RZESZOW UNIVERSITY
OF TECHNOLOGY

THE FACULTY OF
MECHANICAL ENGINEERING
AND AERONAUTICS
RZESZOW UNIVERSITY OF TECHNOLOGY

GAS TURBINES

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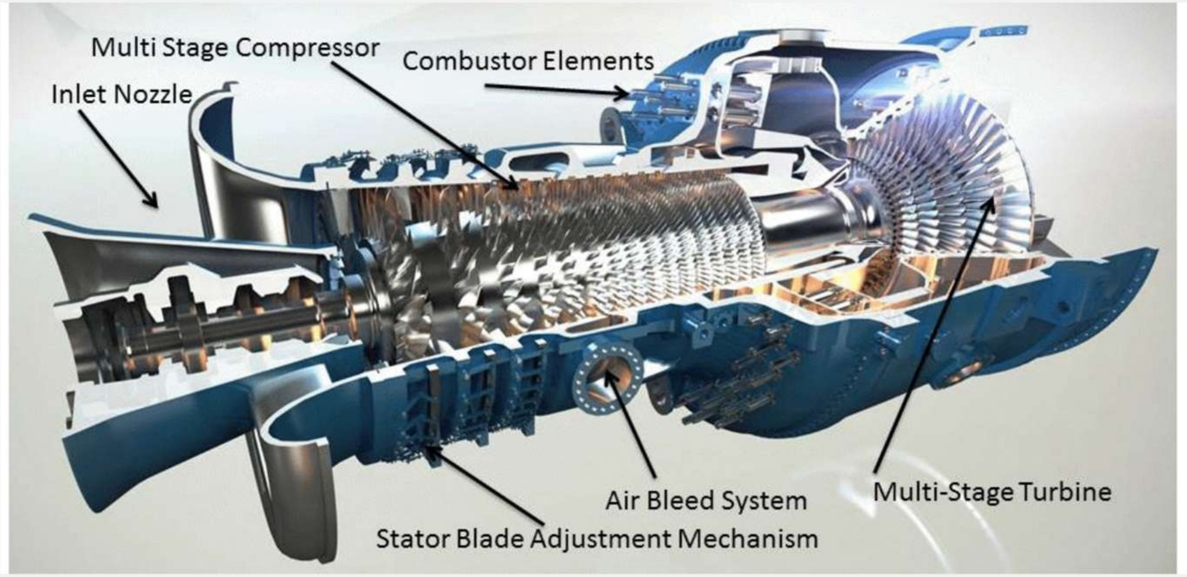
GAS TURBINES

Gas turbines (GT) are power plants within which the chemical energy of the fuel is converted either into mechanical energy in terms of shaft power or into kinetic energy in jet engines.

They produce a great amount of energy for its size and weight

SHAFT POWER GAS TURBINE



Alstom heavy duty power generation gas turbine GT13E2
Output power 202.7 MW (M.T. Schobeiri, Gas Turbine Design, Components and System Design Interation, DOI 10.1007/978-3-319-85378-5_1)

Output Shaft Power:

$$P_{OSH} = EF_R\ (P_T - P_c)$$

Thermal efficiency:

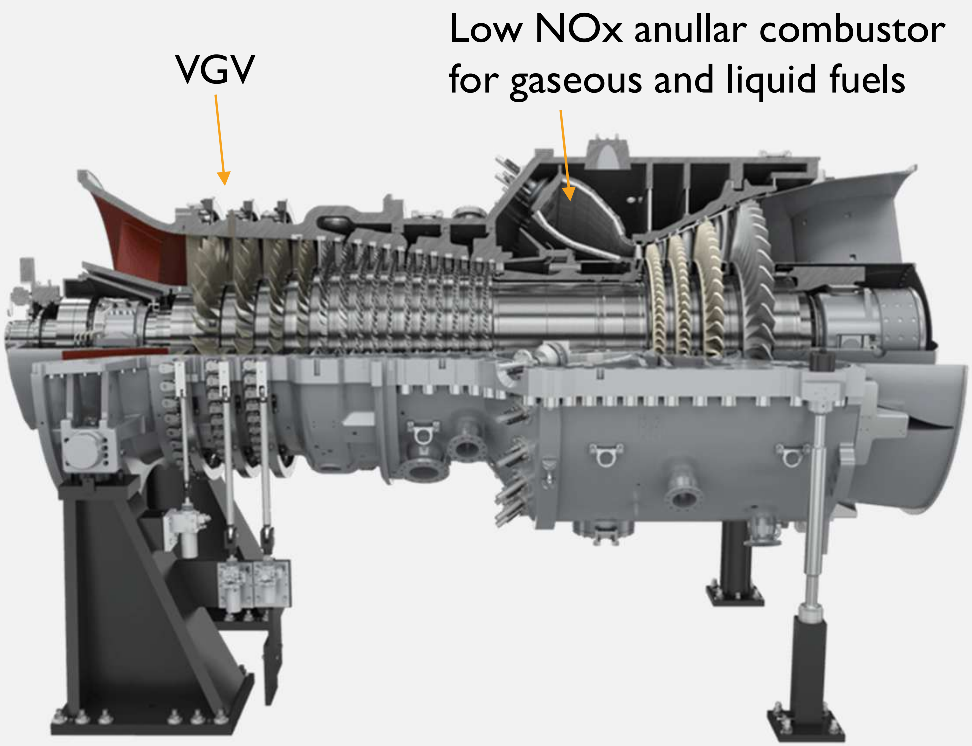
$$\eta_{th} = {}^{P_{OSH}}/_{Q_{ad}}$$

Heat added:

$$Q_{add} = \ \dot{m}_f\ FHV$$

m_f - Fuel mass flow
FHV - Fuel heat value

SIEMENS POWER GAS TURBINE SGT5-4000F

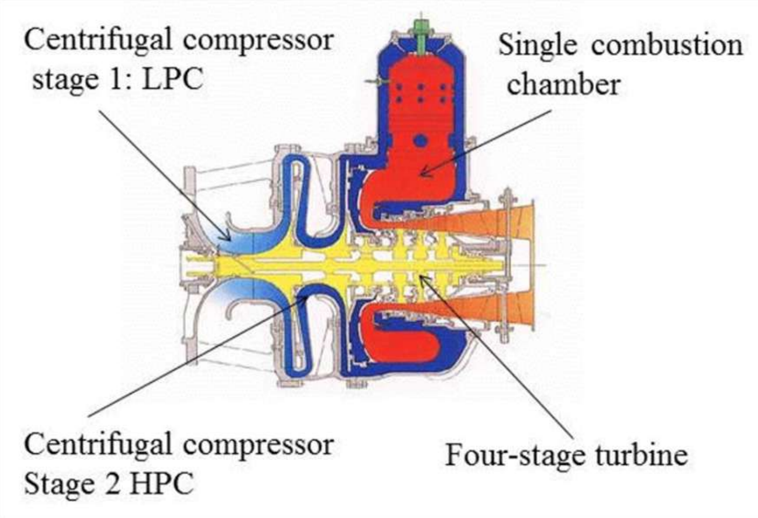


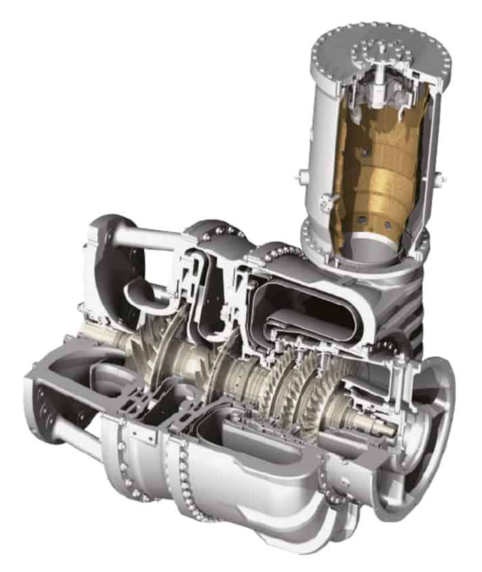
Output shaft power 329 MW
Efficiency 41%
Compressor: 15 stages
CPR= 20.1,
Turbine: 4 stages
Turbine speed 3000 rpm
Mass flow rate = 724 kg/s
Exhaust gas temperature 599° C

VGV – variable guide vane

Source:
https://www.siemens-energy.com/global/en/home/products-services/product/sgt5-4000f.html#Simple-cycle-power-generation-tab-6

KAWASAKI GAS TURBINE GB15D





Gas turbine with single stage side type combustor

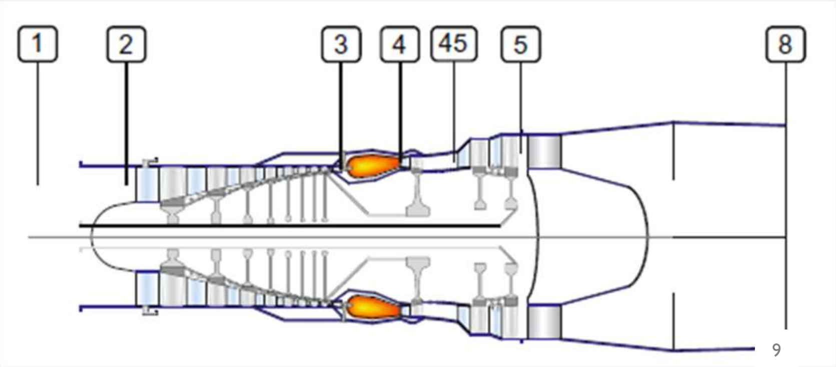
Specifications	Output
Electrical Output	1,450 kW
Heat Rate	15,280 kW
Exhaust Gas Temperature	534 °C
Exhaust Gas Mass Flow	28.5 x10³ kg/hr

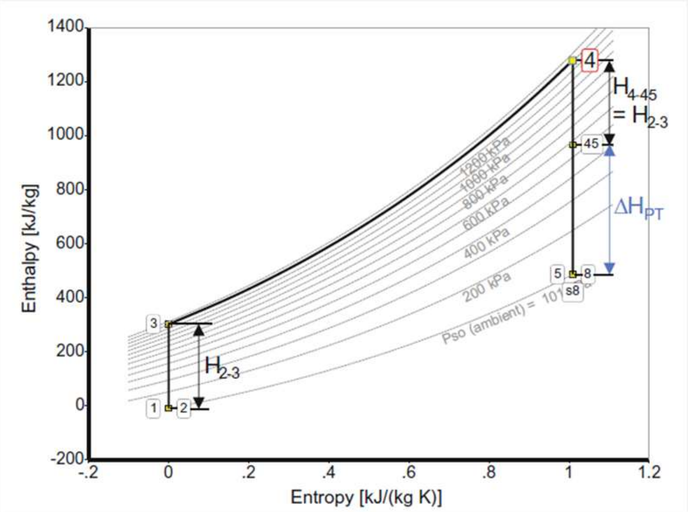
Sources:
https://kga.com.my/product/gbp15d/

