IDEAL TURBOJET

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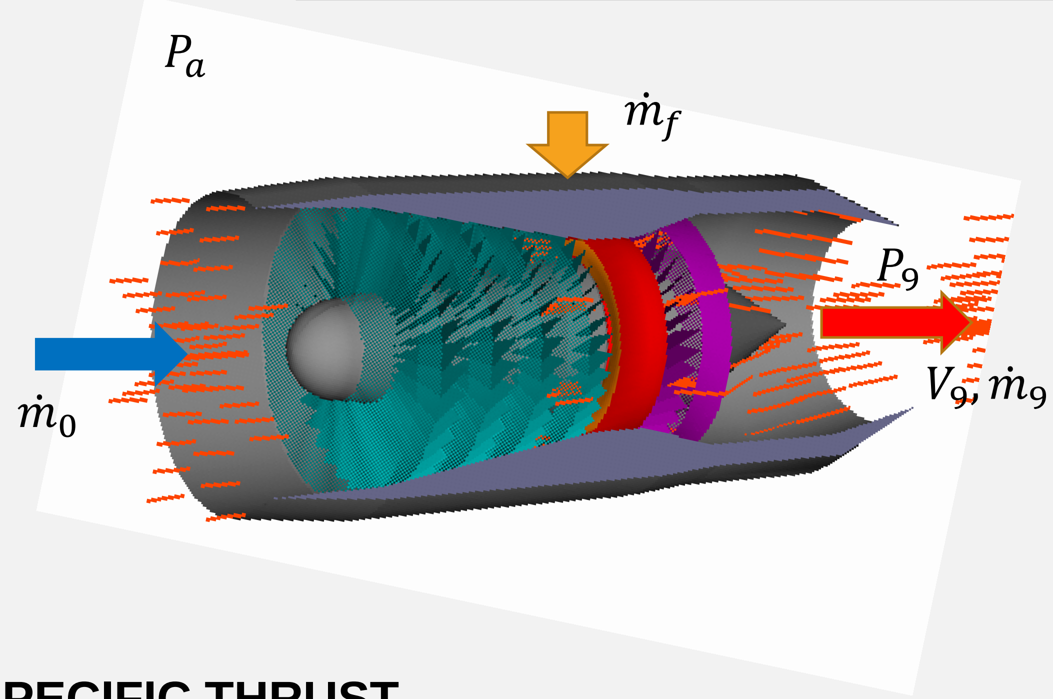
LITERATURE:

- **Jack D. Mattingly, Elements of Propulsion: Gas Turbines and Rockets, AIAA Education Series 2006 (Chapter 5)**
- **Jack D. Mattingly, Elements of Gas Turbine Propulsion, Tata McGraw Hill Education Private Limited, 2013 (Chapter 5)**
- **Gordon C. Oates, Aerothermodynamics of Gas Turbine and Rocket Propulsion, AIAA Education Series, 1997 (Chapter 5)**

ENGINE THRUST AND SPECIFIC PARAMETERS



Flight speed is 0

THTUST / GROS THRUST

$$T = \dot{m}_9 V_9 + A_9(P_9 - P_a)$$

effective exhaust velocity

$$V_{eff} = V_9 + A_9(P_9 - P_a)/\dot{m}_9$$

$$T = \dot{m}_9 V_{eff}$$

Exit pressure = ambient pressure

$$T = \dot{m}_9 V_9$$

Flight speed > 0

THTUST / NET THRUST

$$T = \dot{m}_9 V_9 + A_9(P_9 - P_a) - \dot{m}_0 V_0 = \dot{m}_9 V_{eff} - \dot{m}_0 V_0$$

Net thrust = Gross thrust –Momentum drag

SPECIFIC THRUST

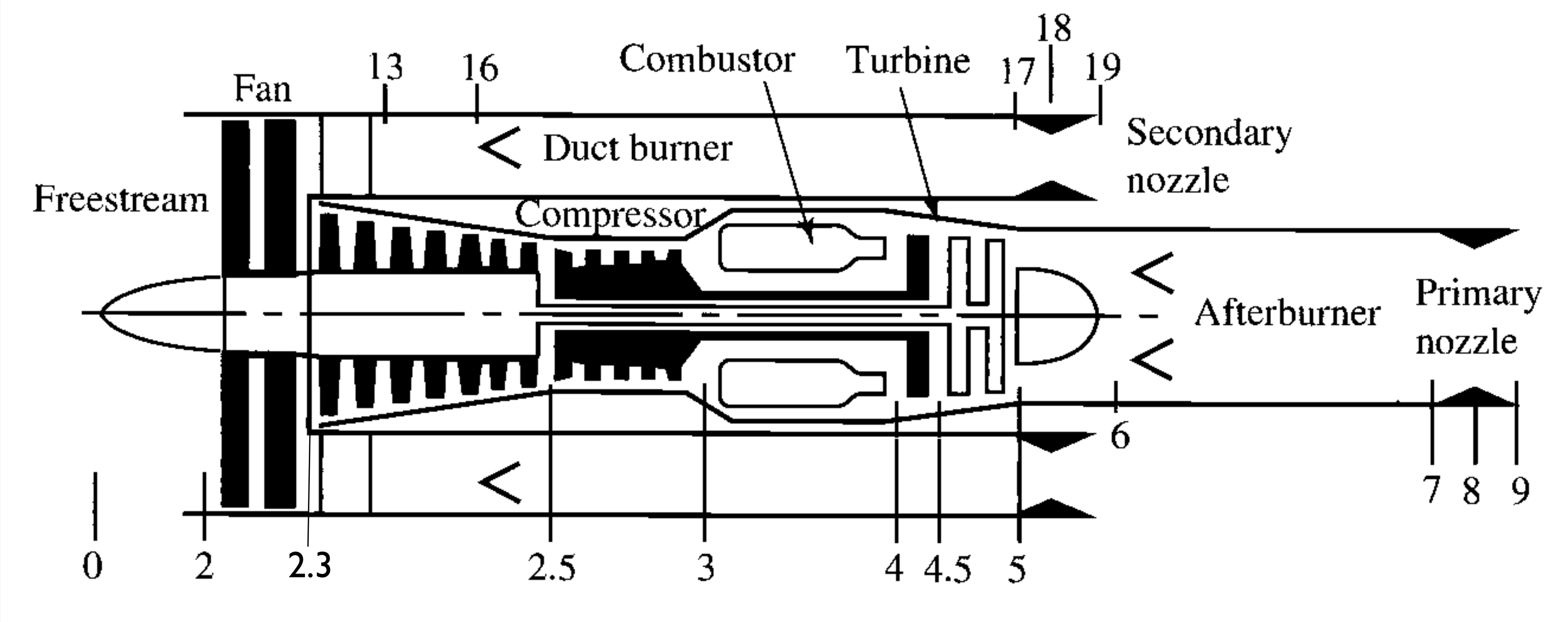
$$ST = T/\dot{m}_0$$

SPECIFIC FUEL CONSUMPTION

$$\mathrm{SFC} = \dot{m}_f/T$$

Details in: **Ahmed F. El-Sayed, Aircraft Propulsion and Gas Turbine Engines, (chapter 2.2)**

SECTION NUMBERING FOR GAS TURBINE ENGINE



$$\pi_a = \frac{\text{total pressure leaving component a}}{\text{total pressure entering component a}}$$

$$\tau_a = \frac{\text{total temperature leaving component a}}{\text{total temperature entering component a}}$$

$$\pi_T = \frac{\text{total pressure entering turbine}}{\text{total pressure leaving turbine}} \quad !$$

D/IN – Inlet diffuser (sections 0-2)
F – Fan (sections 2-13)
C – compressor (2-3)
LPC – Low Pressure Compressor (2.3-2.5)
HPC – High Pressure Compressor (2.5-3)
B – Burner/Combustor (3-4)
T – Turbine (4-5)
HPT – High Pressure Turbine (4-4.5)

LPT – Low Pressure Turbine (4.5-5)
AB – Afterburner (5-7)
N – Nozzle (7-9) (5-9)
ED – External Duct (13-17)
EN – ExternalNozzle(17-19) (13-19)

ENGINE EFFICIENCIES

Thermal efficency

$$\eta_{TH} = \frac{\text{Power imparted to engine airflow}}{\text{Rate of energy supplied in the fuel}}$$

$$\eta_{TH} = \frac{0,5 * (\dot{m}_9 {V_{9e}}^2 - \dot{m}_0 {V_0}^2)}{\dot{m}_f FHV}$$

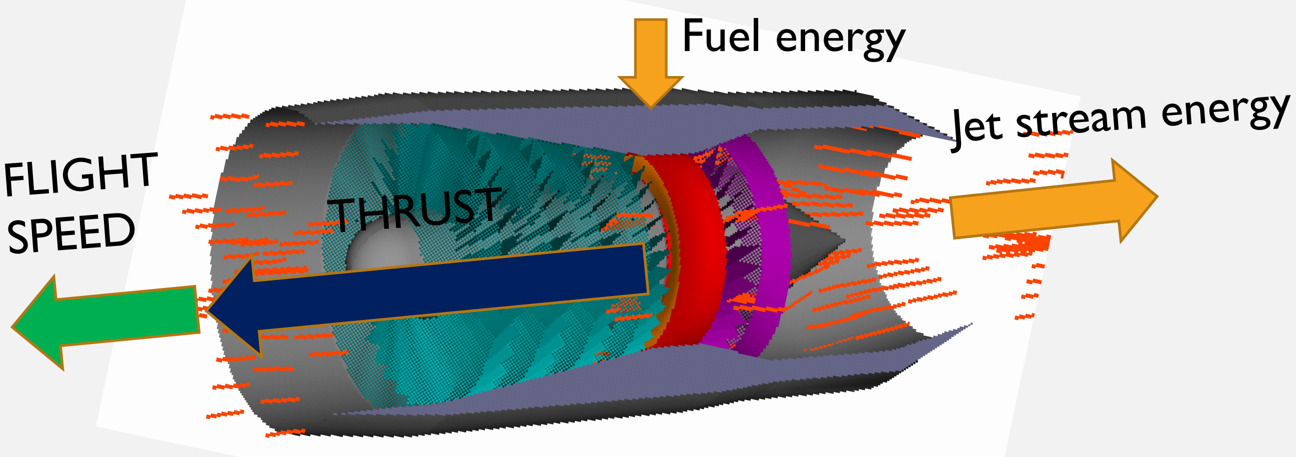
Propulsive efficency

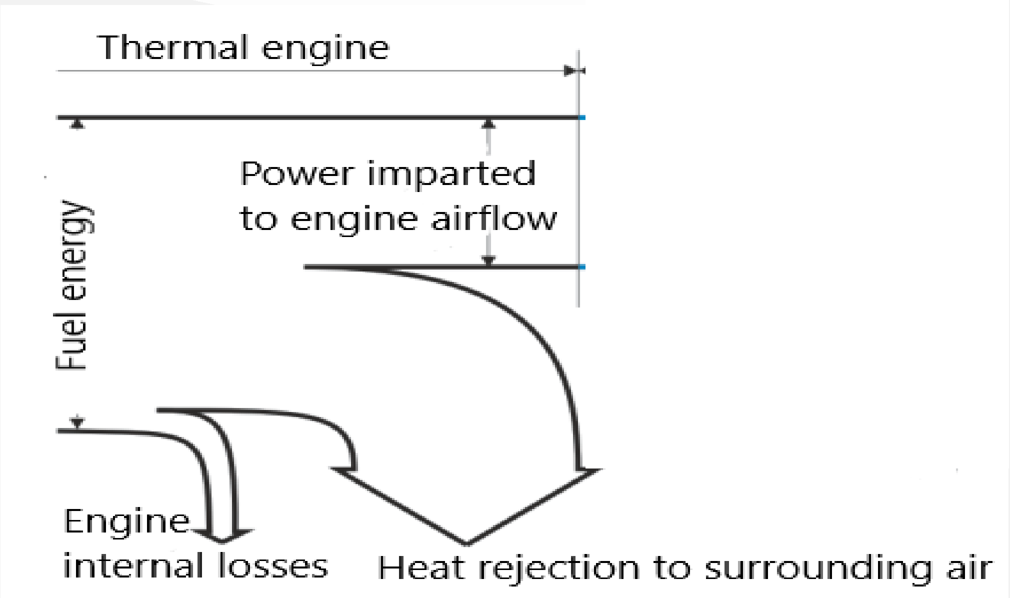
$$\eta_P = \frac{\text{Thrust power}}{\text{Power imparted to engine airflow}}$$

