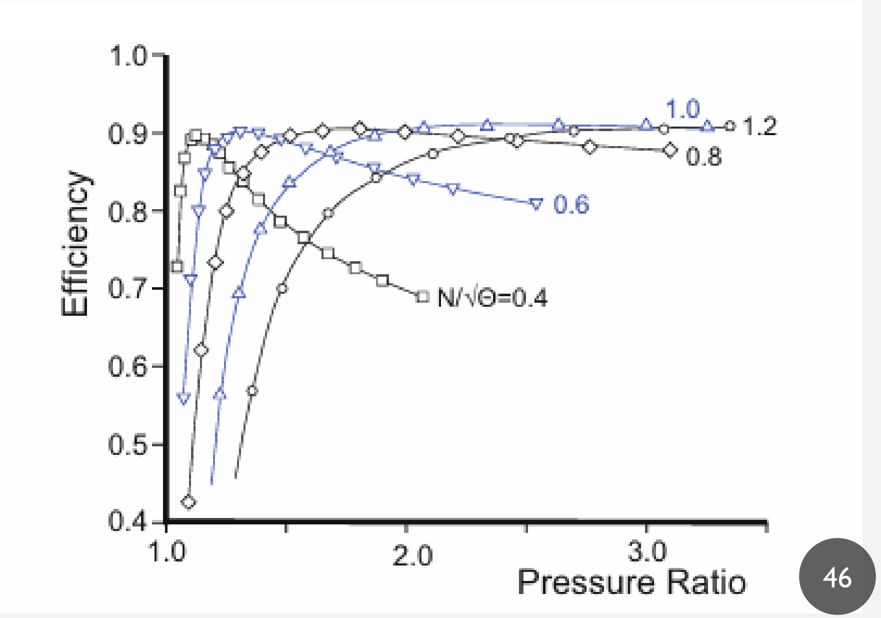


Corrected rotational speed

$$N_{cor} = N/\sqrt{\theta_4} = N/\sqrt{T_{t4}/288}$$

Corrected mass flow

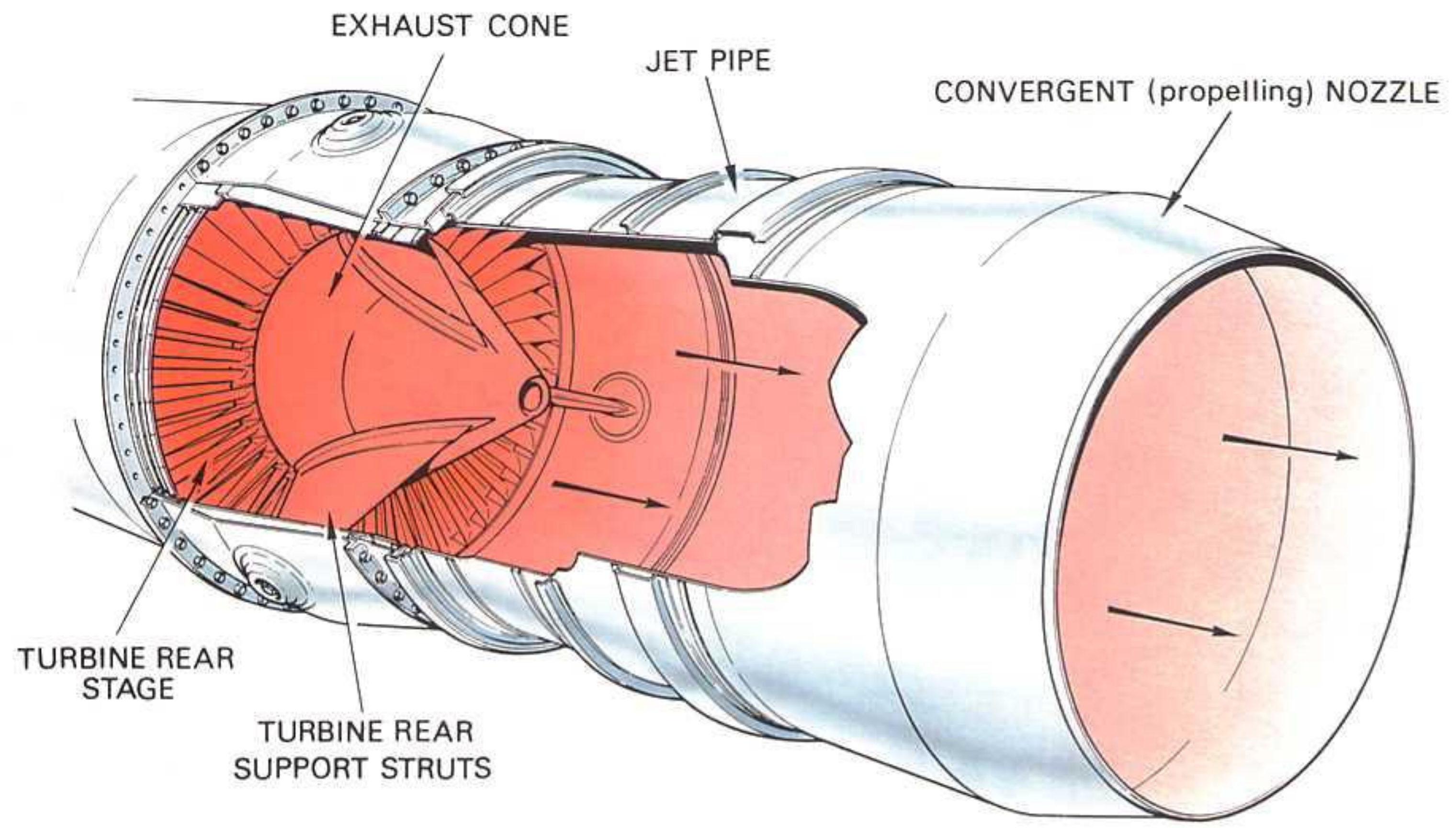
$$m_{cor} = m_4\sqrt{T_{t4}/288}\,\frac{1013.25}{P_{t4}}$$