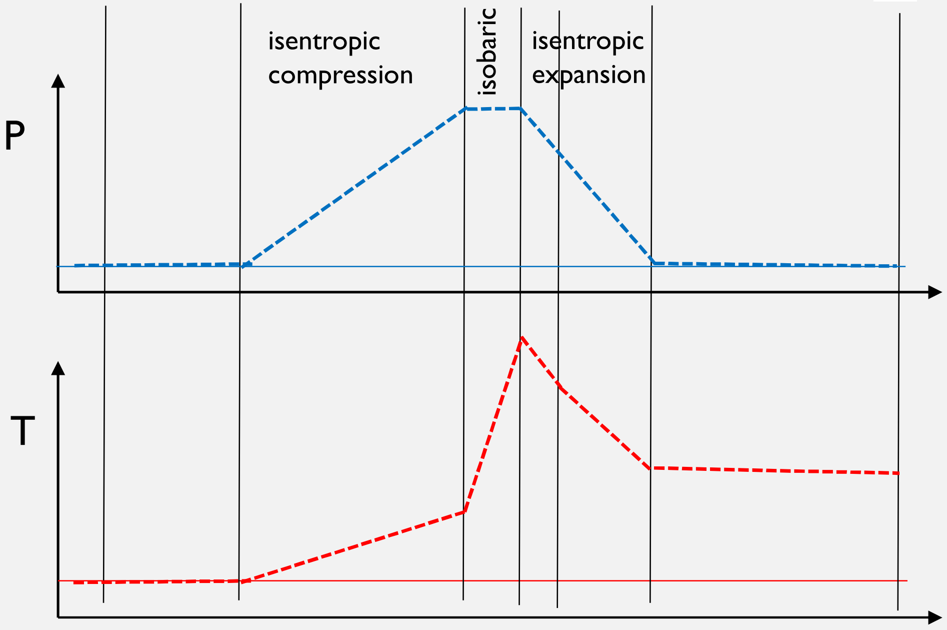
ASSUMPTIONS FOR IDEAL GAS TURBINE

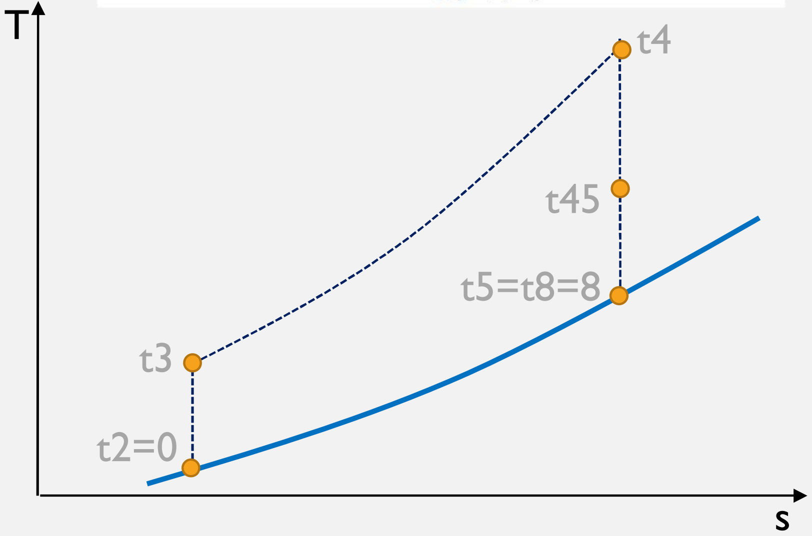
- The gas flowing through the engine is treated as a clorycally perfect.
- The processes in the engine are considered reversible (flow, mechanical, and thermal losses in the engine are neglected).
- The change in the mass of the working fluid flowing through the engine is neglected.
- The parameters of the flowing stream are represented at control points by averaged values.

IDEAL GAS-TURBINE

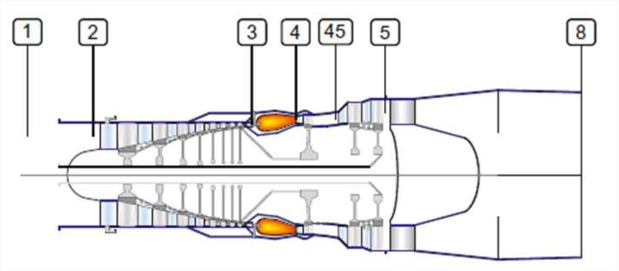


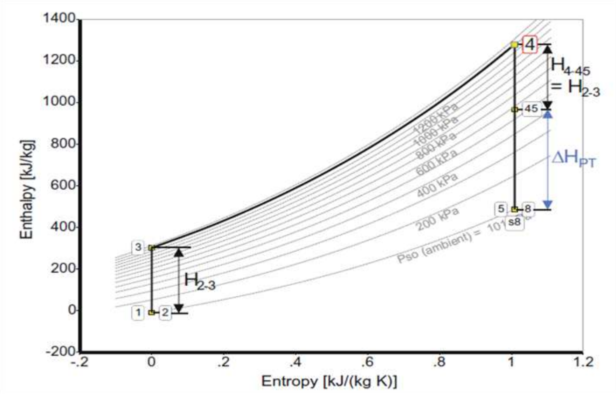






IDEAL GAS TURBINE ANALYSIS (1)





No **INLET** losses → $P_{t2} = P_0$ and $T_{t2} = T_0$

COMPRESSOR (2 – 3)

Compressor work is isentropic

$$\frac{T_{t3}}{T_{t2}} = \left(\frac{P_{t3}}{P_{t2}}\right)^{(k-1)/k} = CPR^{(k-1)/k}$$

Compressor work:

$$W_C = H_{23} = Cp(T_{t3} - T_{t2})$$

Compressor power:

$$P_C = \dot{m}_C W_C = \dot{m}_0 Cp(T_{t3} - T_{t2})$$

COMBUSTOR (3 – 4)

Energy balance

$$\dot{m}_f FHV = \dot{m}_0 Cp_B(T_{t4} - T_{t3})$$

$$\dot{m}_f = \frac{\dot{m}_0 Cp_B(T_{t4} - T_{t3})}{FHV}$$

Fuel air ratio:

$$\text{f} = \frac{\dot{m}_f}{\dot{m}_0} = \frac{Cp_B(T_{t4} - T_{t3})}{FHV}$$

$$\pi_B = 1 \rightarrow P_{t4} = P_{t3}$$

TURBINE (4 - 45)

Compressor turbine power balance equation

$$P_C = \dot{m}_0 Cp(T_{t3} - T_{t2}) = P_T = \dot{m}_T Cp_T(T_{t45} - T_{t5})$$

Turbine mass flow: $\dot{m}_T = \dot{m}_0 + \dot{m}_f$

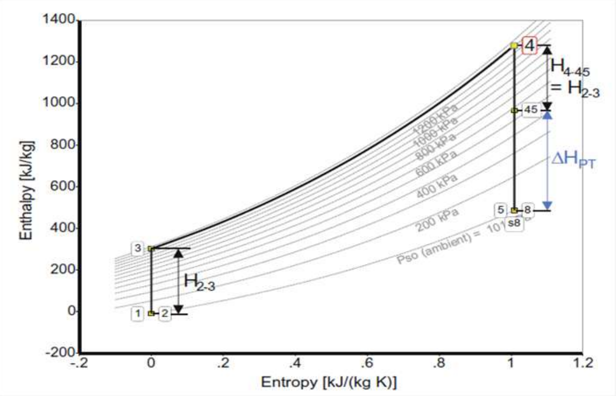
Turbine outlet temperature:

$$T_{t45} = T_{t4} - \frac{Cp(T_{t3} - T_{t2})}{Cp_T(1 + f)}$$

Turbine outlet pressure:

$$P_{t45} = P_{t4}\left(\frac{T_{t45}}{T_{t4}}\right)^{k_t/(k_t-1)}$$

IDEAL GAS TURBINE ANALYSIS (2)



Turbine expansion to ambient pressure → $P_{t5} = P_0$

TURBINE (45 - 5)

Power Turbine outlet temperature:

$$T_{t5} = T_{t45}\left(\frac{P_{t5}}{P_{t45}}\right)^{(k_t-1)/k_t}$$

Power Turbine work:

$$W_{PT} = H_{PT} = Cp_T(T_{t45} - T_{t5})$$

Power Turbine power:

$$P_{PT} = \dot{m}_T W_{PT} = \dot{m}_0 Cp_T(1+f)(T_{t45} - T_{t5})$$

Output Shaft Power:

$$P_{OSH} = EF_G\, P_{PT} = EF_R \dot{m}_{45} Cp_T(T_{t45} - T_{t5})$$

EF_G - Gear efficiency

Thermal efficiency:

$$\eta_{th} = {}^{P_{OSH}}/_{Q_{ad}}$$

Heat added:

$$Q_{add} = \dot{m}_f\, FHV$$

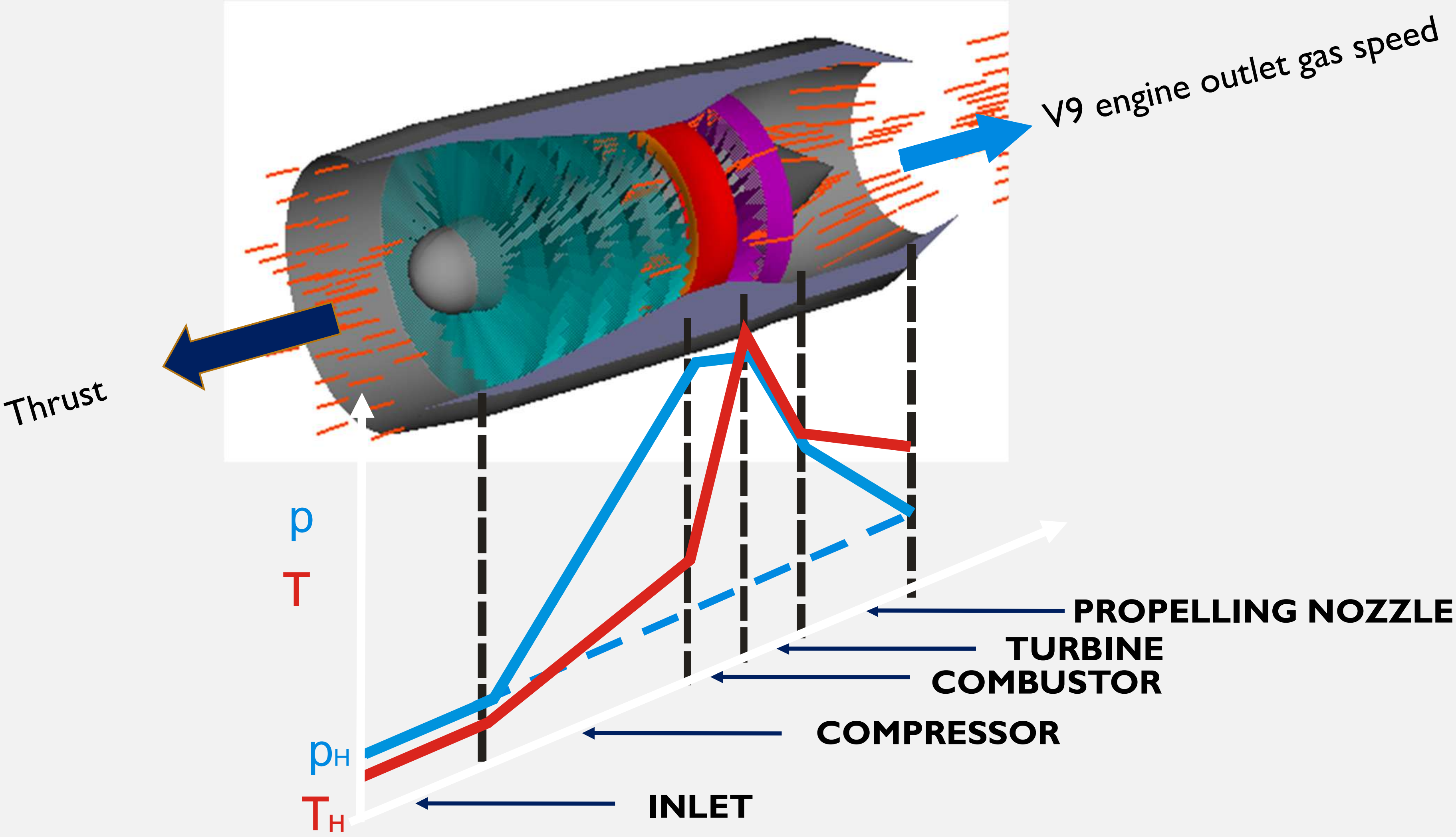
Specific fuel consumption:

SFC= $^{m_f}/_{P_{OSH}} = {}^{f}/_{W_{OSH}}$

Work of output shaft:

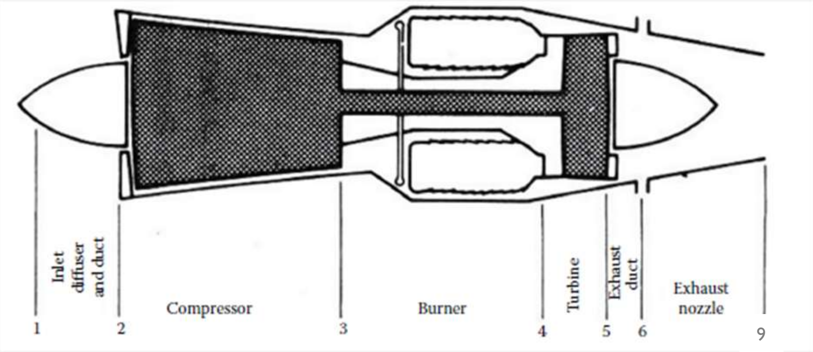
$$W_{OSH} = {}^{P_{OSH}}/_{m_0}$$

TURBOJET ENGINE



IDEAL TURBOJET ENGINE

GIVEN: H – altitude, CPR copmressor pressure ratio,
TIT – turbine inlet temperature (Tt4)