$$\eta_P = \frac{V_0 * T}{0,5 * (\dot{m}_9 {V_{9e}}^2 - \dot{m}_0 {V_0}^2)}$$

Overall efficency

$$\eta_O = \eta_{TH} * \eta_P = \frac{V_0 * T}{\dot{m}_f FHV}$$





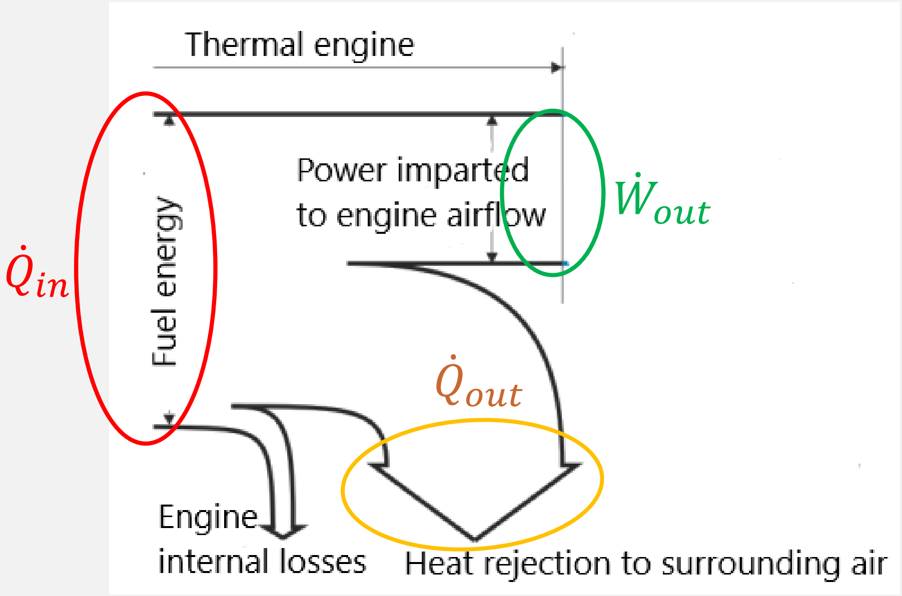
THERMAL EFFICIENCY

$$\eta_{TH} = \frac{\dot{W}_{out}}{\dot{Q}_{in}} \quad \Rightarrow \quad \eta_{TH} = \frac{\dot{Q}_{in} - \dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}}$$

where, $\dot{Q}_{out} = \dot{m}_9 c_p (T_9 - T_0)$

- Wout = net power out of engine (engine work)
- Qin= rate of thermal energy released/suplied in the fuel)

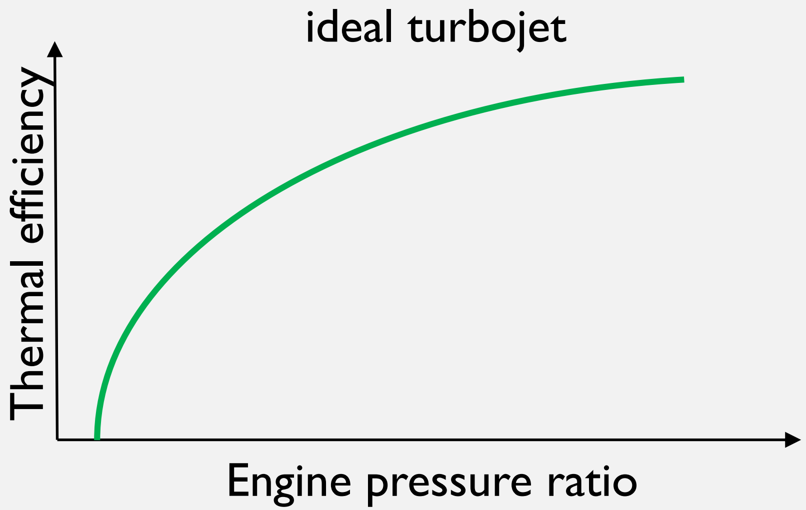
Higher thermal efficiency causes lower specific fuel consumption
Thermal efficiency is higher when exhaust gas temperature is closer to the ambient temperature



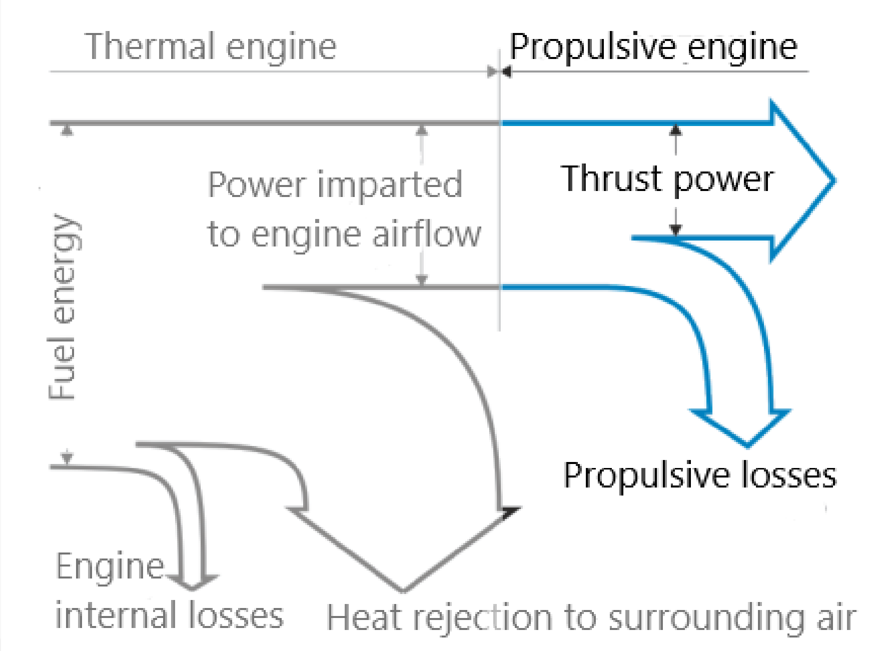
For ideal turbojet engine:

$\eta_{TH} = 1 - \frac{1}{\pi^{(k-1)/k}}$,

where π – engine compression pressure ratio, k- isentropic exponent



PROPULSIVE EFFICIENCY

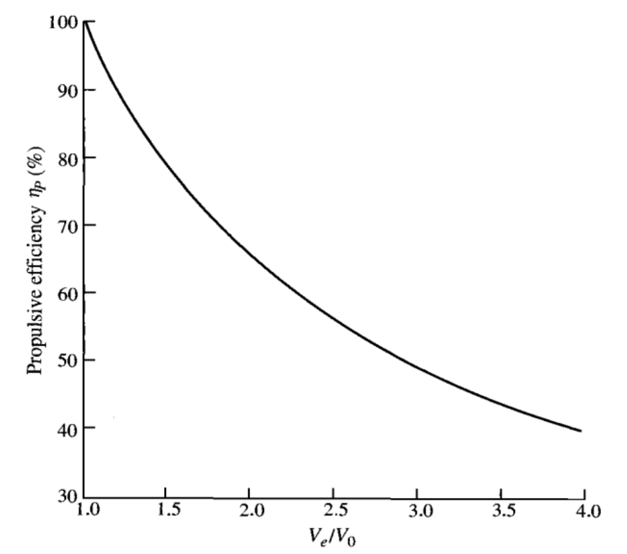


$$\eta_P = \frac{V_0 * T}{0{,}5 * (\dot{m}_9 {V_{9e}}^2 - \dot{m}_0 {V_0}^2)}$$

$$T = \dot{m}_9 V_{9e} - \dot{m}_0 V_0 \quad and \quad \dot{m}_9 = \dot{m}_0$$

$$\eta_P = \frac{V_0 (V_{9e} - V_0)}{0{,}5 * ({V_{9e}}^2 - {V_0}^2)} = \frac{2V_0 (V_{9e} - V_0)}{(V_{9e} - V_0)(V_{9e} + V_0)} = \frac{2}{1 + V_{9e}/V_0}$$

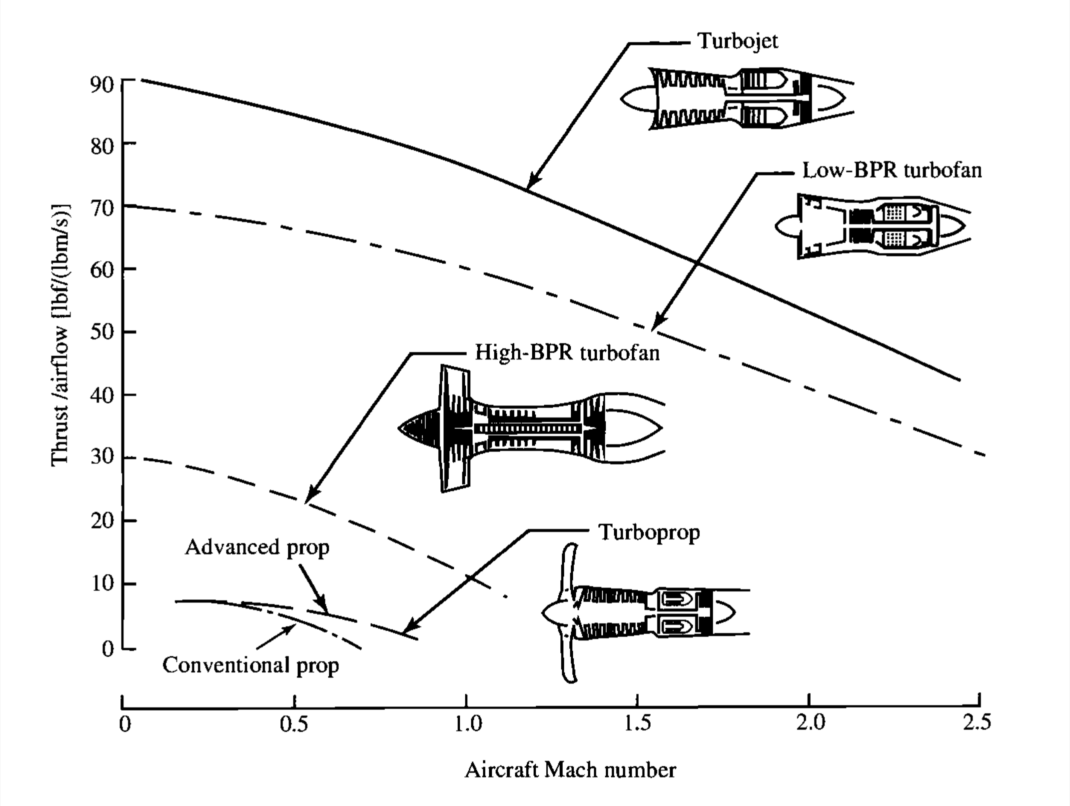
$$\eta_P \Rightarrow 1, gdy \;\; V_{9e} \Rightarrow V_0$$