ENGINE PROPELLING NOZZLE

The exhaust nozzle accelerates the exhaust gases, converting static pressure into velocity. It is the final component responsible for producing thrust in a jet engine.

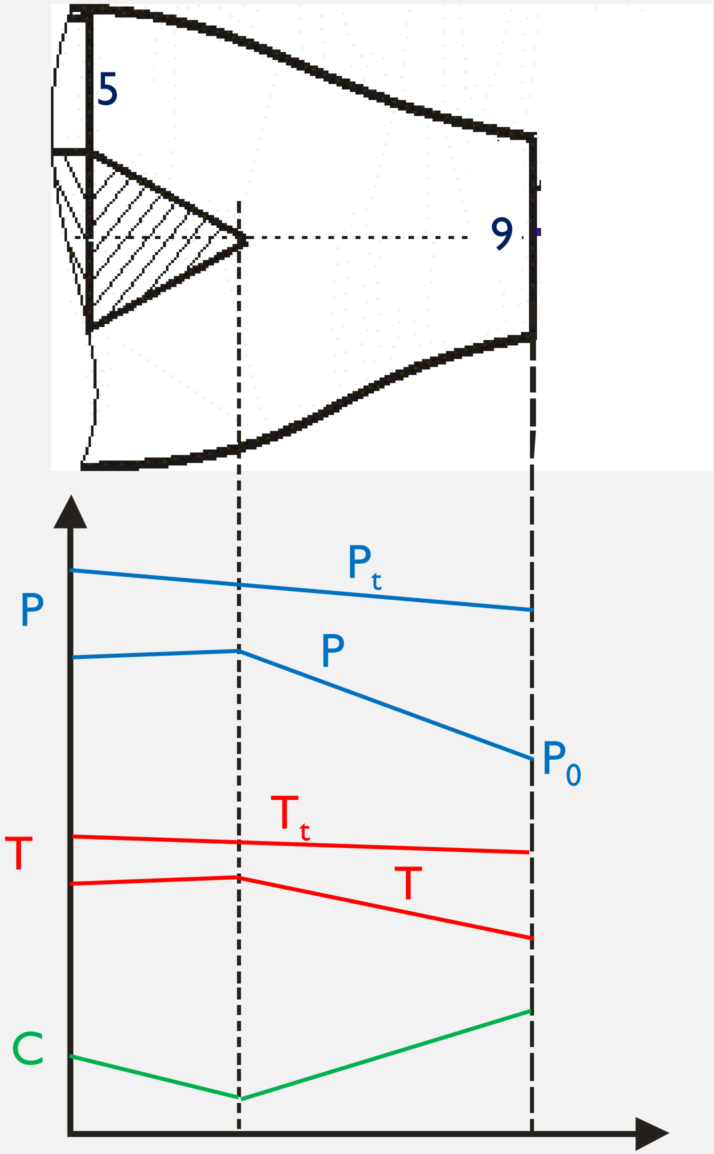
CONVERGENT EXHAUST NOZZLE



Used in engines where exhaust gas velocity is subsonic or only slightly supersonic.
Features:

- Simple, fixed geometry
- High reliability
- Low weight
- No need for variable geometry

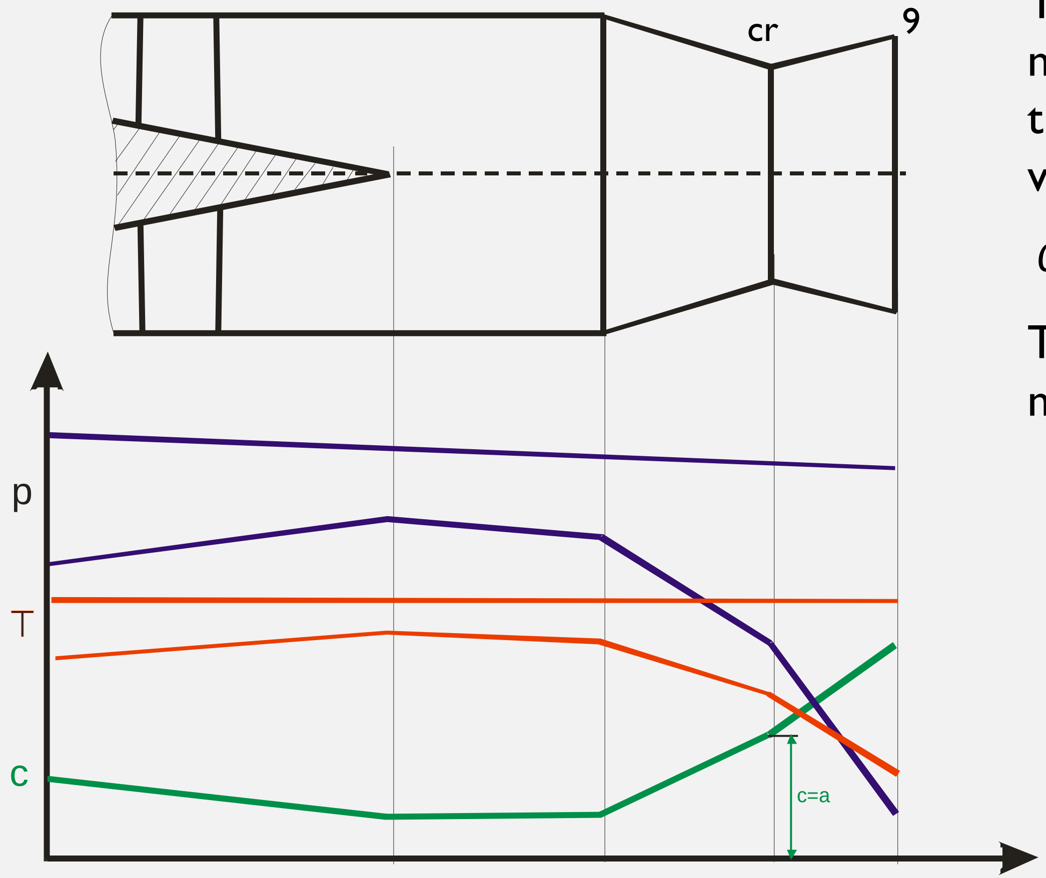
EXHAUST NOZZLE OPERATION



In the nozzle:

- Total pressure decreases due to friction losses
- Static pressure changes depending on nozzle shape
- Ideally, the nozzle expands the flow to ambient pressure
- Total temperature is assumed constant (heat transfer neglected)
- Static temperature decreases as the flow accelerates
- In a convergent nozzle, subsonic flow accelerates; in divergent sections, behavior depends on Mach number