Engine work in static conditions V0=0 → $P_{t0} = P_0,\ \ T_{t0} = T_0$

No **INLET** losses → $P_{t2} = P_{t0}$ and $T_{t2} = T_{t0}$

COMPRESSOR (2 – 3)

Compressor work is isentropic

$$\frac{T_{t3}}{T_{t2}} = \left(\frac{P_{t3}}{P_{t2}}\right)^{(k-1)/k} = {\pi_C}^{(k-1)/k}$$

Compressor work:

$$W_C = Cp(T_{t3} - T_{t2})$$

Compressor power:

$$P_C = \dot{m}_C W_C = \dot{m}_0 Cp(T_{t3} - T_{t2})$$

COMBUSTOR (3 – 4)

Energy balance

$$\dot{m}_f FHV = \dot{m}_0 Cp_B(T_{t4} - T_{t3})$$

$$\dot{m}_f = \frac{\dot{m}_0 Cp_B(T_{t4} - T_{t3})}{FHV}$$

Fuel air ratio:

$$\mathrm{f} = \frac{\dot{m}_f}{\dot{m}_0} = \frac{Cp_B(T_{t4} - T_{t3})}{FHV}$$

$$\pi_B = 1 \rightarrow P_{t4} = P_{t3}$$

TURBINE (4 - 5)

Compressor turbine power balance equation

$$P_C = \dot{m}_0 Cp(T_{t3} - T_{t2}) = P_T = \dot{m}_T Cp_T(T_{t4} - T_{t5})$$

Turbine mass flow: $\dot{m}_T = \dot{m}_0 + \dot{m}_f$

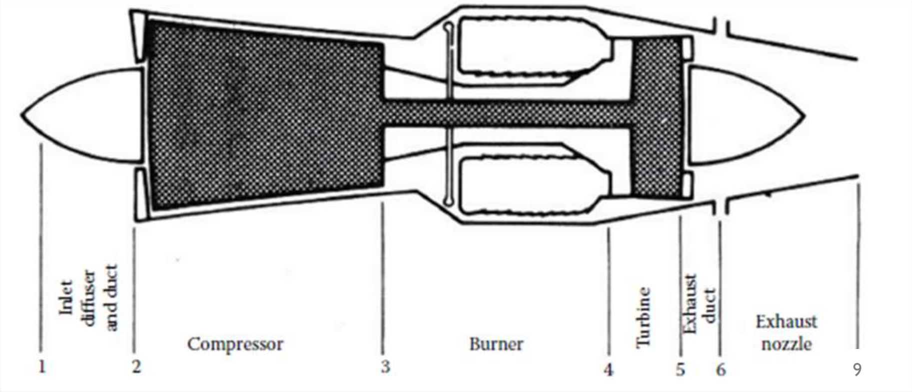
Turbine outlet temperature:

$$T_{t5} = T_{t4} - \frac{Cp(T_{t3} - T_{t2})}{Cp_T(1+f)}$$

Turbine outlet pressure:

$$P_{t5} = P_{t4}\left(\frac{T_{t5}}{T_{t4}}\right)^{k_t/(k_t-1)}$$

IDEAL TURBOJET ENGINE – GASS FULL EXPANSION IN THE NOZZLE



NOZZLE (5-9)

No losses: $\pi_N = 1 \rightarrow \quad P_{t9} = P_{t5}$ and $T_{t9} = T_{t5}$

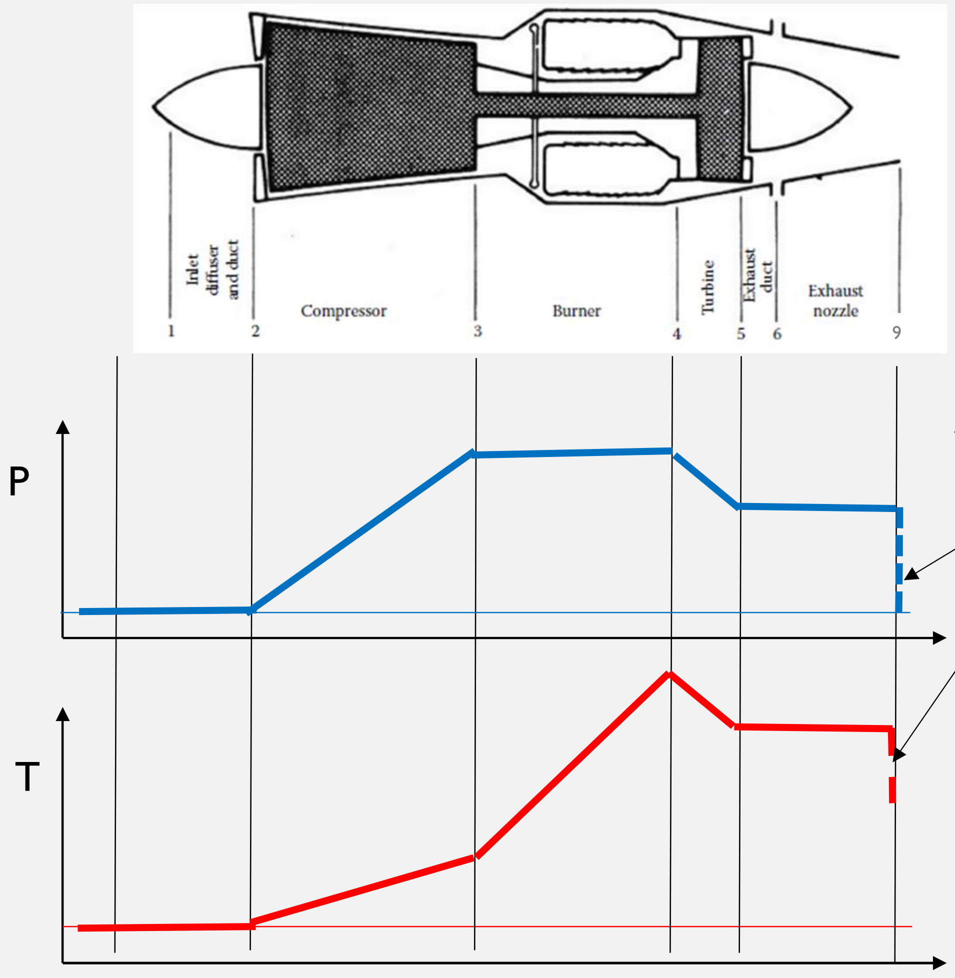
Full expansion: $P_9 = P_0$

$$c_9 = \sqrt{2Cp_t(T_{9t} - T_9)}$$ - for incompressible flow

$$c_9 = a_9 M_9 = \sqrt{k_t R_t T_9} * \sqrt{\frac{2}{k_t - 1}\left(\frac{T_{t9}}{T_9} - 1\right)}$$ - for compressible flow

$$\frac{T_{t9}}{T_9} = \frac{P_{t9}}{P_9}^{(k_t-1)/k_t}$$

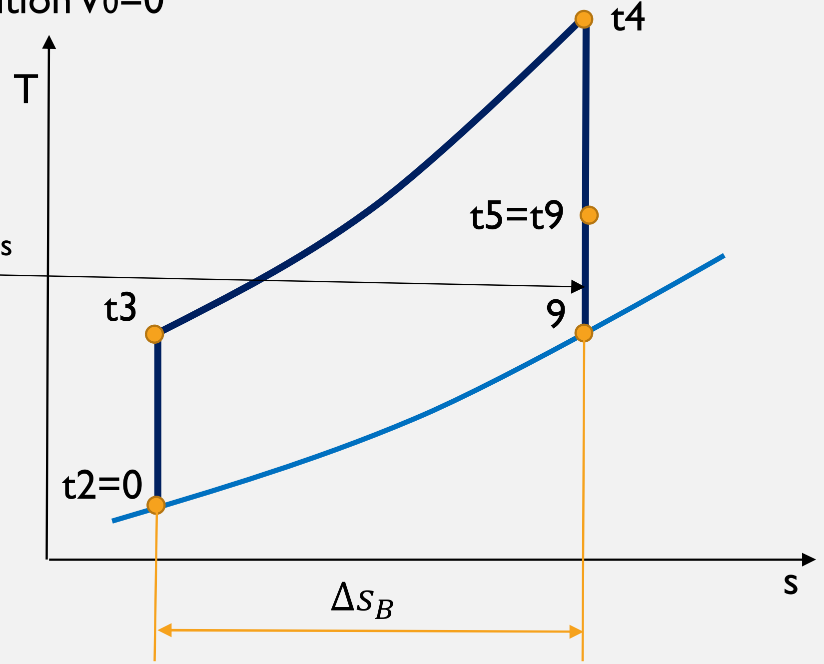
IDEAL TURBOJET ENGINE



Proesses in the inlet (diffuser) burner and nozzle are idel – no pressure losses $\boldsymbol{\pi_D = \pi_B = \pi_N = 1}$