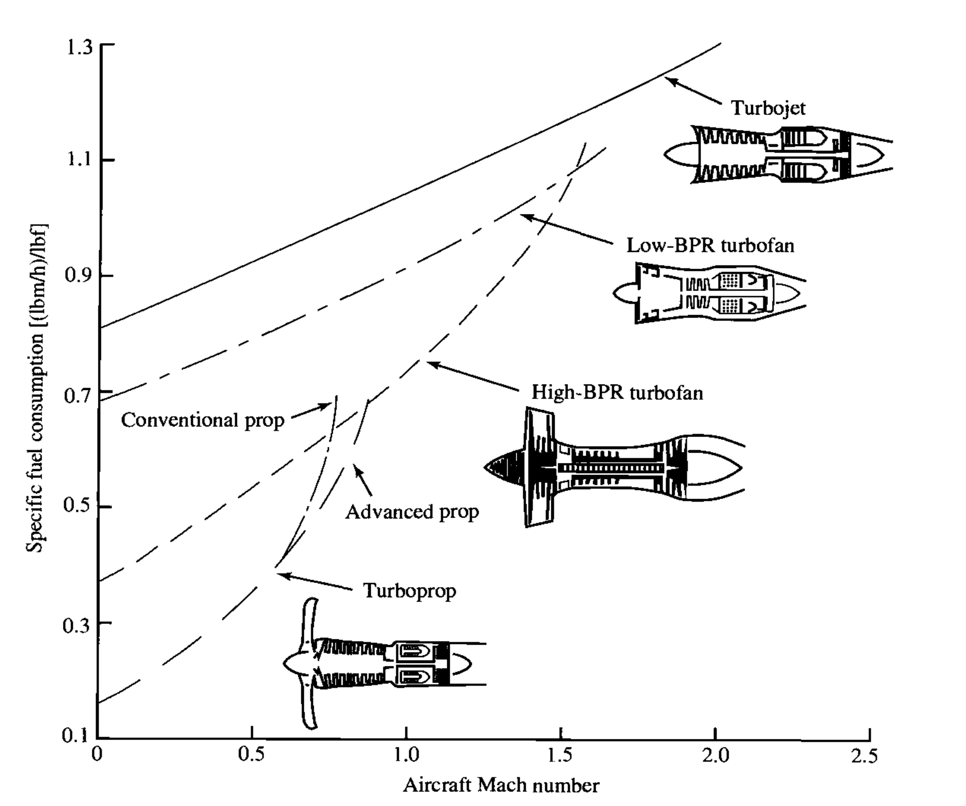
Propulsiv efficiency define the thrust produced for specific flight speed from kinetic energy added to engine airflow

ENGINES PERFORMANCE

Specific thrust vs. flight speed



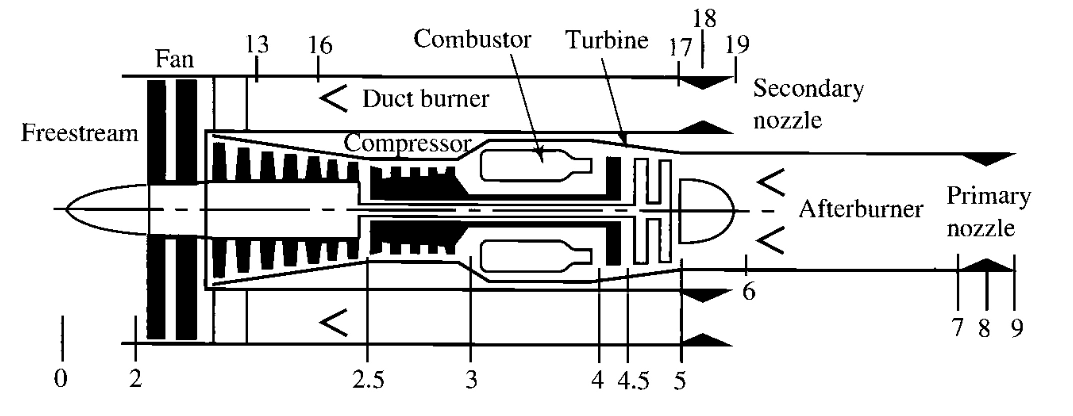
Specific fuel consumption vs. flight speed



Propulsions of high specific thrust have got high specific fuel consumption
Low specific fuel consumption is characteristic for propulsions dedicated for low speed

Figures from: **Jack D. Mattingly, Elements of Propulsion: Gas Turbines and Rockets, AIAA Education Series 2006**

STATIC / TOTAL PARAMETERS



Ram pressure recovery for flight condition ($M_0 > 0$)

$$P_{t0} = P_0\left(1 + \frac{k-1}{2}{M_0}^2\right)^{k/(k-1)}$$

$$T_{t0} = T_0\left(1 + \frac{k-1}{2}{M_0}^2\right)$$

P_2 - static pressure in section 2
P_{t2} - total pressure in section 2

$$P_{t2} = P_2\left(1 + \frac{k-1}{2}{M_2}^2\right)^{k/(k-1)}$$

T_2 - static temperature in section 2
T_{t2} - total temperature in section 2

$$T_{t2} = T_2\left(1 + \frac{k-1}{2}{M_2}^2\right)$$

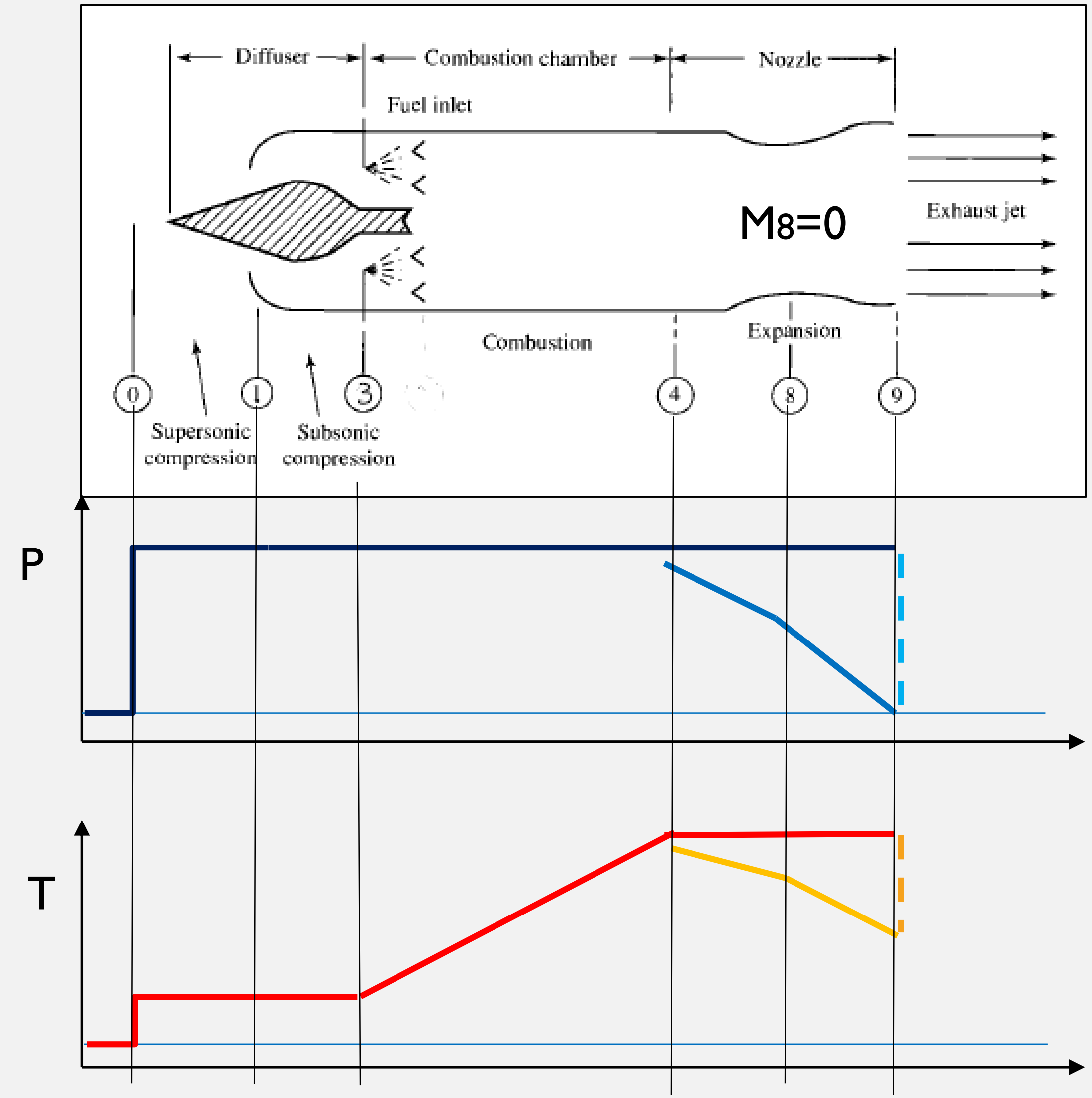
For IDEAL ENGINE losses are omitted

$$\boldsymbol{\pi_D = \pi_B = \pi_N = \pi_{EN} = 1}$$

Additional assumption:

Perfect gas:
Cp=1005 J/kg/K, k=1.4, R=287 J/kg/K – for air
Cpt =1170 J/kg/K, kt=1.33, R=290 J/kg/K – for fume
CpB=1200 J/kg/K
CpAB=1250 J/kg/K

IDEAL RAMJET



Air compression is provided in diffuser for $V_0>1$

Ram pressure recovery for flight condition ($M_0 > 0$)

$$P_{t0} = P_0\left(1 + \frac{k-1}{2}{M_0}^2\right)^{k/(k-1)}$$

$$T_{t0} = T_0\left(1 + \frac{k-1}{2}{M_0}^2\right)$$

where: $M_0 = \frac{V_0}{\sqrt{k*R*T_0}}$

No diffuser losses

$$\pi_D = 1, \tau_D = 1, \qquad P_{t3} = P_{t1} = P_{t0} \qquad T_{t3} = T_{t1} = T_{t0}$$

Ideal combustion process fo specific T_{t4}, $\pi_B = 1$

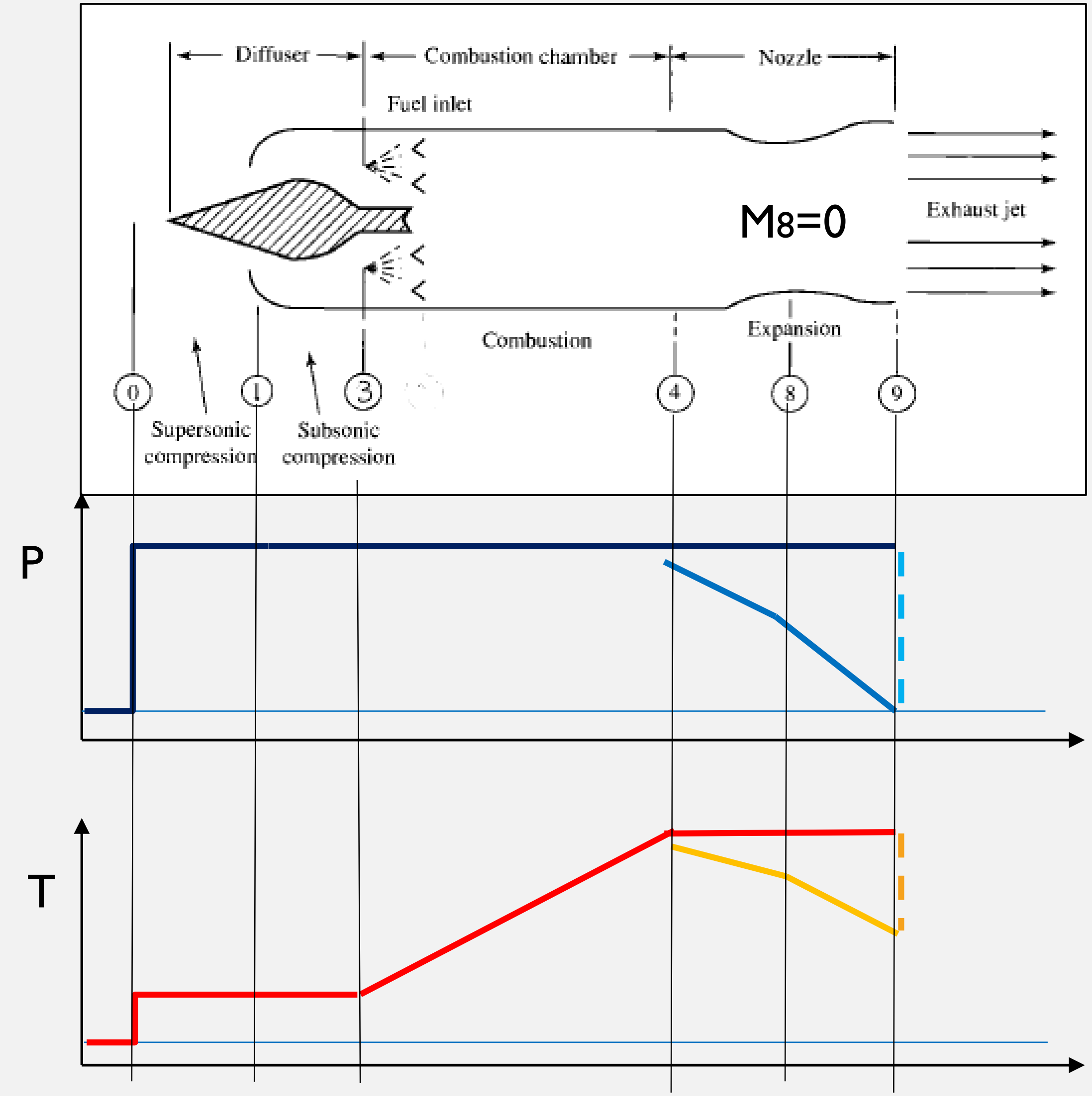
$$P_{t4} = \pi_B P_{t3} \qquad f * FHV = Cp_B(T_{t4} - T_{t3}),$$