CONVERGENT–DIVERGENT NOZZLE OPERATION



The throat is the minimum-area section where the flow reaches sonic velocity.

$$c_{kr} = a$$

The throat limits the maximum mass flow (chocked nozzle)

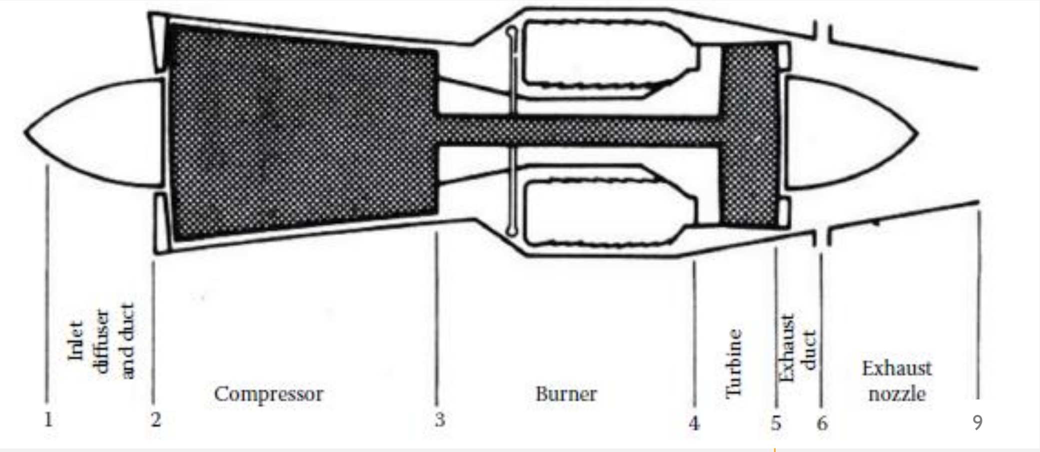
If the exit pressure matches ambient pressure, the nozzle produces fully expanded supersonic flow,

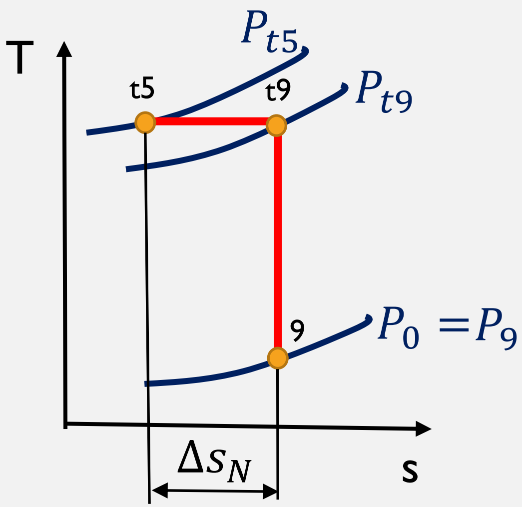
$$C_5 > C_{kr} \qquad P_9 \approx P_0$$

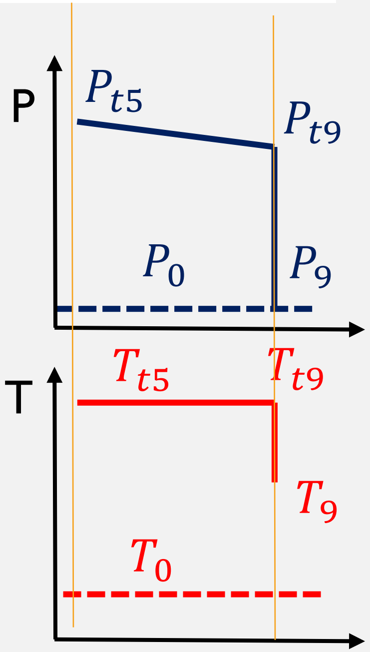
$$c_9 = \sqrt{2c_pT_{t5}\left(1-\left(\frac{P_9}{\pi_N P_{t5}}\right)^{\frac{k-1}{k}}\right)}$$

π_N - pressure losess in the propelling nozzle

FULL EXPANSION IN THE NOZZLE WITH LOSESS







NOZZLE (5-9)