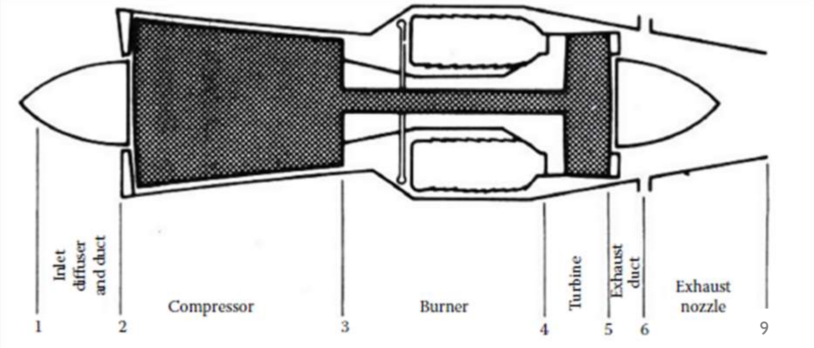
Static condition V0=0

total to static process



Entropy increase in a burner $\Delta s_B = cp_B ln \frac{T_{t4}}{T_{t3}}$

IDEAL TURBOJET ENGINE

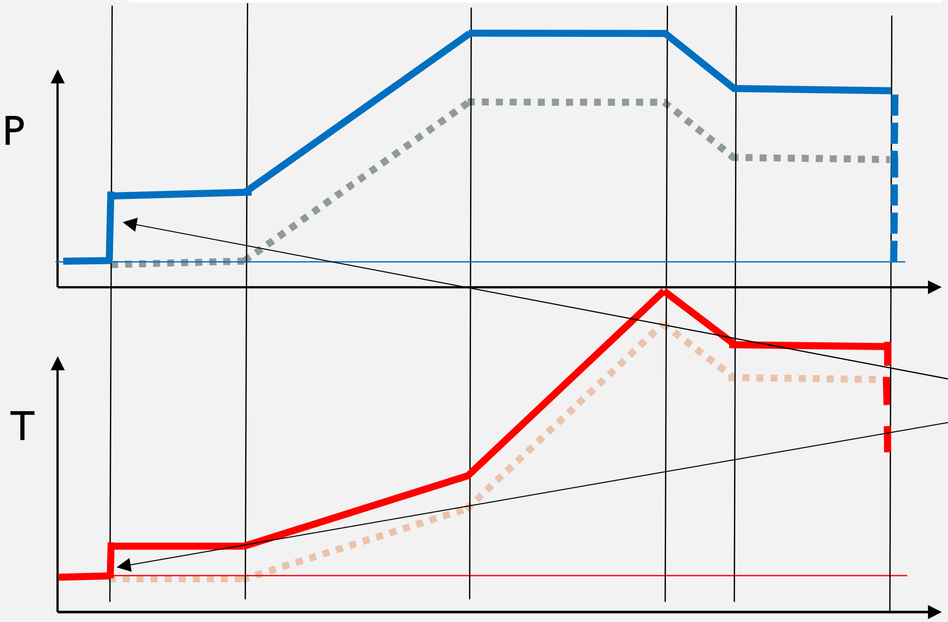


Flight condition V0>0

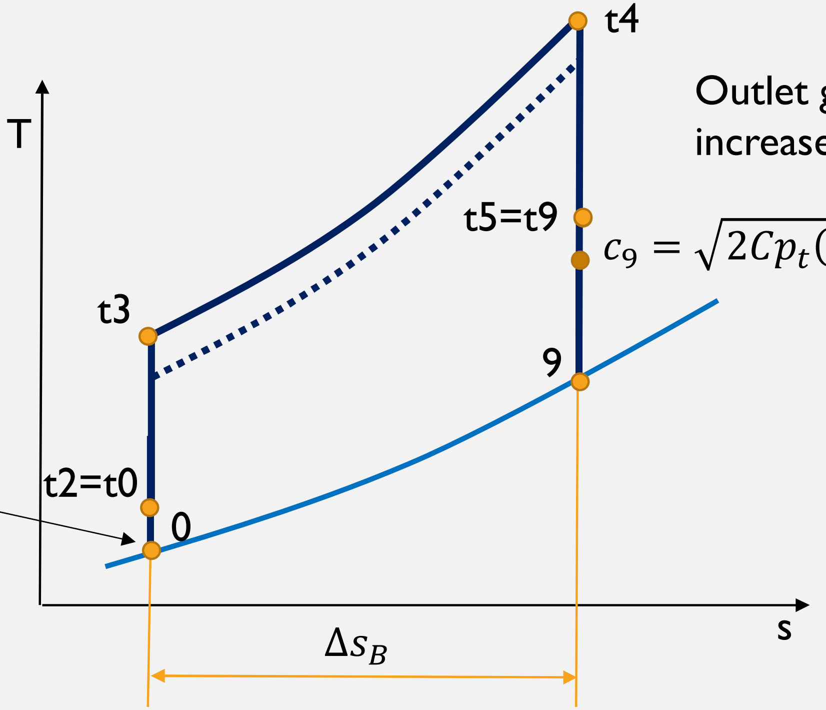
Ram pressure recovery for flight condition ($M_0 > 0$)

$$P_{t0} = P_0 \left(1 + \frac{k-1}{2} {M_0}^2\right)^{k/(k-1)}$$

$$T_{t0} = T_0 \left(1 + \frac{k-1}{2} {M_0}^2\right)$$



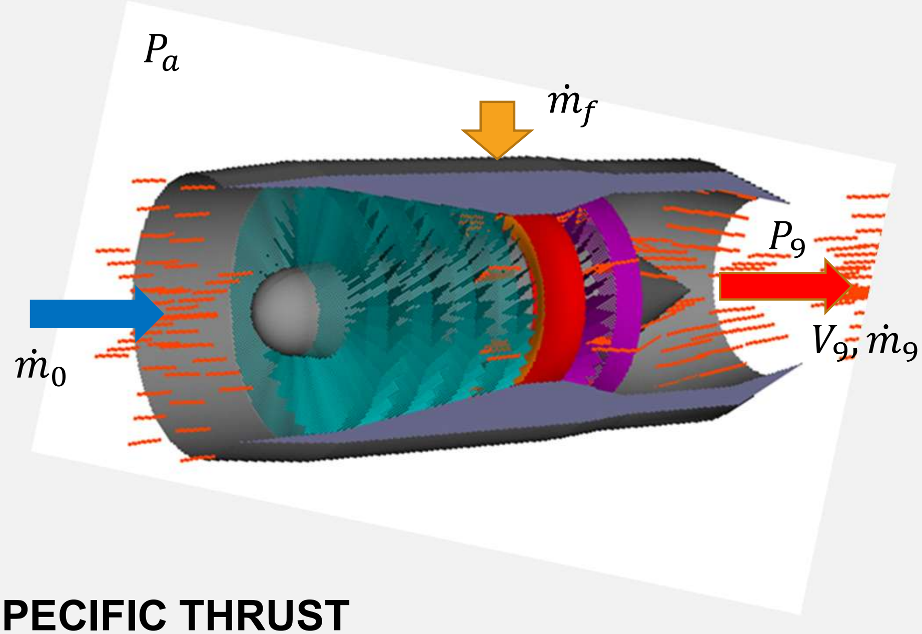
Ram effect



Outlet gas speed increase

$$c_9 = \sqrt{2Cp_t(T_{9t} - T_9)}$$

ENGINE THRUST AND SPECIFIC PARAMETERS



SPECIFIC THRUST

$$ST = T/\dot{m}_0$$

SPECIFIC FUEL CONSUMPTION

$$\mathrm{SFC} = \dot{m}_f/T$$

THTUST / GROS THRUST

$$T = \dot{m}_9 V_9 + A_9(P_9 - P_a)$$

effective exhaust velocity

$$V_{eff} = V_9 + A_9(P_9 - P_a)/\dot{m}_9$$

$$T = \dot{m}_9 V_{eff}$$

Exit pressure = ambient pressure

$$T = \dot{m}_9 V_9$$

Flight speed > 0

THTUST / NET THRUST

$$T = \dot{m}_9 V_9 + A_9(P_9 - P_a) - \dot{m}_0 V_0 = \dot{m}_9 V_{eff} - \dot{m}_0 V_0$$

Net thrust = Gross thrust –Momentum drag

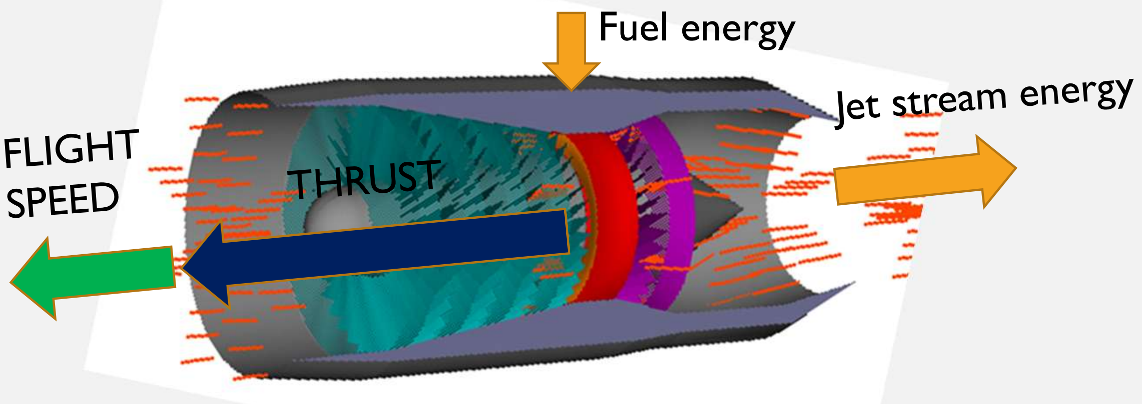
Details in: **Ahmed F. El-Sayed, Aircraft Propulsion and Gas Turbine Engines, (chapter 2.2)**

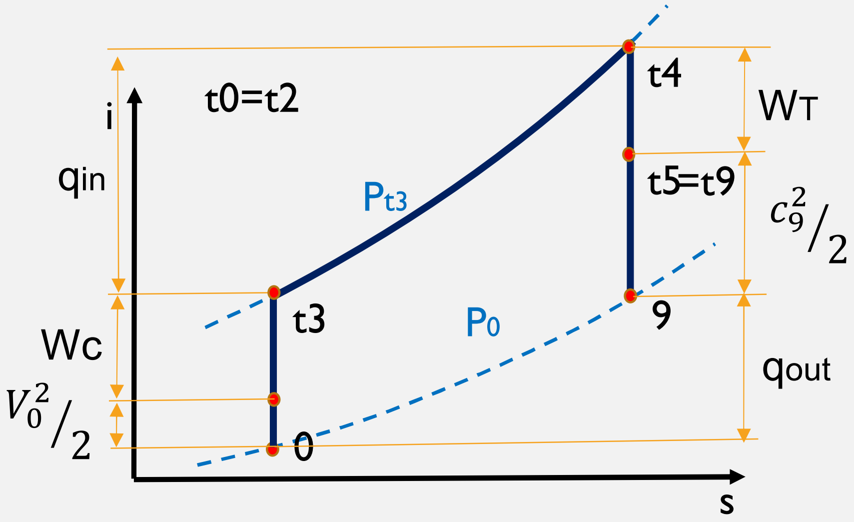
ENGINE EFFICIENCIES

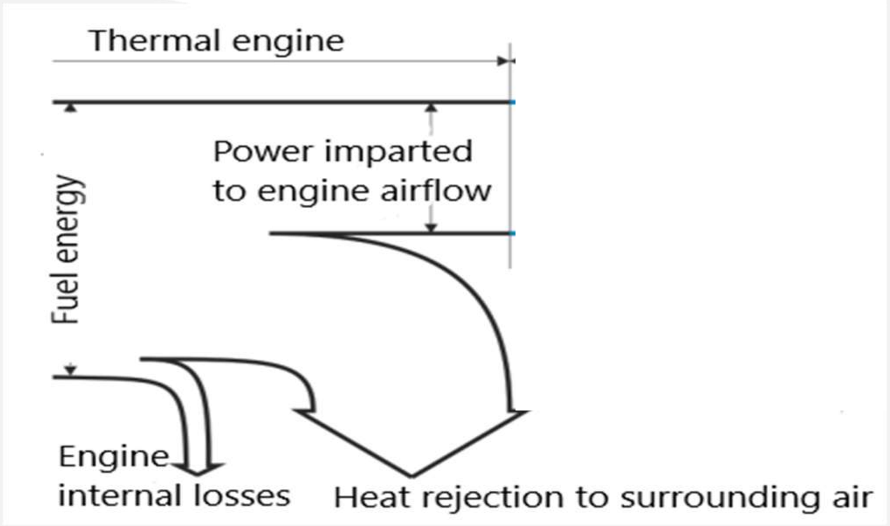
Thermal efficency

$$\eta_{TH} = \frac{\text{Power imparted to engine airflow}}{\text{Rate of energy supplied in the fuel}}$$

$$\eta_{TH} = \frac{0{,}5 * (\dot{m}_9 {V_{9e}}^2 - \dot{m}_0 {V_0}^2)}{\dot{m}_f FHV}$$





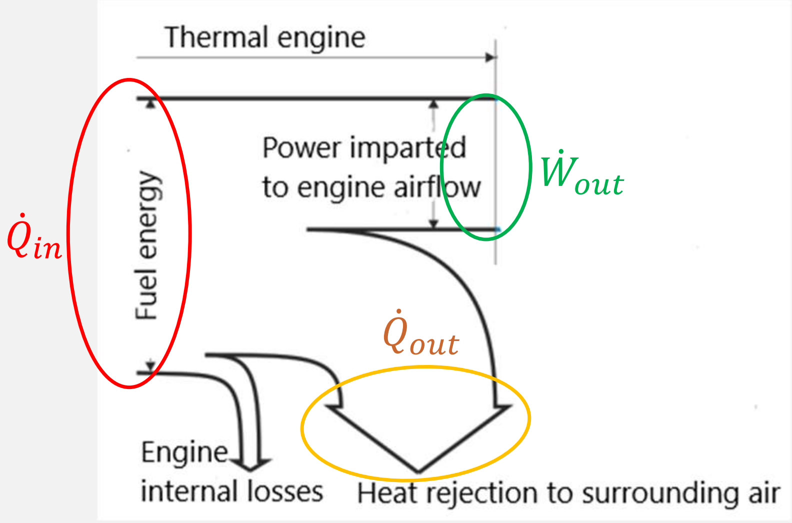


THERMAL EFFICIENCY

$$\eta_{TH} = \frac{\dot{W}_{out}}{\dot{Q}_{in}} \Rightarrow \eta_{TH} = \frac{\dot{Q}_{in} - \dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} \quad \text{where, } \dot{Q}_{out} = \dot{m}_9 c_p (T_9 - T_0)$$

- Wout = net power out of engine (engine work)
- Qin= rate of thermal energy released/suplied in the fuel)