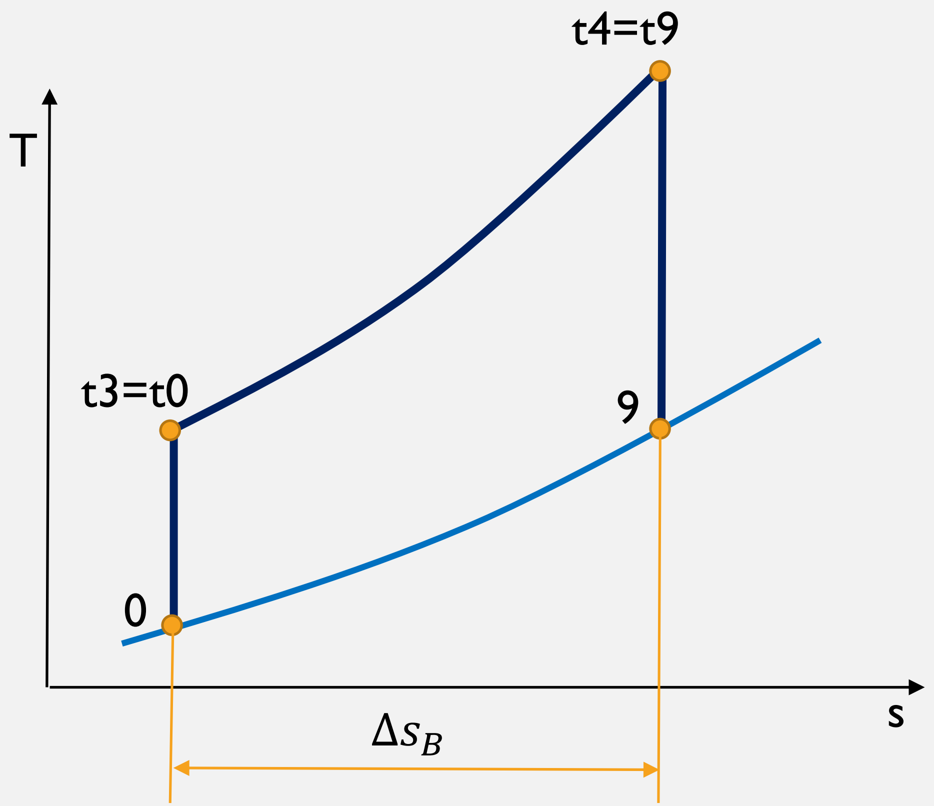
$f = \dot{m}_f/\dot{m}_0$ - Fuel air ratio

Ideal nozzle expansion $\pi_N = 1, \tau_N = 1,$

$$P_{t9} = P_{t4} \qquad T_{t9} = T_{t4} \qquad P_9 = P_0 \qquad T_9 = T_{9t}(P_9/P_{9t})^{(kt-1)/kt}$$

IDEAL RAMJET – THERMODYNAMIC CYCLE





Entropy increase in a burner

$$\Delta s_B = cp_B ln \frac{T_{t4}}{T_{t3}}$$

IDEAL RAMJET CYCLE PARAMETHERS CALCULATION

Propelling nozzle outlet gas speed c9

$$c_9 = \sqrt{2Cp_t(T_{9t} - T_9)}$$

- for incompressible flow

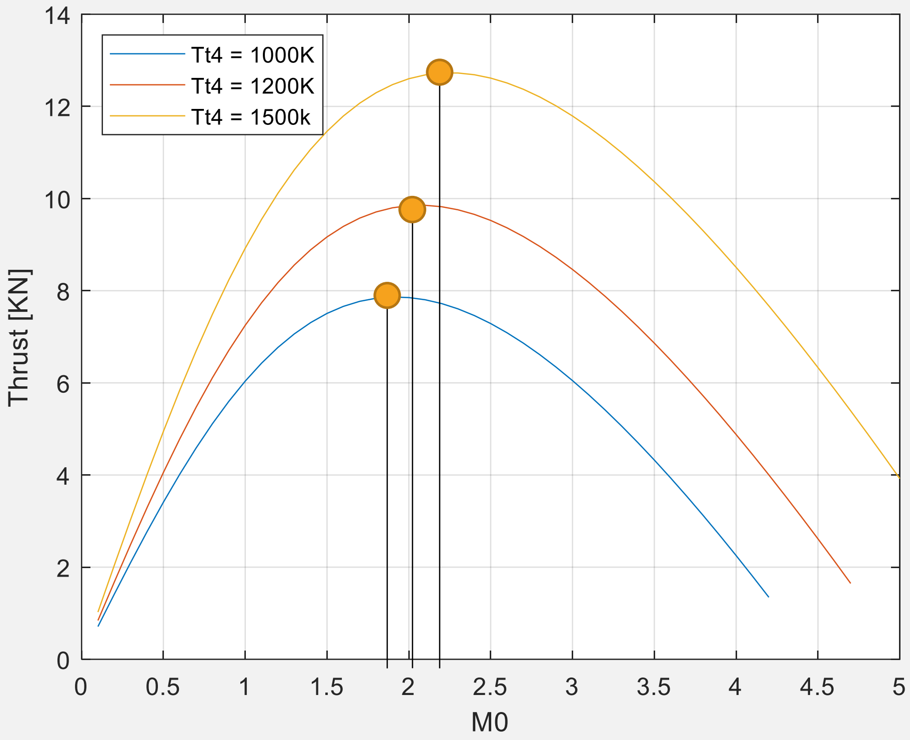
$$c_9 = a_9 M_9 = \sqrt{k_t R_t T_9} * \sqrt{\frac{2}{k_t - 1}\left(\frac{T_{t9}}{T_9} - 1\right)}$$

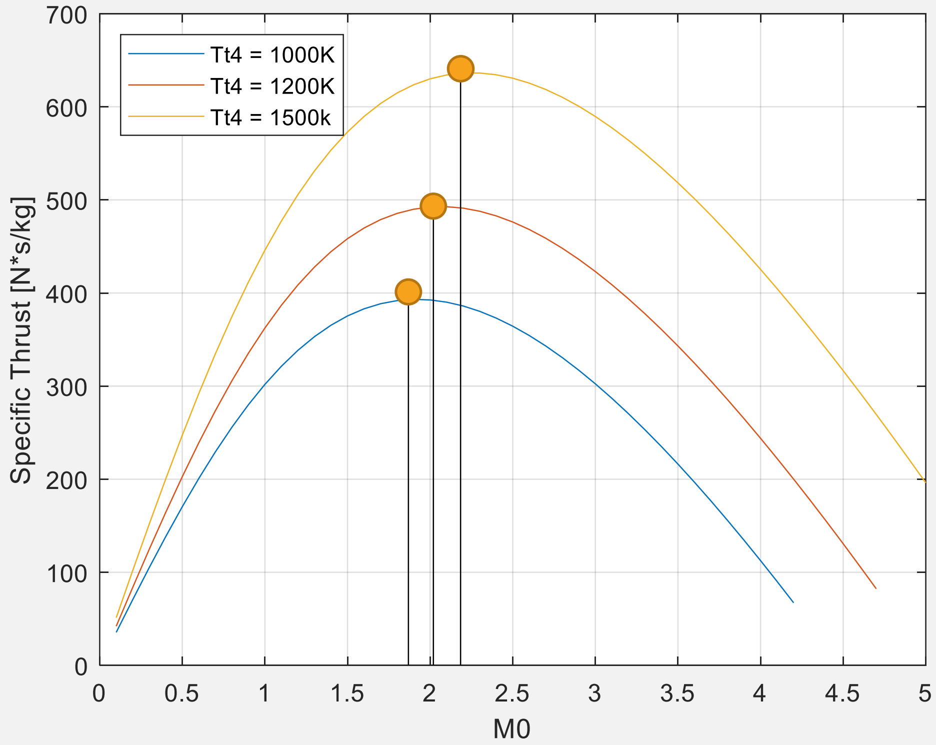
- for compressible flow

CYCLE PARAMETHERS CALCULATION AND ENGINE PERFORMANCE PARAMETHERS are presented in: Ideal_ramjet_example.pdf

IDEAL RAMJET PERFORMANCE - THRUST

Calculation dome for H=11 km, m_0=20 kg/s and T_{t4}=1000, 1200, 1500K; $T = \dot{m}_9 V_9 - \dot{m}_0 V_0$; $ST = T/\dot{m}_0$