Pressure losses: $\pi_N = \frac{P_{t9}}{P_{t5}} < 1 \quad P_{t9} = \pi_N P_{t5}$

No heat losess $\quad T_{t9} = T_{t5}$

Full expansion: $\quad P_9 = P_0$

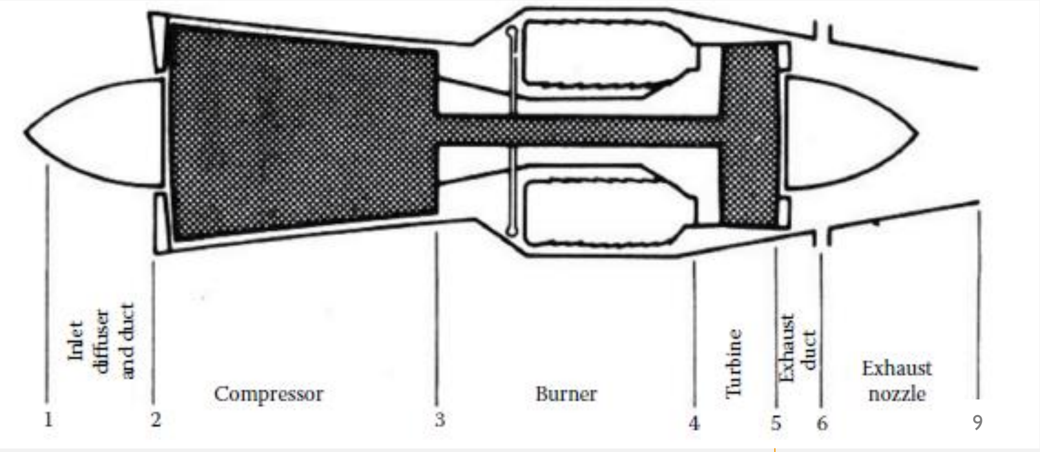
Entropy growth: $\Delta s_N = R_t ln \frac{1}{\pi_N}$

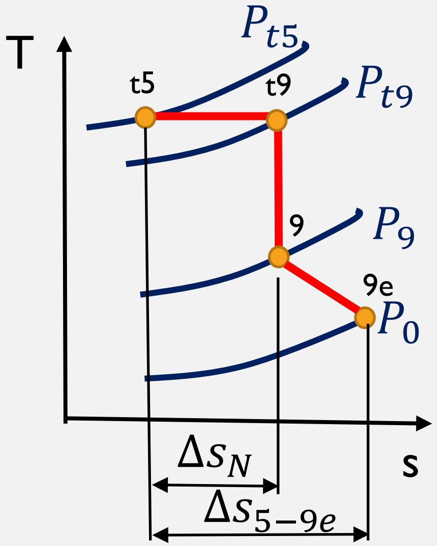
$c_9 = \sqrt{2Cp_t(T_{9t} - T_9)}$ - for incompressible flow

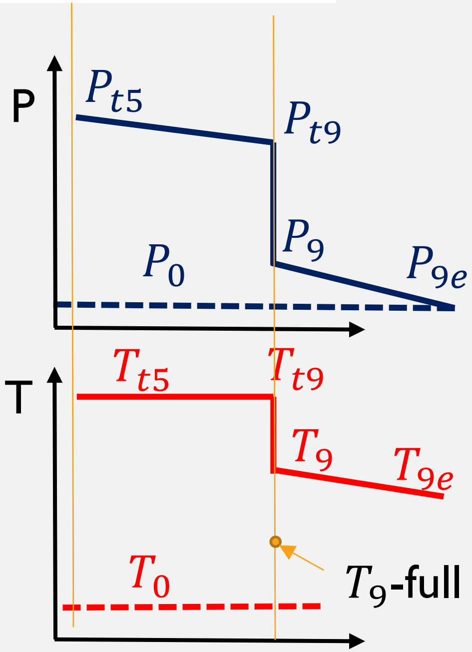
$c_9 = a_9 M_9 = \sqrt{k_t R_t T_9} * \sqrt{\frac{2}{k_t - 1}\left(\frac{T_{t9}}{T_9} - 1\right)}$ - for compressible flow

$\frac{T_{t9}}{T_9} = \frac{P_{t9}}{P_9}^{(k_t-1)/k_t}$ - isentropic relation betwen total and static paramethers

UNDER EXPANDED FLOW IN THE NOZZLE WITH LOSESS







T_9-full expansion

NOZZLE (5-9)