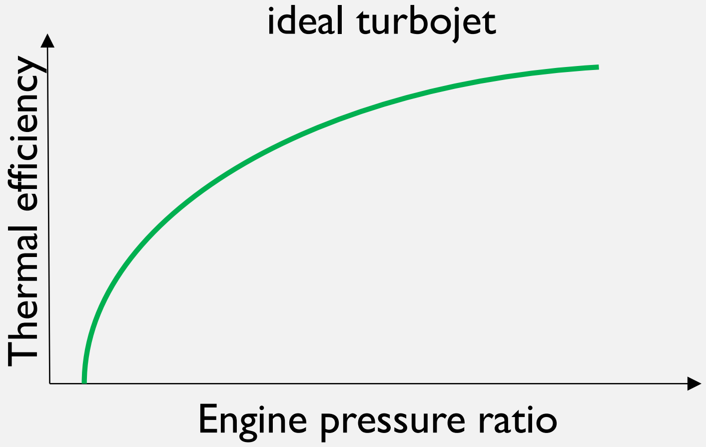
Higher thermal efficiency causes lower specific fuel consumption
Thermal efficiency is higher when exhaust gas temperature is closer to the ambient temperature



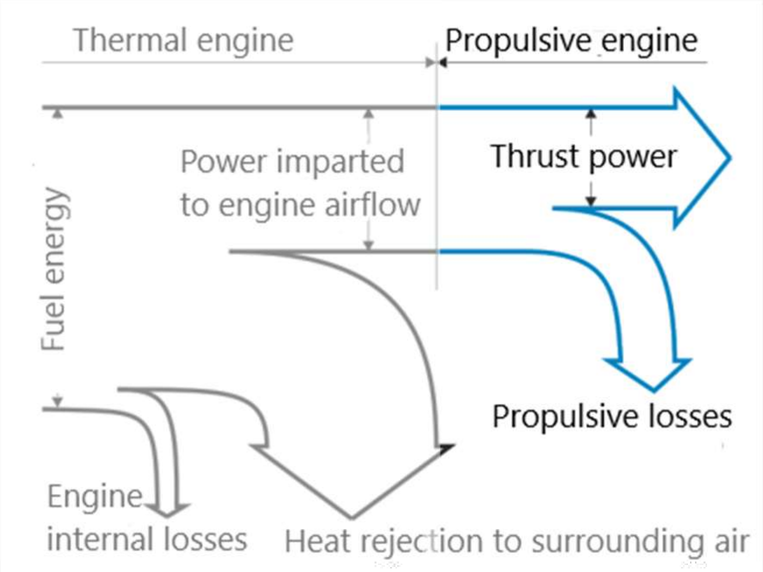
For ideal turbojet engine:

$\eta_{TH} = 1 - \frac{1}{\pi^{(k-1)/k}}$,

where π – engine compression pressure ratio, k- isentropic exponent



PROPULSIVE EFFICIENCY



$$\eta_P = \frac{V_0 * T}{0,5 * (\dot{m}_9 {V_{9e}}^2 - \dot{m}_0 {V_0}^2)}$$

$$\Downarrow$$

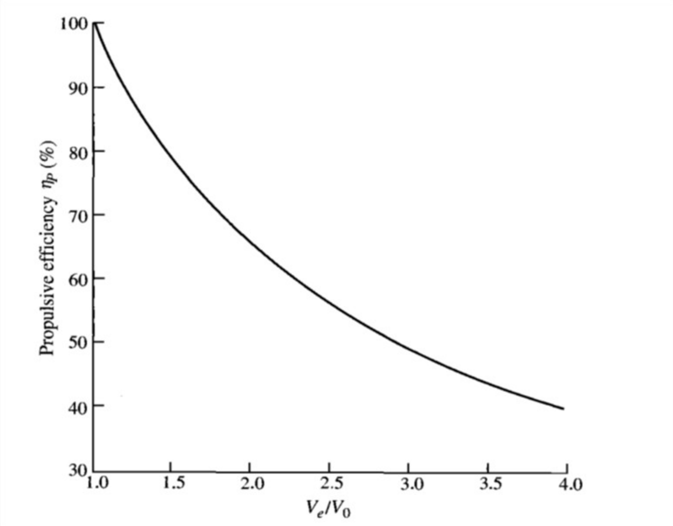
$$T = \dot{m}_9 V_{9e} - \dot{m}_0 V_0 \quad and \quad \dot{m}_9 = \dot{m}_0$$

$$\Downarrow$$

$$\eta_P = \frac{V_0(V_{9e} - V_0)}{0,5 * ({V_{9e}}^2 - {V_0}^2)} = \frac{2V_0(V_{9e} - V_0)}{(V_{9e} - V_0)(V_{9e} + V_0)} = \frac{2}{1 + V_{9e}/V_0}$$

$$\eta_P \Rightarrow 1,\ for\ \ V_{9e} \Rightarrow V_0$$

Propulsive efficiency define the thrust produced for specific flight speed from kinetic energy added to engine airflow



ENGINE EFFICIENCIES

Thermal efficency

$$\eta_{TH} = \frac{\text{Power imparted to engine airflow}}{\text{Rate of energy supplied in the fuel}}$$

$$\eta_{TH} = \frac{0{,}5 * (\dot{m}_9 {V_{9e}}^2 - \dot{m}_0 {V_0}^2)}{\dot{m}_f FHV}$$

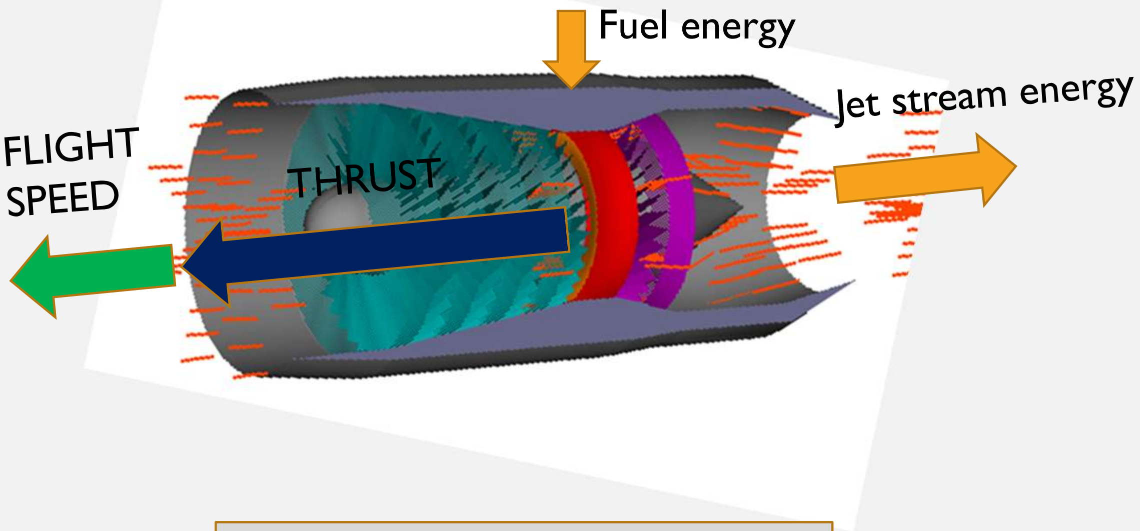
Propulsive efficency

$$\eta_P = \frac{\text{Thrust power}}{\text{Power imparted to engine airflow}}$$

$$\eta_P = \frac{V_0 * T}{0{,}5 * (\dot{m}_9 {V_{9e}}^2 - \dot{m}_0 {V_0}^2)}$$

Overall efficency

$$\eta_O = \eta_{TH} * \eta_P = \frac{V_0 * T}{\dot{m}_f FHV}$$



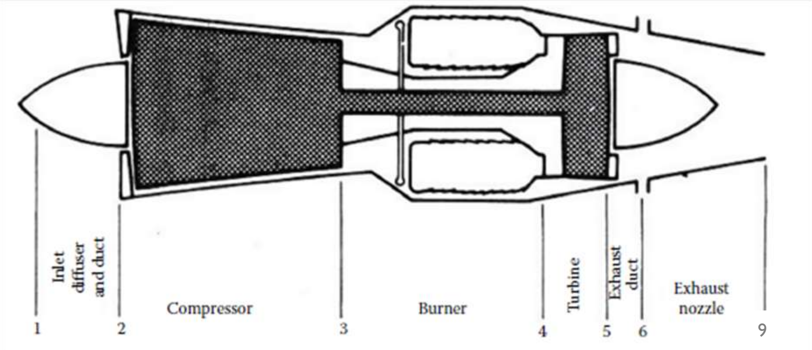
For V_0=0

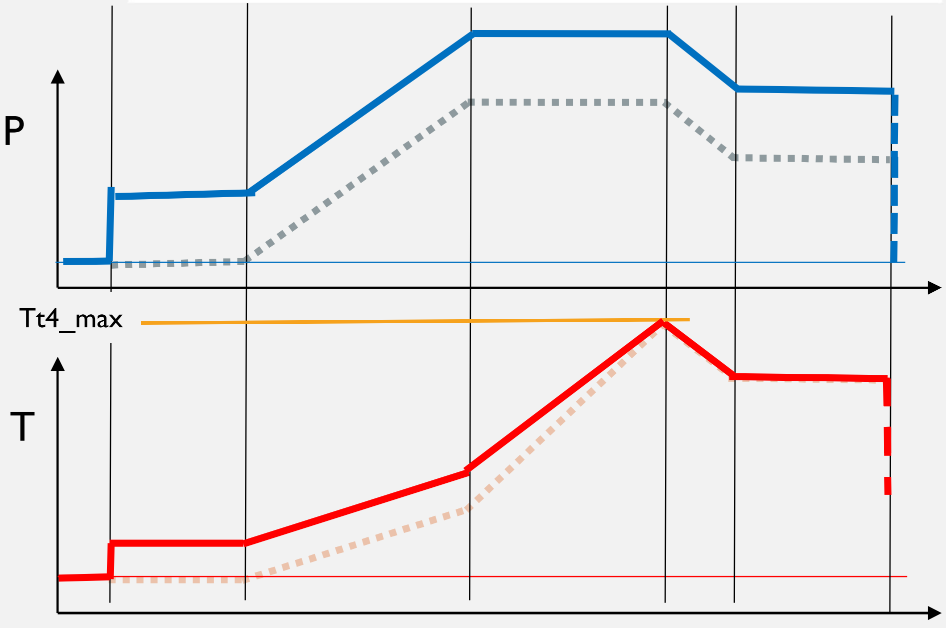
$$\eta_O = 0$$
$$\eta_P = 0$$

For V_0>0

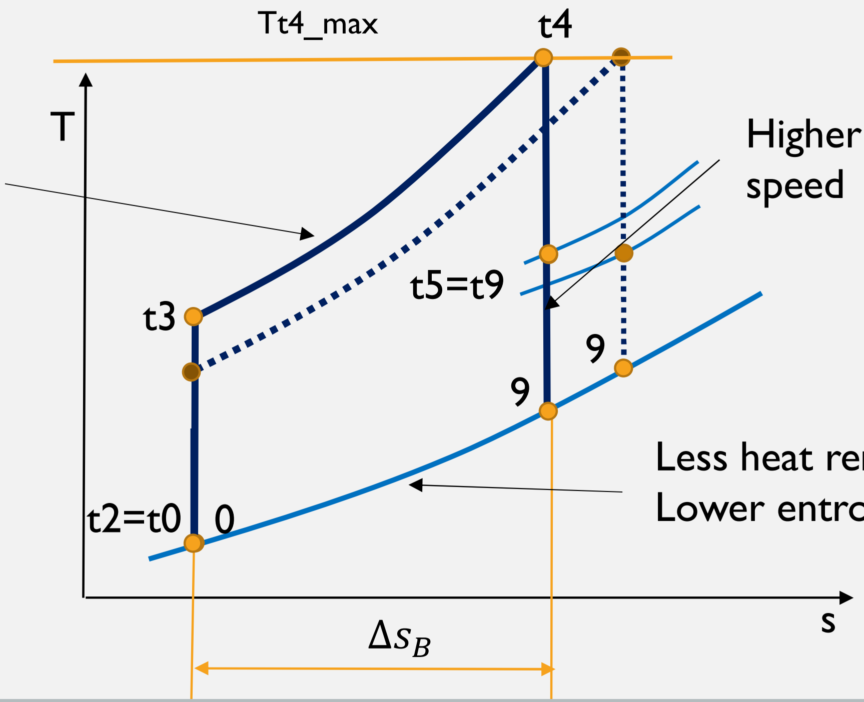
$$\eta_O < \eta_P$$
$$\eta_O < \eta_{TH}$$