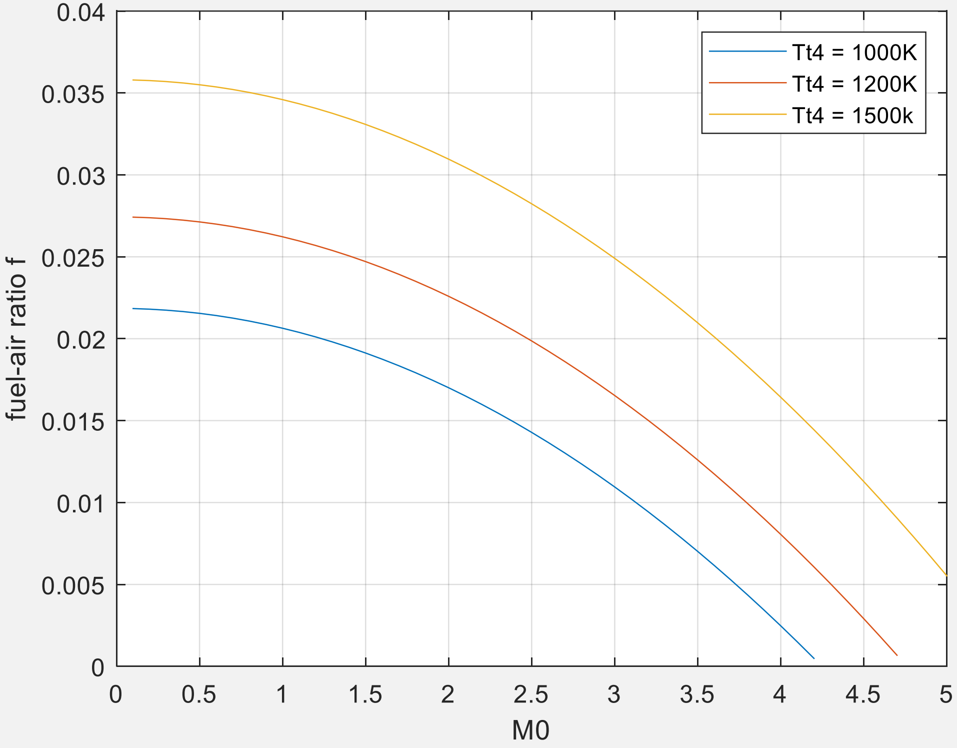


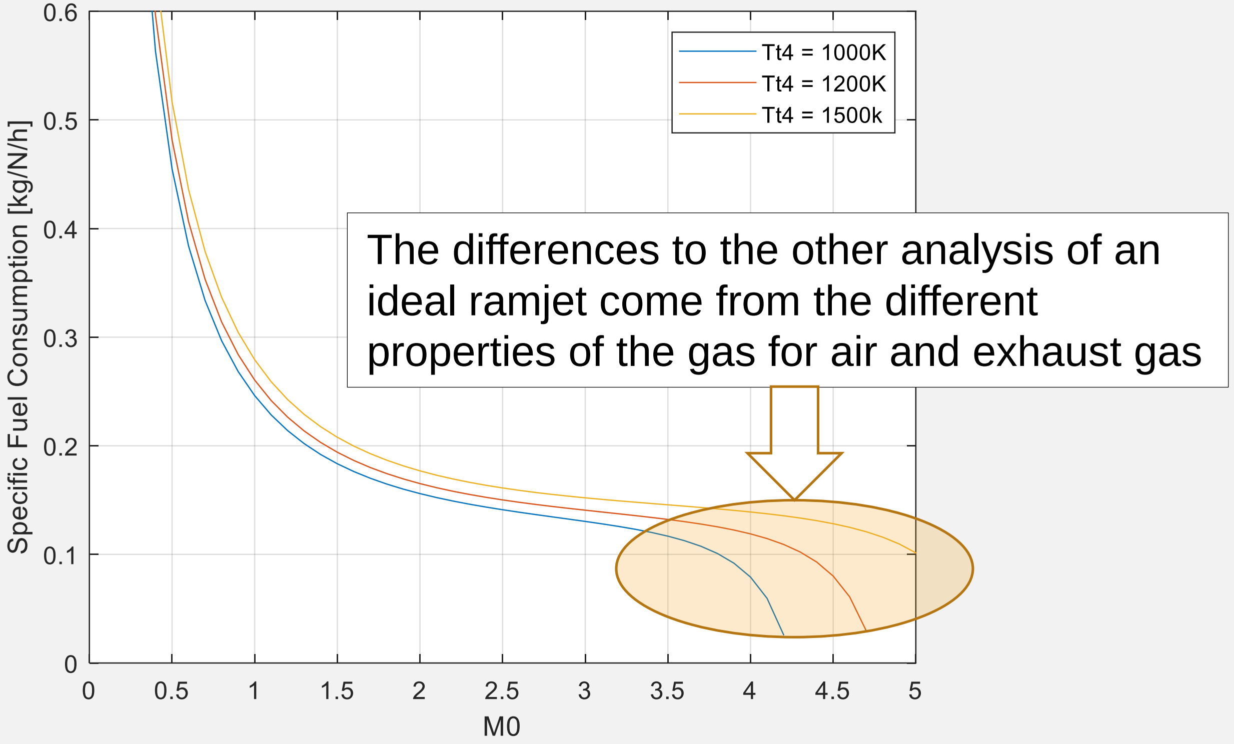


- Ramjet engine thrust increases to its maximum with flight speed and then decreases.
- Specific thrust vs flight speed relation looks similar
- For higher Tt4 thrust is higher and reaches its maximum value for higher flight speed.

IDEAL RAMJET PERFORMANCE - SFC

Calculation dome for H=11 km, m_0=20 kg/s and T_{t4}=1000, 1200, 1500K; $SFC = f/ST$



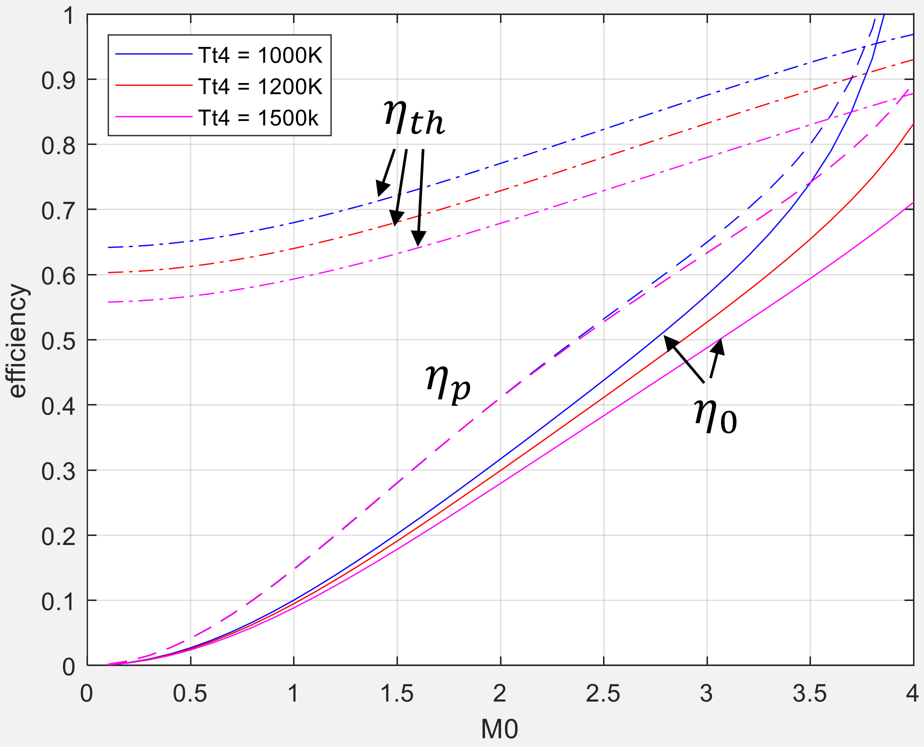


The differences to the other analysis of an ideal ramjet come from the different properties of the gas for air and exhaust gas

- The fuel to air ratio goes down with flight speed.
- The fuel to air ratio is lower for the lower Tt4.
- The range of flight speed applicability is lower when Tt4 is lower (the end of range when f reaches 0)
- SFC goes down for higher flight speed

IDEAL RAMJET PERFORMANCE - EFFICIENCIES

Calculation dome for H=11 km, m_0=20 kg/s and T_{t4}=1000, 1200, 1500K;



Thermal, propulsive and overall efficiency of ideal ramjet grow with flight speed

Thermal efficiency depends on flight speed only (Tt4 doesn't influence on it). For higher Ma thermal efficiency for Tt4=1000K is different than for higher Tt4s due to other air and exhaust gas parameters

Propulsive and overall efficiencies depend on flight speed and Tt4, and for higher Tt4 they are lower.

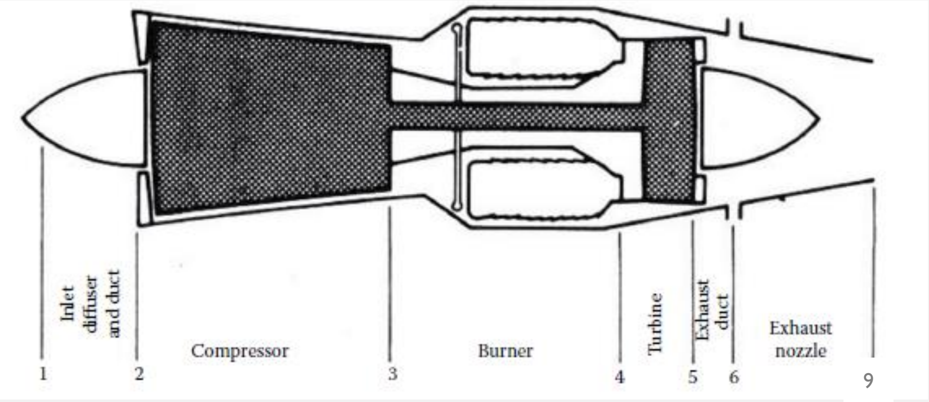
IDEAL TURBOJET ENGINE

Engine work in static conditions V0=0 → $P_{t0} = P_0,\ \ T_{t0} = T_0$

Ram pressure recovery for flight condition ($M_0 > 0$)

$$P_{t0} = P_0\left(1 + \frac{k-1}{2}{M_0}^2\right)^{k/(k-1)} \quad T_{t0} = T_0\left(1 + \frac{k-1}{2}{M_0}^2\right)$$

No **INLET** losses → $P_{t2} = P_{t0}$ and $T_{t2} = T_{t0}$



COMPRESSOR (2 – 3)

Compressor work is isentropic

$$\frac{T_{t3}}{T_{t2}} = \left(\frac{P_{t3}}{P_{t2}}\right)^{(k-1)/k} = {\pi_C}^{(k-1)/k}$$

Compressor work:

$$W_C = Cp(T_{t3} - T_{t2})$$

Compressor power:

$$P_C = \dot{m}_C W_C = \dot{m}_0 Cp(T_{t3} - T_{t2})$$

COMBUSTOR (3 – 4)

Energy balance

$$\dot{m}_f FHV = \dot{m}_0 Cp_B(T_{t4} - T_{t3})$$

$$\dot{m}_f = \frac{\dot{m}_0 Cp_B(T_{t4} - T_{t3})}{FHV}$$

Fuel air ratio:

$$\mathrm{f} = \frac{\dot{m}_f}{\dot{m}_0} = \frac{Cp_B(T_{t4} - T_{t3})}{FHV}$$

$$\pi_B = 1 \rightarrow P_{t4} = P_{t3}$$

TURBINE (4 - 5)

Compressor turbine power balance equation

$$P_C = \dot{m}_0 Cp(T_{t3} - T_{t2}) = P_T = \dot{m}_T Cp_T(T_{t4} - T_{t5})$$

Turbine mass flow: $\dot{m}_T = \dot{m}_0 + \dot{m}_f$

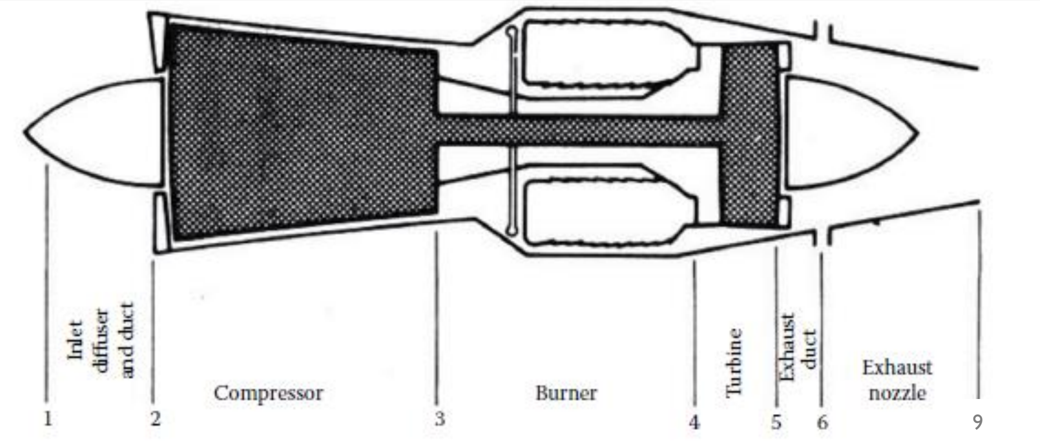
Turbine outlet temperature:

$$T_{t5} = T_{t4} - \frac{Cp(T_{t3} - T_{t2})}{Cp_T(1+f)}$$

Turbine outlet pressure:

$$P_{t5} = P_{t4}\left(\frac{T_{t5}}{T_{t4}}\right)^{k_t/(k_t-1)}$$

IDEAL TURBOJET ENGINE – GASS FULL EXPANSION IN THE NOZZLE



NOZZLE (5-9)

No losses: $\pi_N = 1 \rightarrow \quad P_{t9} = P_{t5}$ and $T_{t9} = T_{t5}$

Full expansion: $P_9 = P_0$

$$c_9 = \sqrt{2Cp_t(T_{9t} - T_9)}$$

- for incompressible flow

$$c_9 = a_9 M_9 = \sqrt{k_t R_t T_9} * \sqrt{\frac{2}{k_t - 1}\left(\frac{T_{t9}}{T_9} - 1\right)}$$