Pressure losses: $\pi_N = \frac{P_{t9}}{P_{t5}} < 1 \quad P_{t9} = \pi_N P_{t5}$

No heat losess $\quad T_{t9} = T_{t5}$

Incomploete expansion: $\quad P_9 > P_0$

Entropy growth: $\Delta s_{5-9e} = Cp_t ln\frac{T_{9e}}{T_{t5}} - R_t ln\frac{P_0}{P_{t5}}$

$$c_9 = a_9 M_9 = \sqrt{k_t R_t T_9} * \sqrt{\frac{2}{k_t - 1}\left(\frac{T_{t9}}{T_9} - 1\right)}$$

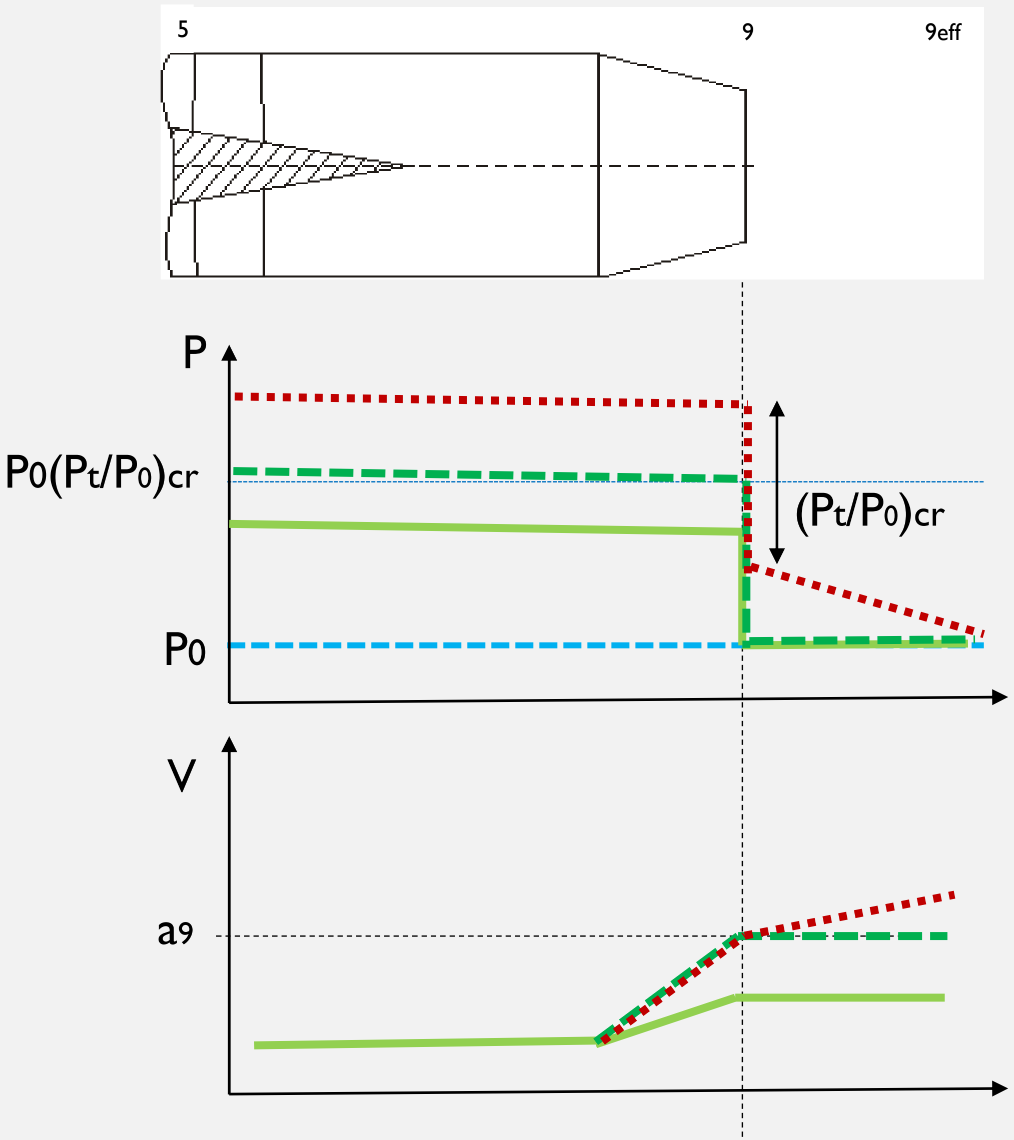
$$c_{9e} = c_9 + \frac{A_9(P_9 - P_o)}{\dot{m}_9} = c_9 + \frac{(P_9 - P_o)}{\rho_9 c_9}$$