IDEAL TURBOJET ENGINE – FLIGHT SPEED INFLUENCE





Flight condition V0>0 and Tt4 limit Tt4_max



Less heat added

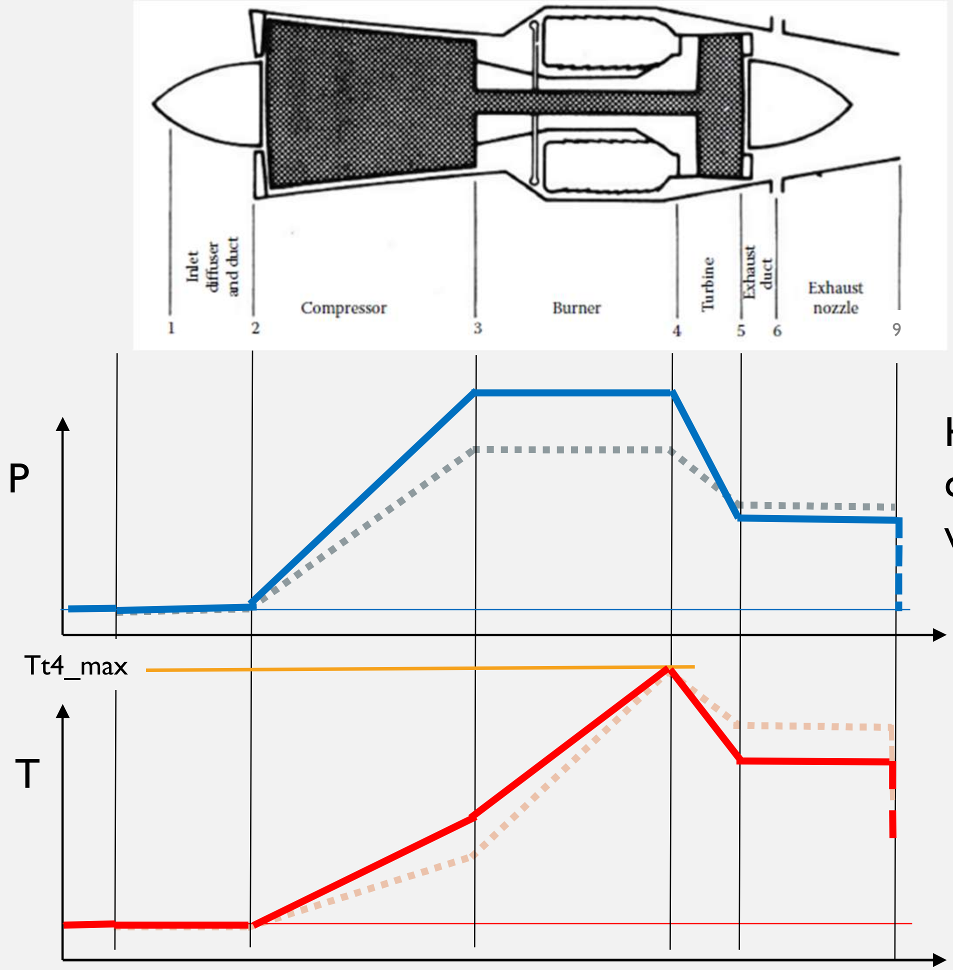
Higher gas outlet speed

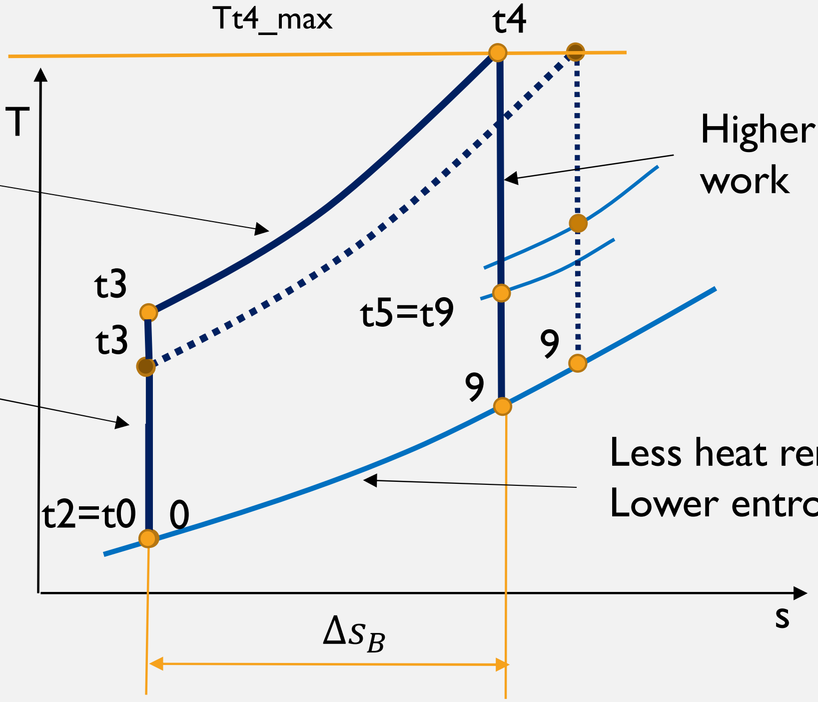
Less heat removed
Lower entropy grow

Engine thermal efficiency grow (less heat removed)
Lower fuel consumption (less heat added)

IDEAL TURBOJET ENGINE – COMPRESSOR PRESSURE RATIO INFLUENCE

CPR (compressor pressure ratio) growing and Tt4 is limited Tt4_max





Less heat added

Higher compressor work

Higher turbine work

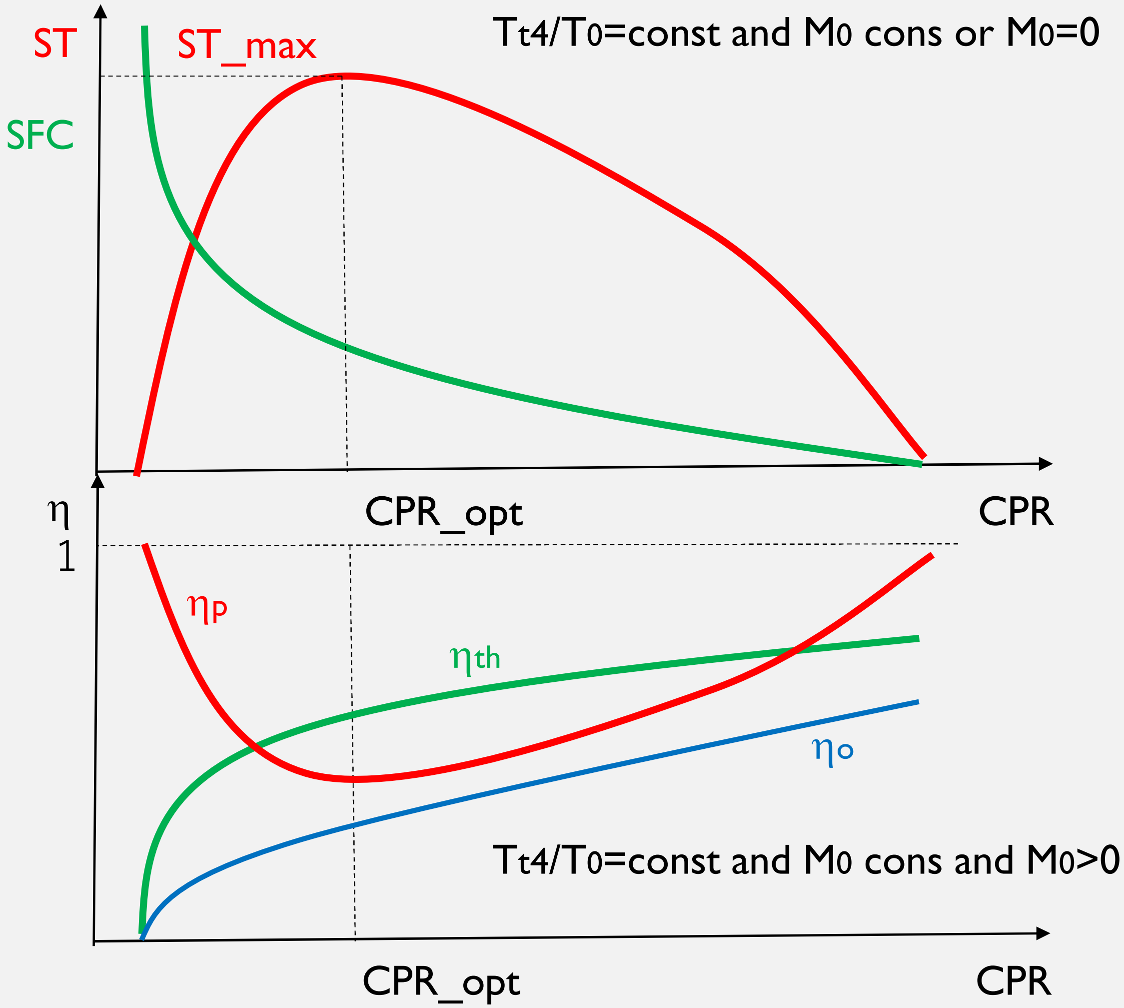
Less heat removed
Lower entropy grow

Engine thermal efficiency grow (less heat removed)
Lower fuel consumption (less heat added) and lower SFC

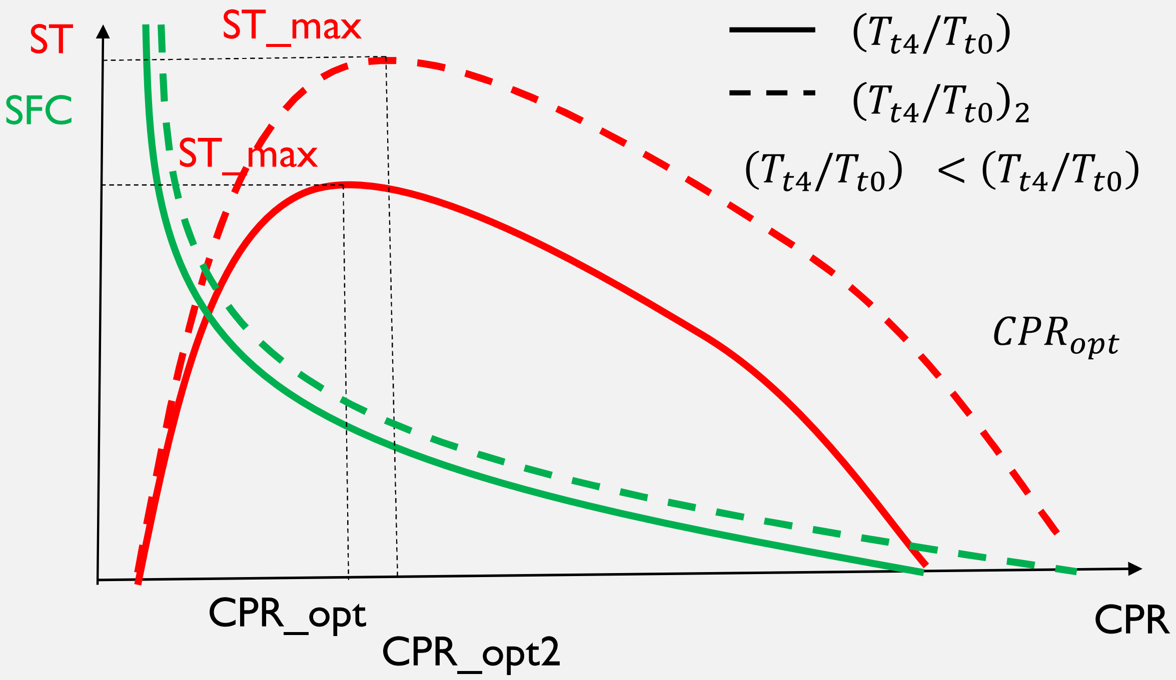
IDEAL TURBOJET CYCLE OPTIMISATION

SUMMARY:

- Specific thrust (ST) grows with compressor pressure ratio increasing, achieves maximum for optimal CPR than it goes down
- Specific fuel consumption decreases with CPR growing
- Propulsive efficiency as a function of CPR represents opposite relation to ST, it is minimal for optimal CPR and achieves 1 for ST=0.
- Thermal and overall efficiencies grow with CPR
- The presented relationship between efficiencies and CPR are right for M0>0, for M0=0, propulsive and overall efficiency are 0