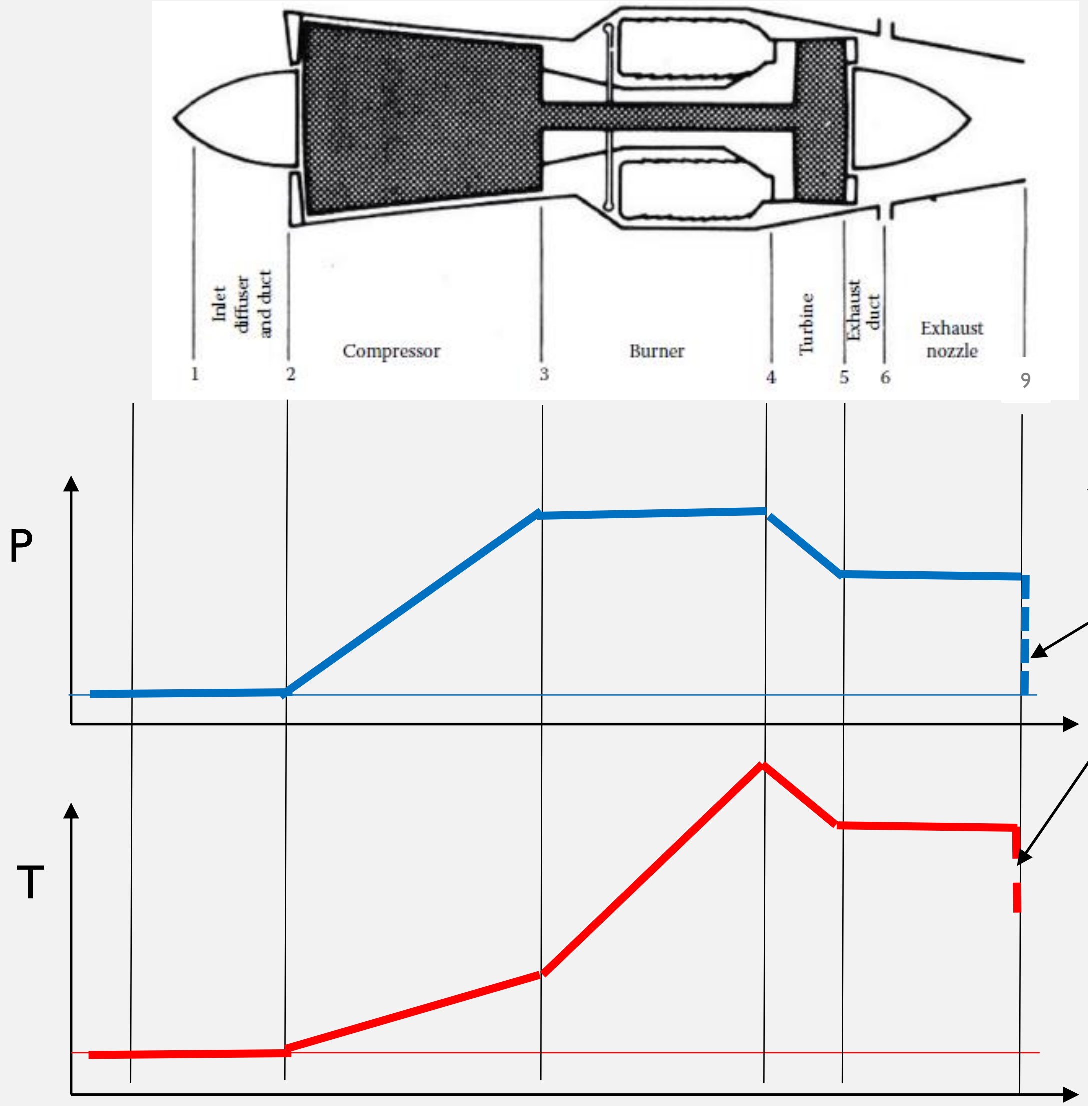
- for compressible flow

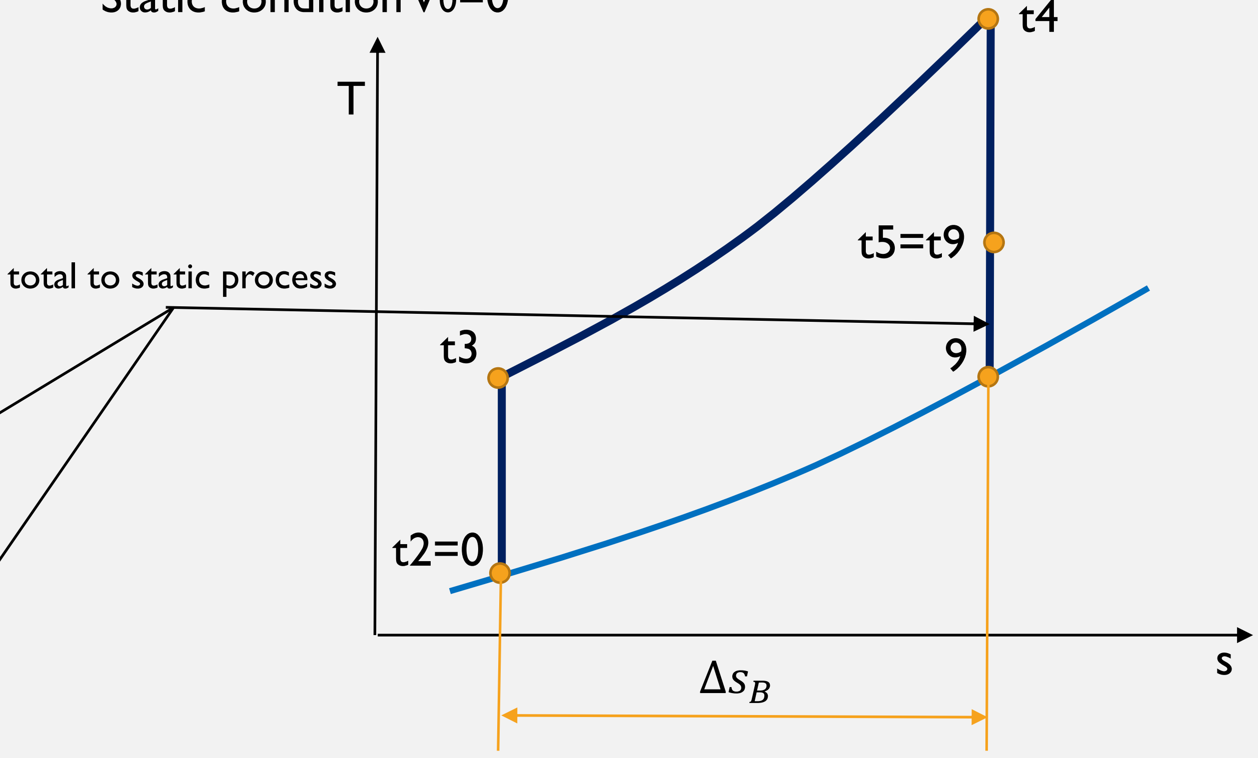
$$\frac{T_{t9}}{T_9} = \frac{P_{t9}}{P_9}^{(k_t-1)/k_t}$$

IDEAL TURBOJET ENGINE

Proesses in the inlet (diffuser) burner and nozzle are idel – no pressure losses $\boldsymbol{\pi_D = \pi_B = \pi_N = 1}$

Static condition V0=0





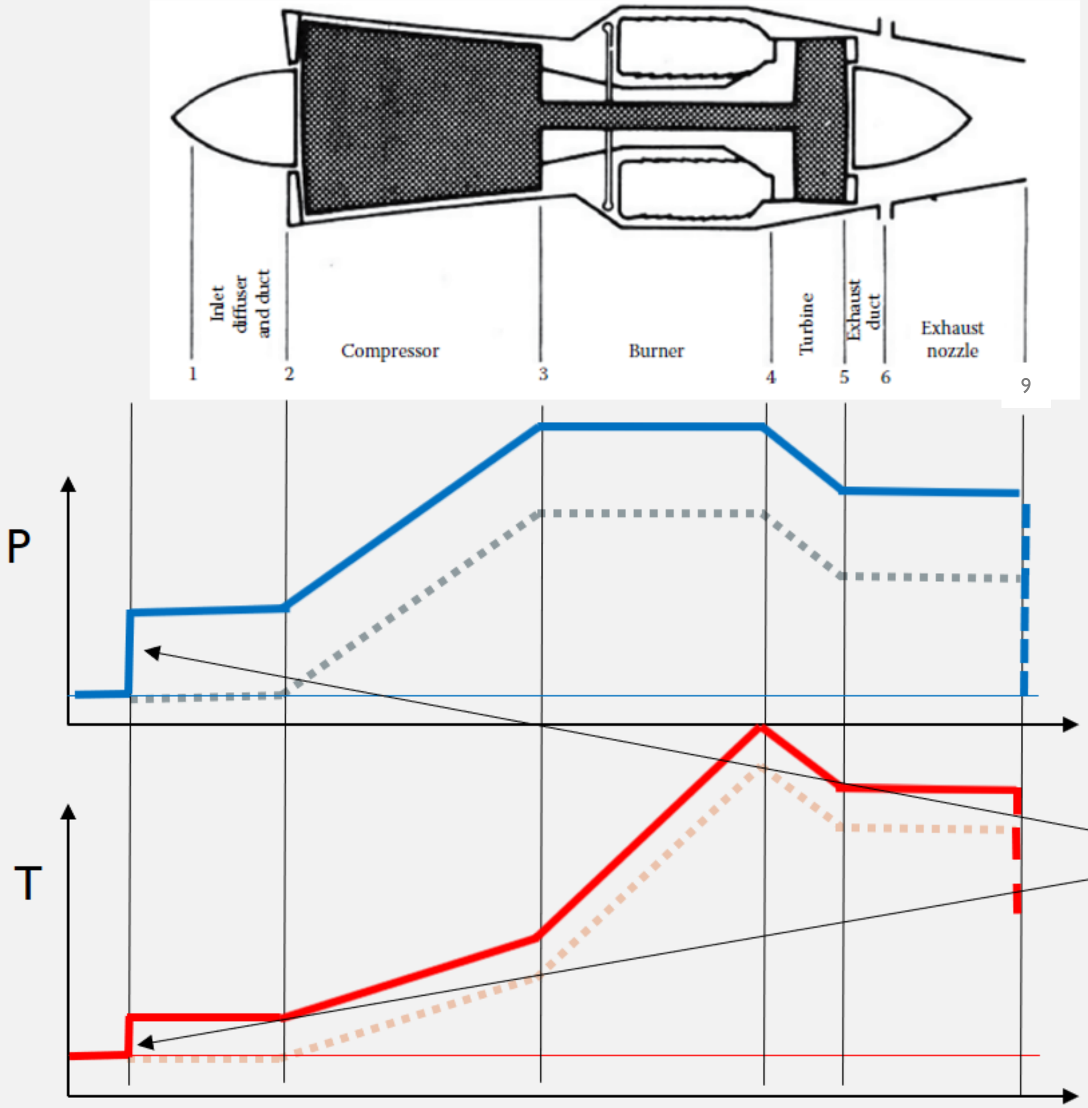
Entropy increase in a burner $\Delta s_B = cp_B ln\frac{T_{t4}}{T_{t3}}$