$$= c_9 + \frac{R_9 T_9 (P_9 - P_o)}{P_9 c_9}$$

$$\frac{T_{t9}}{T_9} = \left(\frac{P_{t9}}{P_9}\right)^{(k_t - 1)/k_t}$$

continuity equation:
$\dot{m}_9 = \rho_9 \rho_9 c_9$

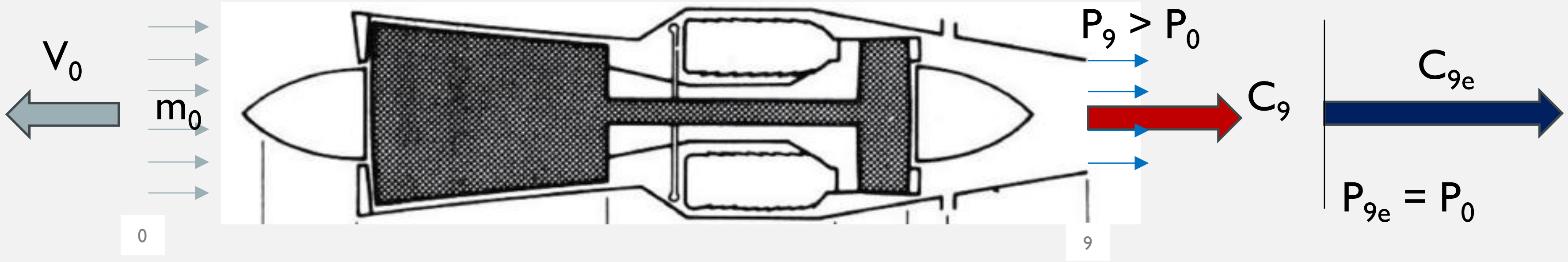
TURBOJET ENGINE WITH SUBSONIC (CONVERGENT) NOZZLE



$$\left(\frac{P_t}{P}\right)_{cr} = \left(\frac{1+k_t}{2}\right)^{\frac{k_t}{k_t-1}}$$

- Full gas expansion in the nozzle is available for subsonic flow speed ($C_9<a_9$)
- When total to static pressure ratio is critical the flow velocity in nozzle outlet is equal speed of sound
- When total to static pressure ratio is higher than critical the nozzle outlet velocity is equal speed of sound – chocked nozzle. Gas expansion process is continued outside the nozzle

TURBOJET ENGINE PERFORMANCE



$$c_{9e} = c_9 + \frac{A_9(P_9 - P_o)}{\dot{m}_9}$$

- Thrust

$$T = m_9 C_9 - m_0 V_0 + A_9(P_9 - P_0) = m_9 V_{9e} - m_0 V_0$$

- Specific Thrust

$$ST = \frac{T}{m_0} = (1 + f_B) C_{9e} - V_0$$

- Specific Fuel Consumption

$$SFC = \frac{m_f}{T} = \frac{f_B}{(1 + f_B) V_{9e} - V_0}$$

- Thermal efficiency

$$\eta_{th} = \frac{m_9 {C_{9e}}^2 - m_0 {V_0}^2}{2\, m_f\, FHV} = \frac{(1 + f_B){C_{9e}}^2 - {V_0}^2}{2\, f_B FHV}$$

- Propulsive efficiency

$$\eta_p = \frac{2\, T\, V_0}{m_9 {C_{9e}}^2 - m_0 {V_0}^2} = \frac{2\, ST\, V_0}{(1 + f_B){C_{9e}}^2 - {V_0}^2}$$

- Overall efficiency

$$\eta_o = \frac{T\, V_0}{m_f\, FHV} = \frac{ST\, V_0}{f_B\, FHV} = \eta_{th}\eta_p$$

THANKS FOR YOUR ATENTION

Questions and Comments ?

1. …….
2. …….
3. …….