IDEAL TURBOJET CYCLE OPTIMISATION FOR DIFFERENT ENGINE TEMPERATURE RATIO



For ideal cycle:

$$CPR_{opt} = T_{t4}/T_{t0}^{\frac{k}{2(k-1)}} \qquad CPR_{max} = CPR_{opt}^{\ 2}$$

SUMMARY:

- Specific thrust (ST) is higher for higher engine themperature ratio Tt4/Tt0 and achieve ST_max for higher CPR (higher CPR_opt)
- Specific fuel consumption decreases with CPR growing, but for highrt Tt4/Tt0 is higher
- Range of available CPR increases for higher Tt4/Tt0

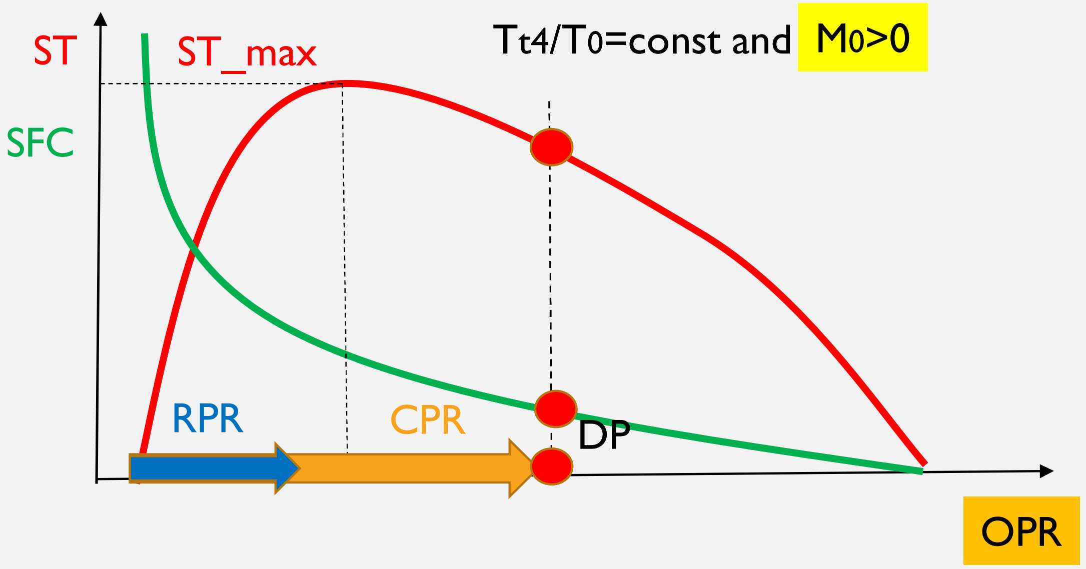
Tt4/Tt0	CPR_opt	CPR_max
4	11,3	128
5	16,7	279,5
6	23	529

IDEAL TURBOJET CYCLE OPTIMISATION FOR VARIOUS V0

Flight speed M0	RPR (ideal)
1	1,89
1,5	3,67
2	7,82
2,5	17,09
3	36,73

SUMMARY:

- An increase in flight speed causes an increase in the ram pressure ratio (RPR).
- When optimizing the engine cycle for increasing flight speeds, RPR should be included in OPR, and CPR should be reduced accordingly.

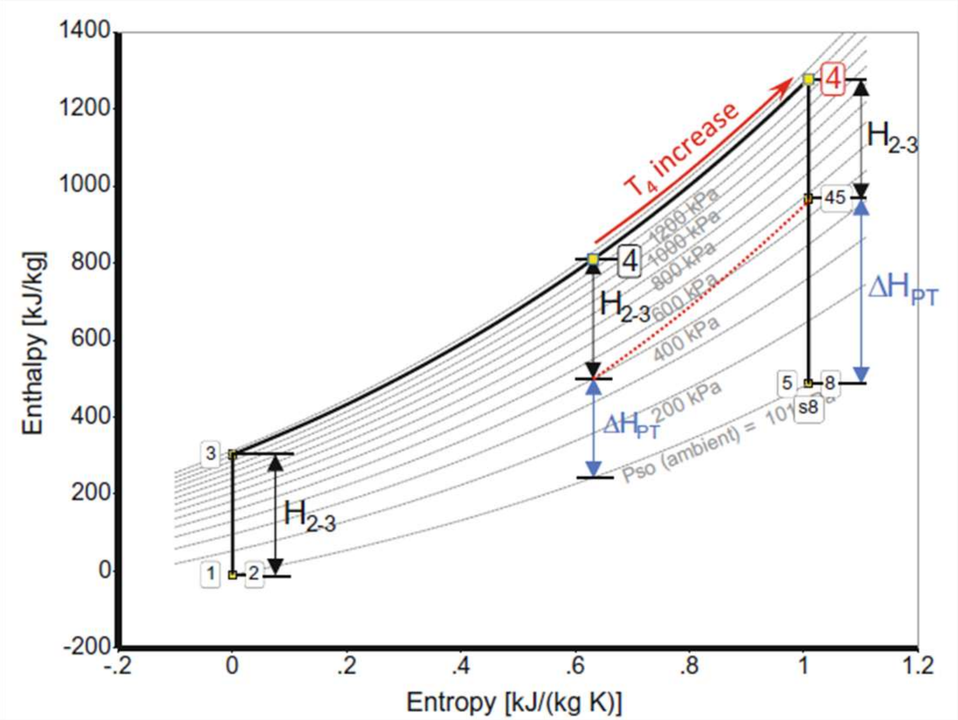


$$OPR = RPR * CPR$$

$$RPR = \left(1 + \frac{k-1}{2} M_0^2\right)^{\frac{k}{k-1}}$$

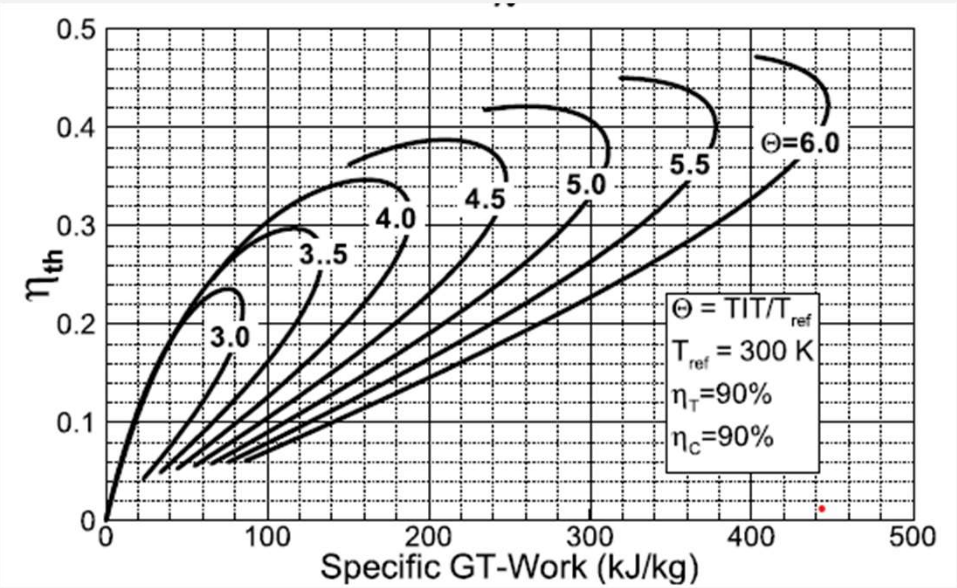
INCREASE OF OUTPUT SHAFT POWER

- **Increase of turbine inlet temperature (TIT)**



Gas turbine entalpy – entropy diagram for TIT rise

J. Kurzke, I. Halliwell, Propulsion and Power An Exploration of Gas Turbine Performance Modeling, https://doi.org/10.1007/978-3-319-75979-1

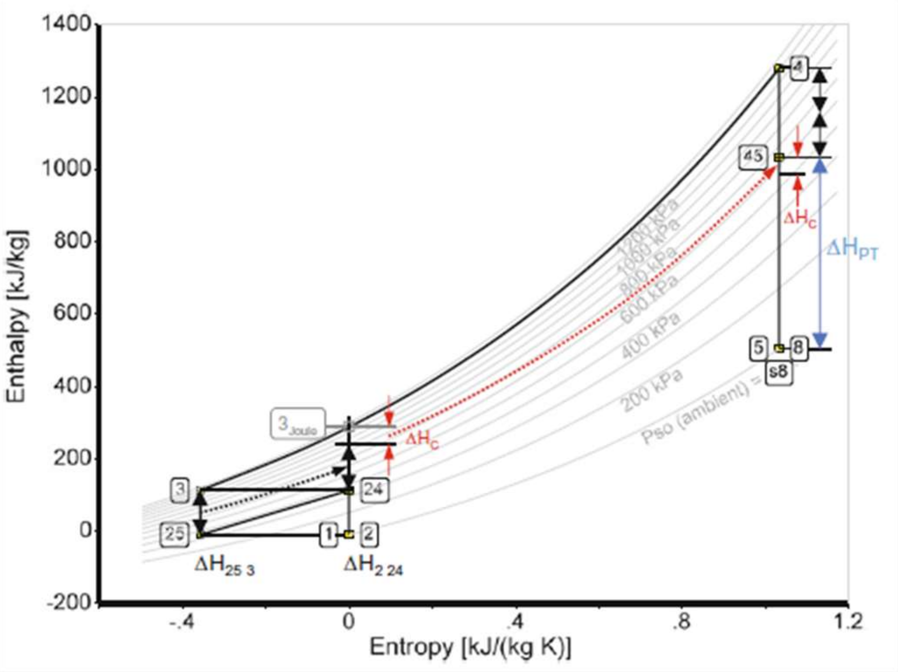


Thermal efficiency vs. Specific work of GT

M.T. Schobeiri, Gas Turbine Design, Components and System Design Interation, DOI 10.1007/978-3-319-85378-5_1

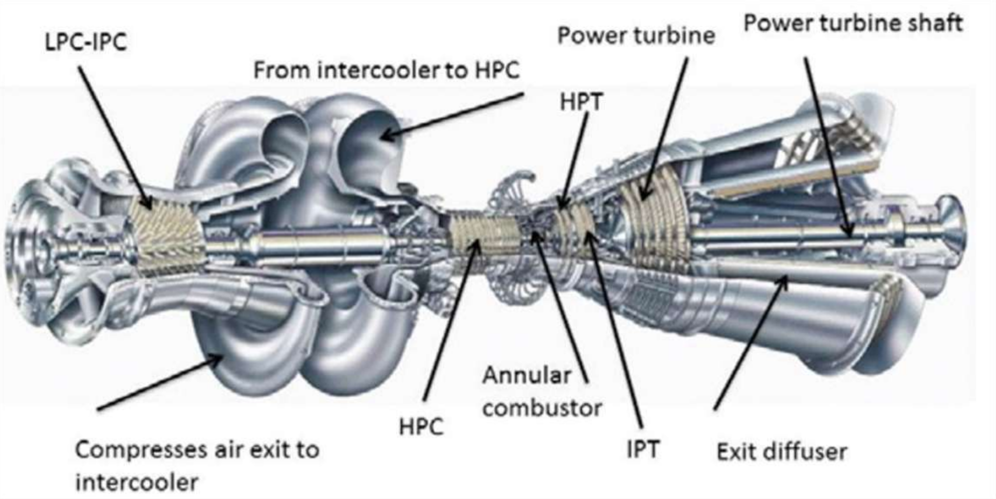
INCREASE OF OUTPUT SHAFT POWER

- **Compressor intercooling**



Gas turbine entalpy – entropy diagram for compressor intercooling

J. Kurzke, I. Halliwell, Propulsion and Power An Exploration of Gas Turbine Performance Modeling, https://doi.org/10.1007/978-3-319-75979-1