IDEAL TURBOJET ENGINE

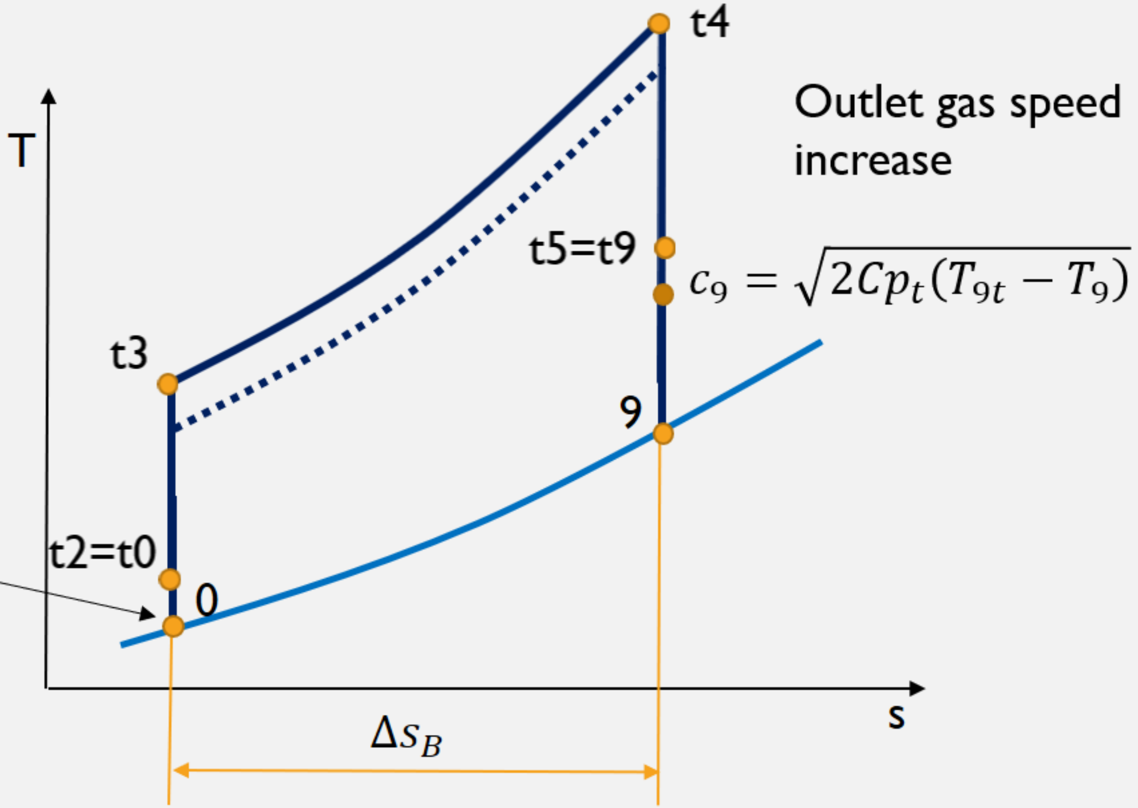
Flight condition V0>0

$$P_{t0} = P_0\left(1 + \frac{k-1}{2}M_0^{\,2}\right)^{k/(k-1)}$$

$$T_{t0} = T_0\left(1 + \frac{k-1}{2}M_0^{\,2}\right)$$



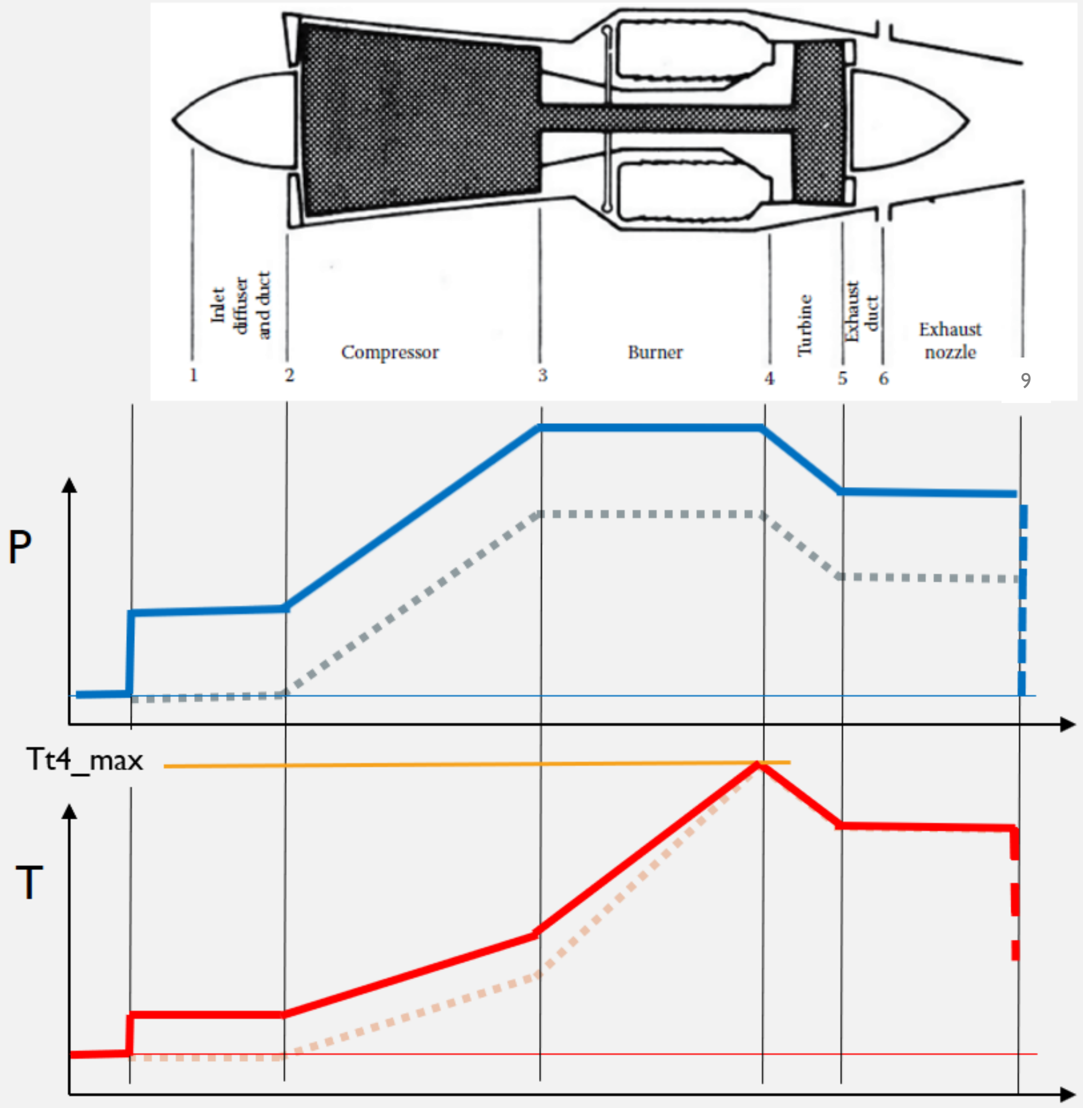
Ram effect



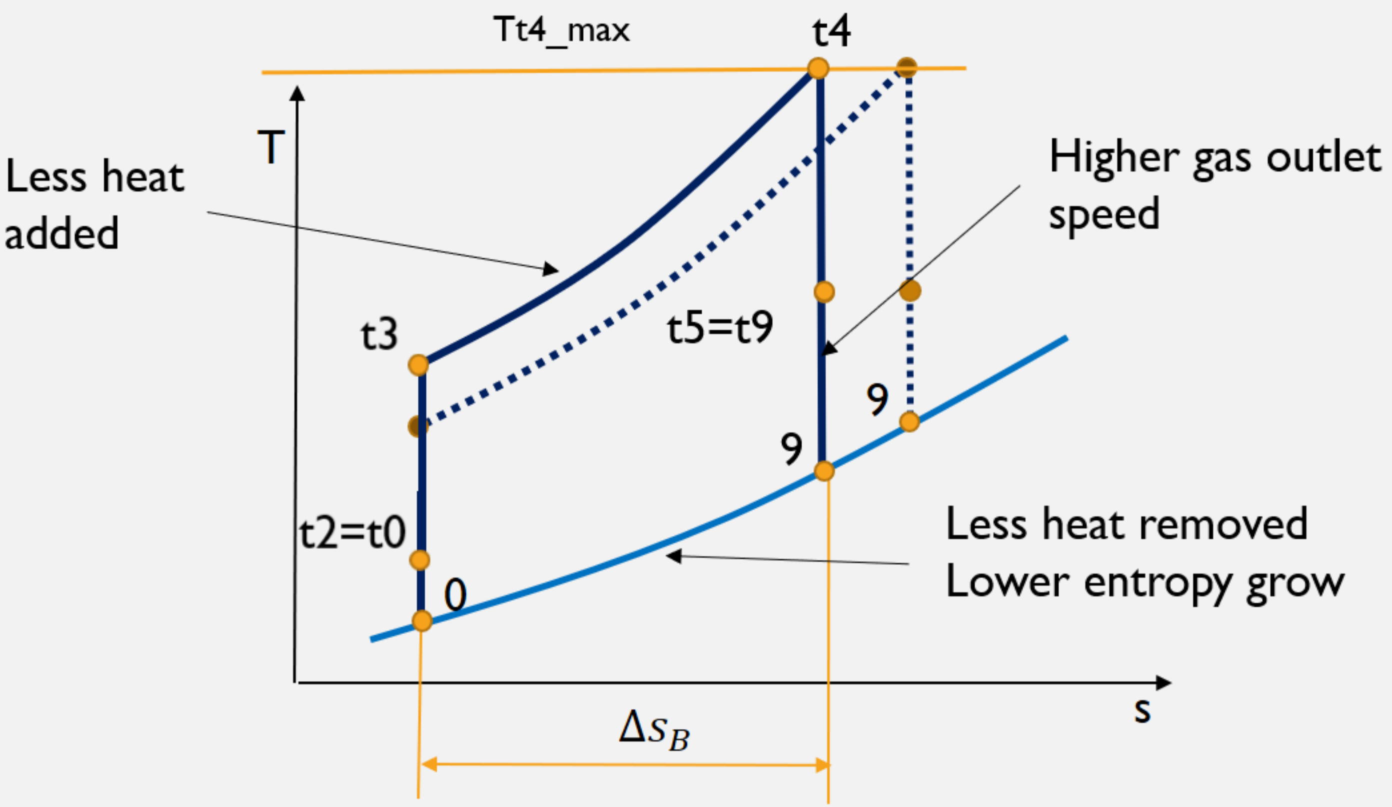
Outlet gas speed increase

$$c_9 = \sqrt{2Cp_t(T_{9t} - T_9)}$$

IDEAL TURBOJET ENGINE – FLIGHT SPEED INFLUENCE



Flight condition $V_0>0$ and Tt4 limit Tt4 max



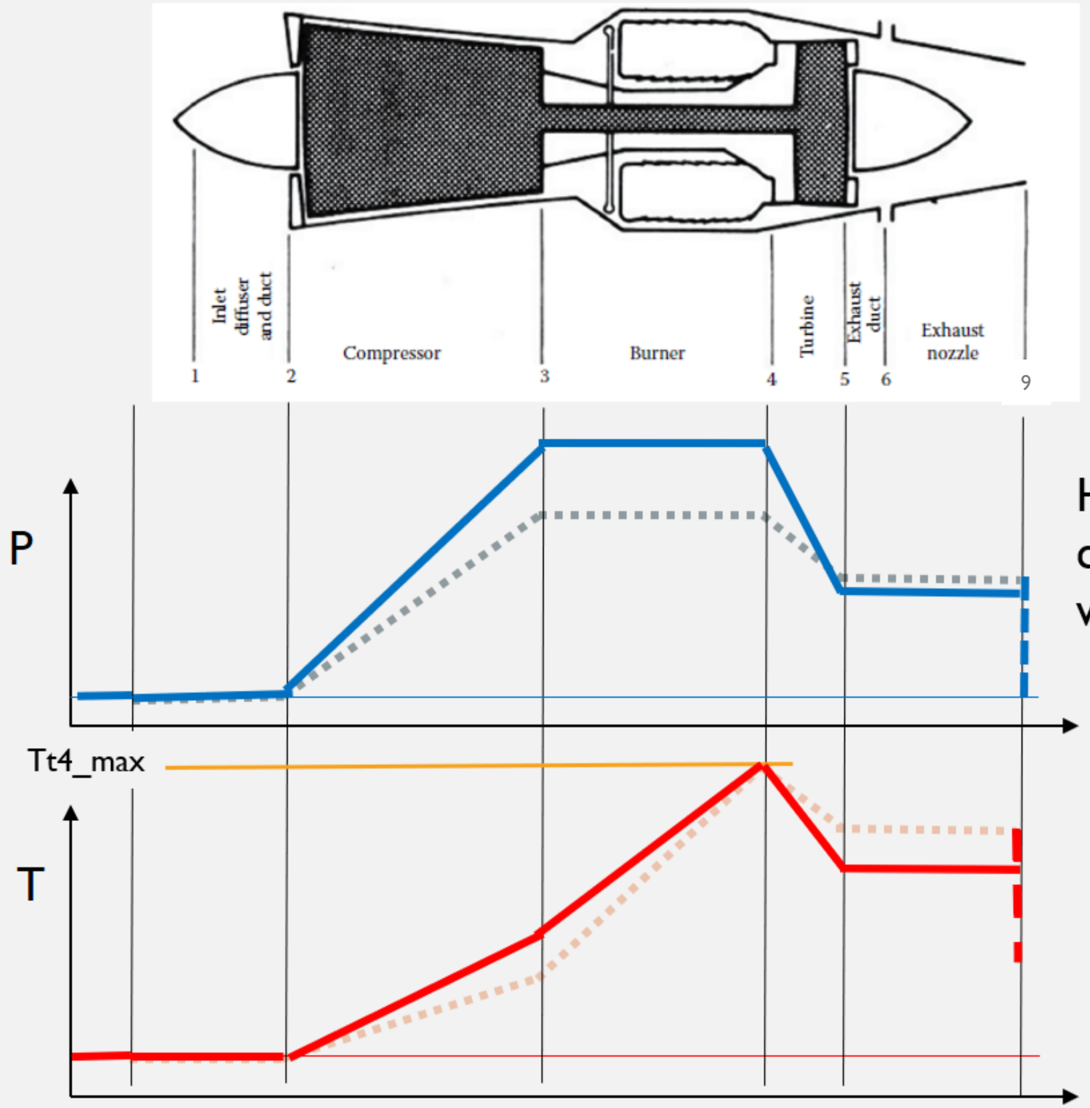
Less heat added

Higher gas outlet speed

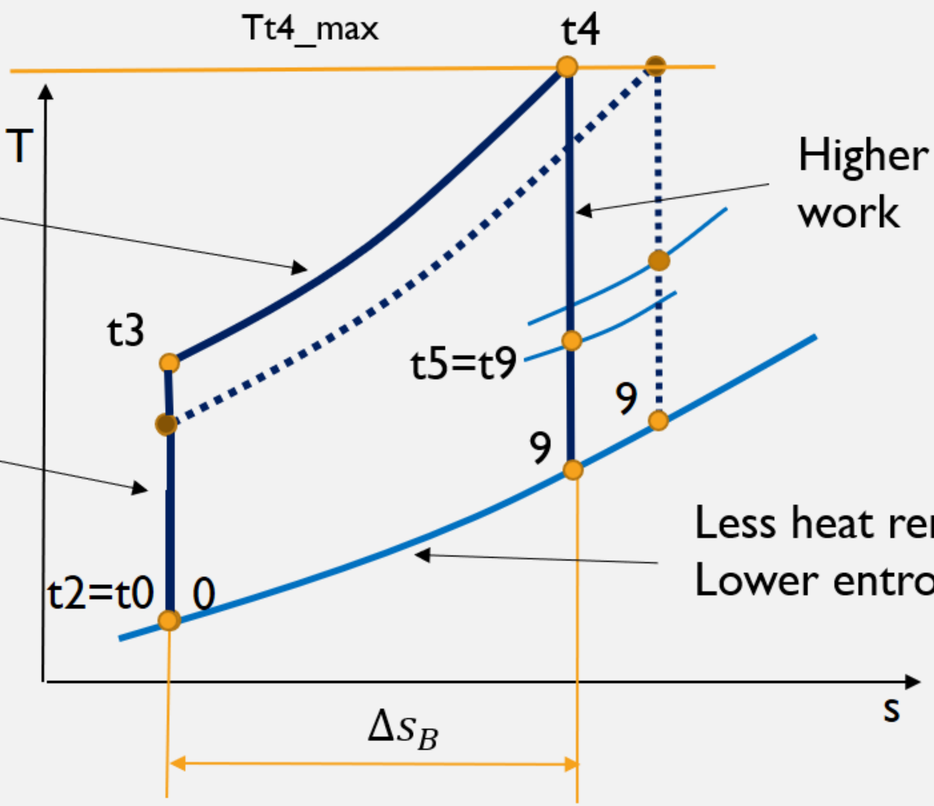
Less heat removed
Lower entropy grow

Engine thermal efficiency grow (less heat removed)
Lower fuel consumption (less heat added)

IDEAL TURBOJET ENGINE – COMPRESSOR PRESSURE RATIO INFLUENCE



CPR (compressor pressure ratio) growing and Tt4 is limited Tt4_max



Less heat added

Higher compressor work

Higher turbine work

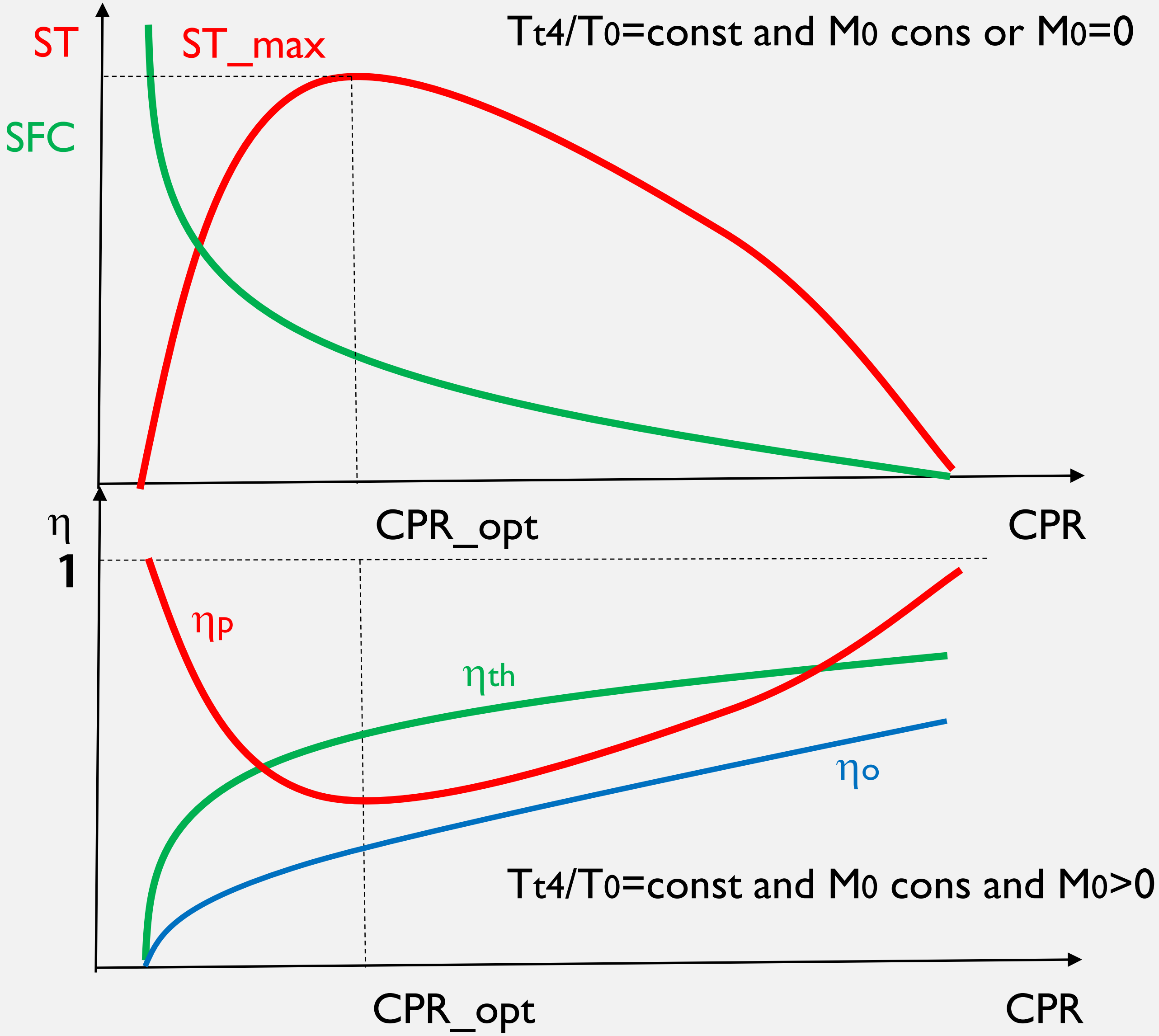
Less heat removed
Lower entropy grow

Engine thermal efficiency grow (less heat removed)
Lower fuel consumption (less heat added) and lower SFC

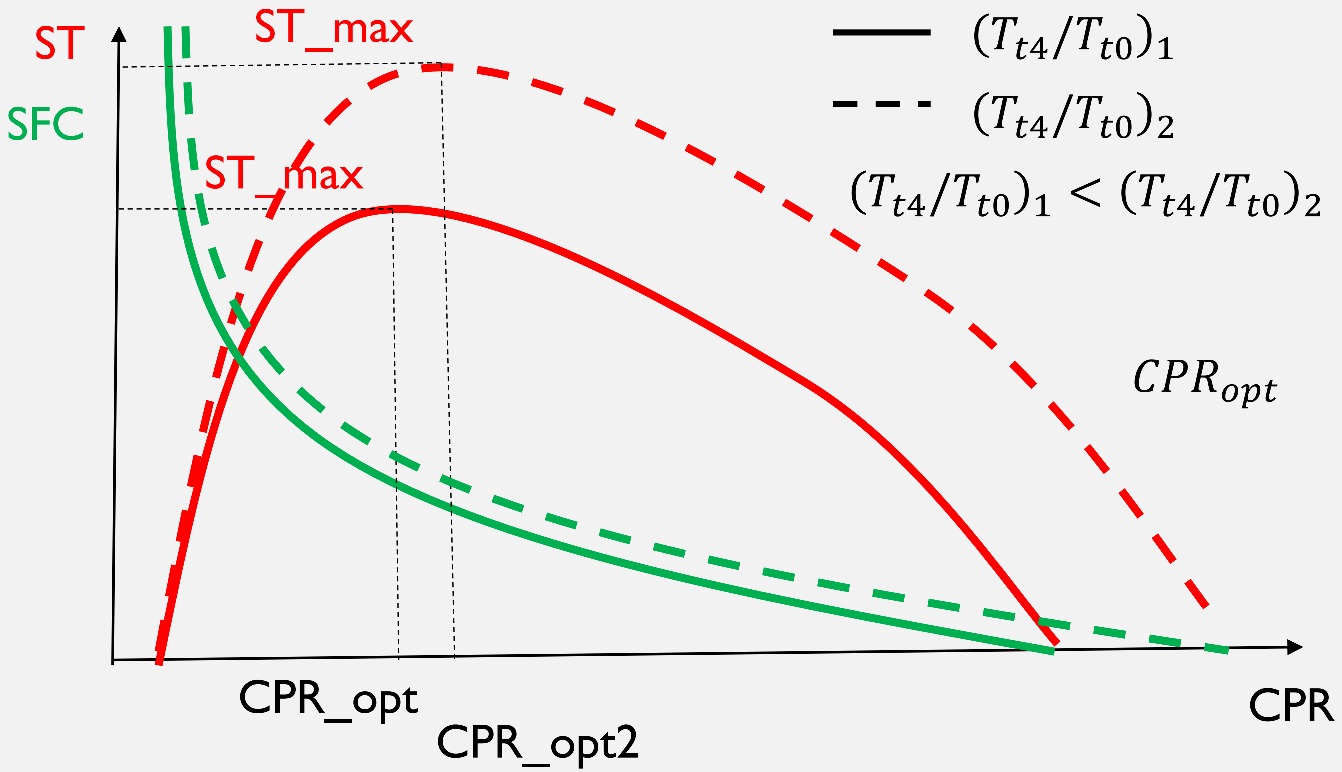
IDEAL TURBOJET CYCLE OPTIMISATION

SUMMARY:

- Specific thrust (ST) grows with compressor pressure ratio increasing, achieves maximum for optimal CPR than is goes down
- Specific fuel consumption decreases with CPR growing
- Propulsive efficiency as a function of CPR represents oposit relation to ST, it is minimal for optimal CPR and achieves 1 for ST=0.
- Thermal and overall efficiencies grow with CPR
- The presented relationship between efficiencies and CPR are right for M0>0, for M0=0, propulsive and overall efficiency are 0



IDEAL TURBOJET CYCLE OPTIMISATION FOR DIFFERENT ENGINE TEMPERATURE RATIO



For ideal cycle:

$$CPR_{opt} = T_{t4}/T_{t0}^{\frac{k}{2(k-1)}} \qquad CPR_{max} = CPR_{opt}^{\;2}$$

SUMMARY:

- Specific thrust (ST) is higher for higher engine themperature ratio Tt4/Tt0 and achieve ST_max for higher CPR (higher CPR_opt)
- Specific fuel consumption decreases with CPR growing, but for highrt Tt4/Tt0 is higher
- Range of available CPR increases for higher Tt4/Tt0

Tt4/Tt0	CPR_opt	CPR_max
4	11,3	128
5	16,7	279,5
6	23	529

THANKS FOR YOUR ATENTION

Questions and Comments ?

1. …….
2. …….
3. …….