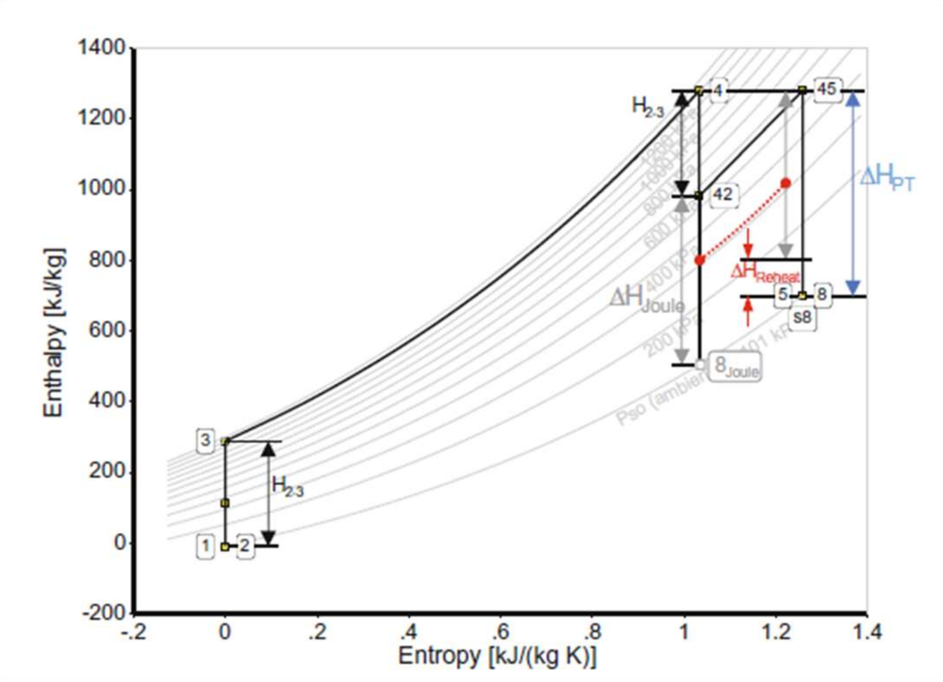


Gas turbine LS100 (GE)

M.T. Schobeiri, Gas Turbine Design, Components and System Design Interation, DOI 10.1007/978-3-319-85378-5_1

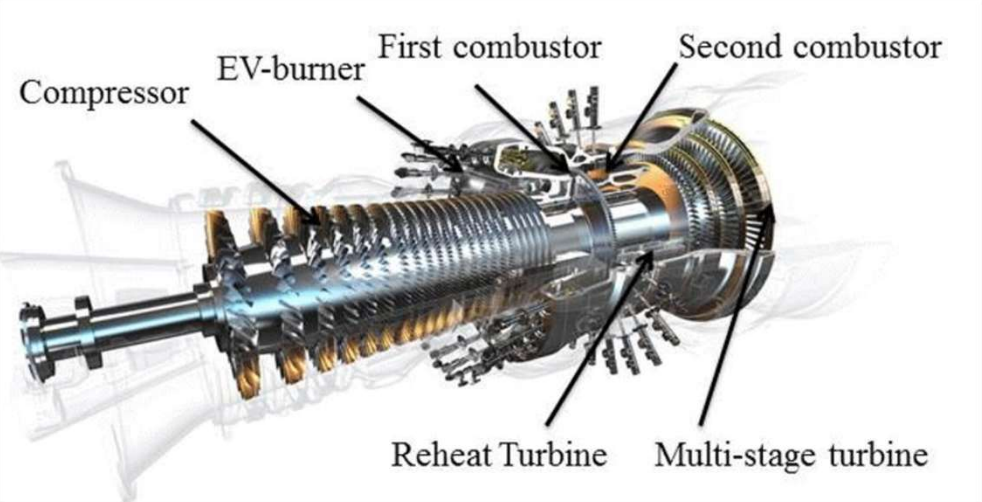
INCREASE OF OUTPUT SHAFT POWER

- **Reheating – inter turnine burner (ITB)**



Gas turbine with ITB entalpy – entropy diagram

J. Kurzke, I. Halliwell, Propulsion and Power An Exploration of Gas Turbine Performance Modeling, https://doi.org/10.1007/978-3-319-75979-1

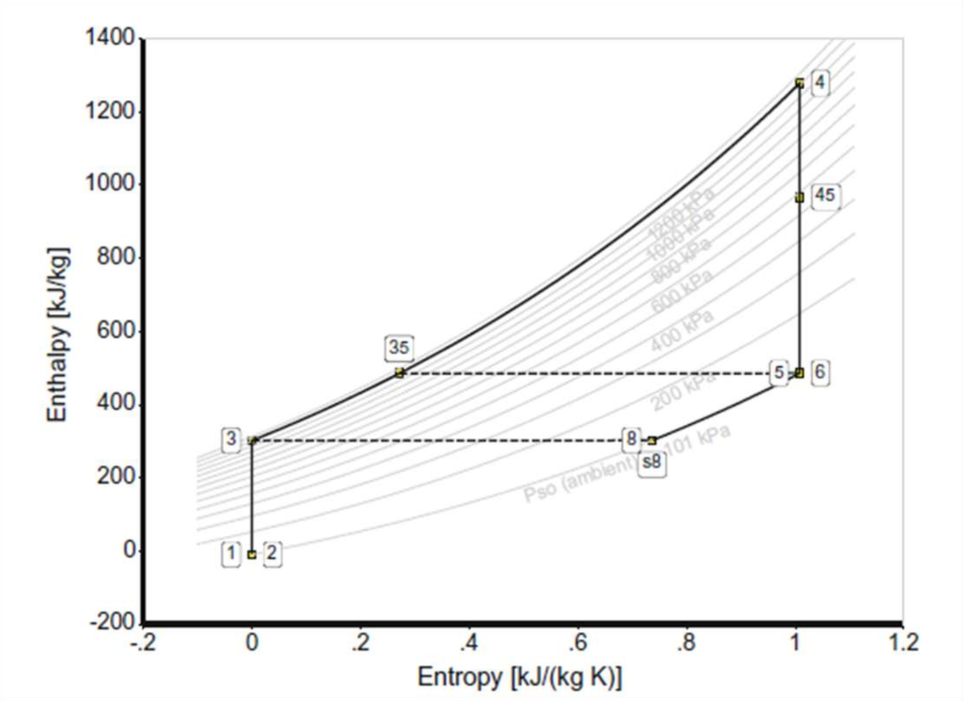


GT with ITB – GT 24/26

M.T. Schobeiri, Gas Turbine Design, Components and System Design Interation, DOI 10.1007/978-3-319-85378-5_1

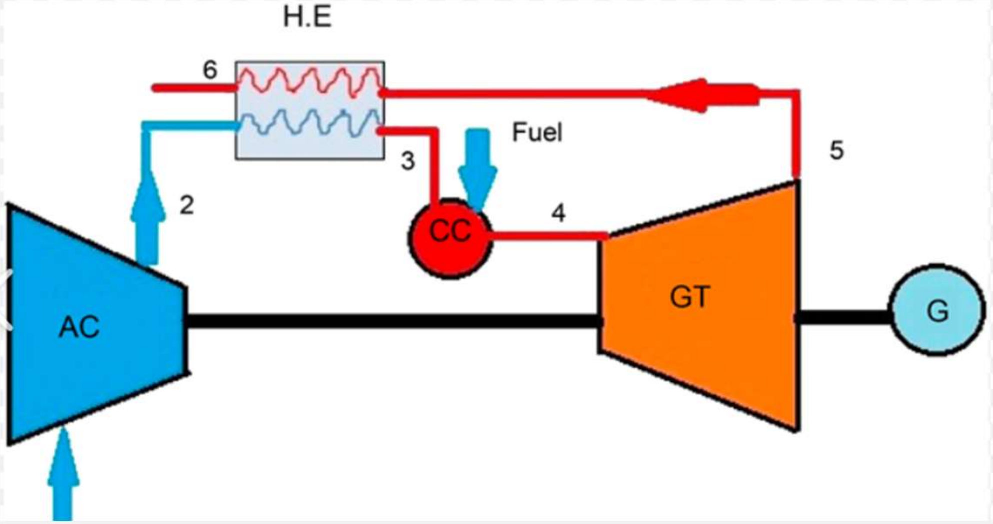
REDUCE FUEL CONSUMPTION

- **GT with heat exchanger (HE)**



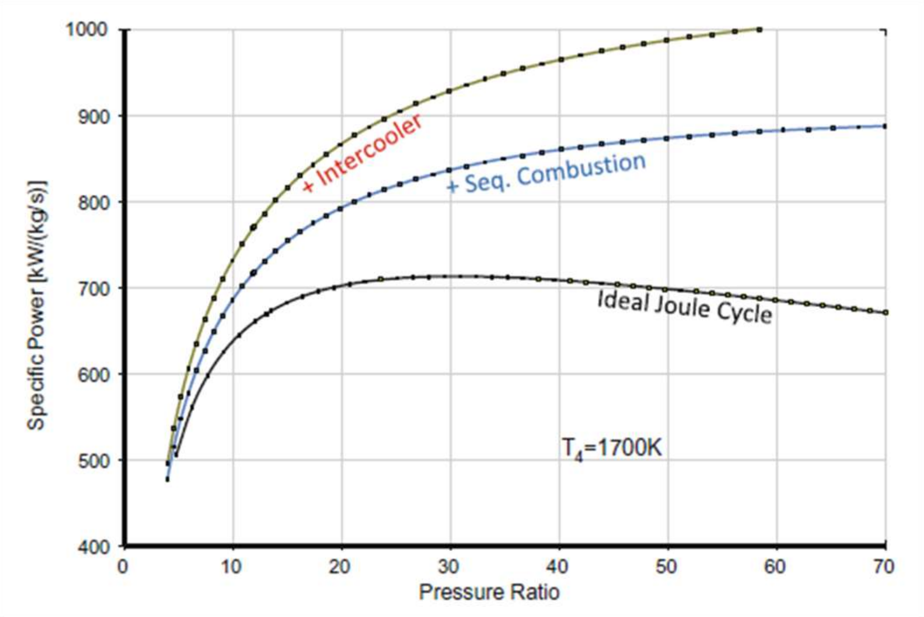
Gas turbine with heah exchanger entalpy – entropy diagram

J. Kurzke, I. Halliwell, Propulsion and Power An Exploration of Gas Turbine Performance Modeling, https://doi.org/10.1007/978-3-319-75979-1

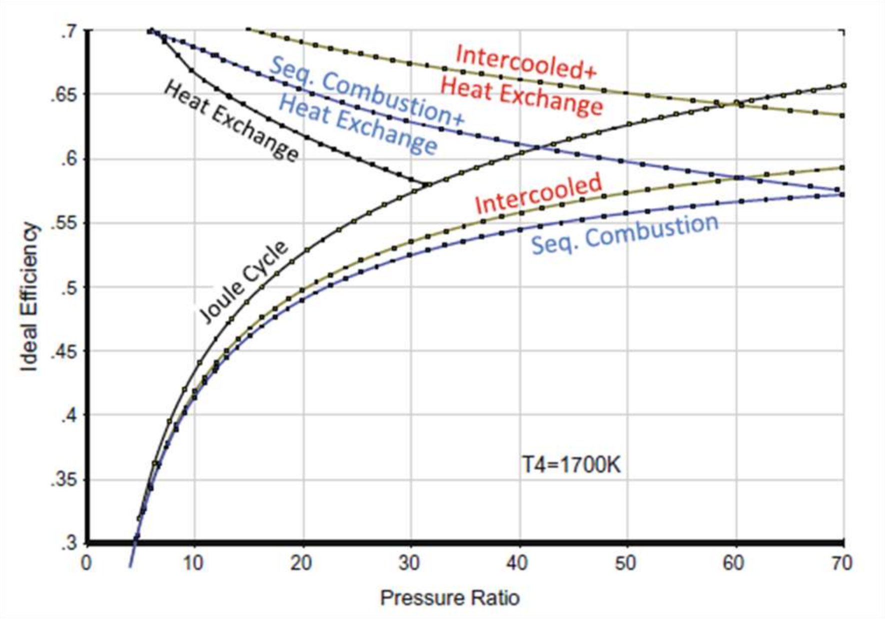


Scheme of GT with HE

GAS TURBINES AND MODIFIED GT



Specific power for ideal cycles



Efficiency of ideal cycles for shaft power generation

J. Kurzke, I. Halliwell, Propulsion and Power An Exploration of Gas Turbine Performance Modeling, https://doi.org/10.1007/978-3-319-75979-1

Higher efficiency means lower specific fuel consumption!

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THANKS FOR YOUR ATENTION

DZIĘKUJĘ ZA UWAGĘ