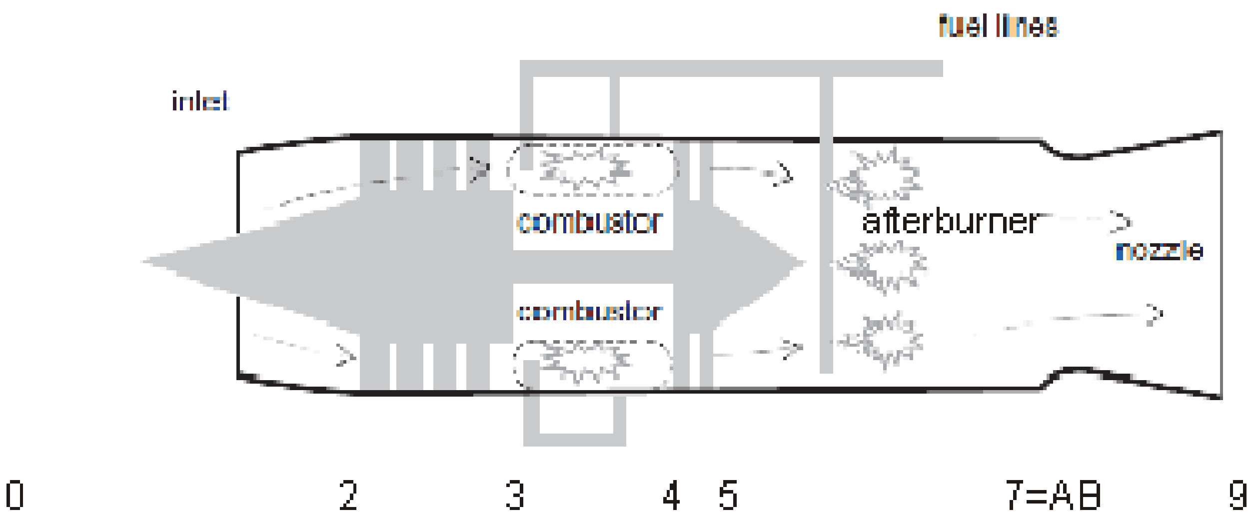
Aircraft Engine Construction - real turbojet engine

Elaborated by phd Robert JAKUBOWSKI, Rzeszow University of Technology,

Table of Contents

TURBOJET ENGINE with LOSSES

En example of turbojet engin calculation is presented below. Three cases were calculated: turbojet engine, turbojet engine with afterburner and turbojet engine with incomplete expansion in propeling nozzle.



Given

T0=217 K, P0=22 kPa, M0=0.9, compressor pressure ratio 12, Turbine inlet temperature Tt4=1300 K, mass flow m=20 kg/s. For Afterburner ON: TtAB=1750 K

inlet pressure losses coefficient σ_{IN} 0.97, burner pessure losses coefficient σ_B 0.98, nozzle pressure losses coefficient σ_N 0.96, compressor efficeincy η_C 0.83, turbine efficiency η_T 0.9, burner efficiency η_B 0.98, mechanical efficiency η_M= 0.99. For AB ON state afterburner efficiency η_{AB} 0.95, additional afterburner pressure losses coeficient σ_{AB} 0.98

Gas parameters:

Air: k=1.4; cp=1005 J/kg/K, R=287 J/kg/K, Fumes in turbine and nozzle kt=1.33, cpt=1170 J/kg/K, Rt=290 J/kg/K,

Fumes for Afterburner and nozzle in AB ON mode kAB=1.3, cpAB=1200 J/kg/K, RAB=297 J/kg/K,

For combustion in combustor cpB=1200 J/kg/K,

Fuel heat value: FHV=43 MJ/kg

Flight Mach No

```
M0 = 0.9000
```

Air Mass flow [kg/s]

```
m0 = 20
```

Turbine inlet temperature [K]

```
Tt4 = 1300
```

Compressor pressure ratio

```
CPR = 12
```

Afterburner temperature [K]

```
TtAB = 1750
```

Ambient conditions

Static temperature [K]

```
T0 = 217
```

Static pressure   [Pa]

```
P0 = 22000
```

TURBOJET ENGINE WITHOUT AFTERBURNER (AB-OFF) - section 7 is disregarded

Section 0

Total temperature   [K]

$T_{t0} = T_0\left(1 + \frac{k-1}{2}M_0^2\right)$ - like for ideal engine

```
Tt0 = 252.1540
```

Total pressure   [Pa]

$P_{t0} = P_0\left(1 + \frac{k-1}{2}M_0^2\right)^{\frac{k}{k-1}}$ - like for ideal engine

```
Pt0 = 3.7209e+04
```

Speed of sound   [m/s]

$a_0 = \sqrt{k * R * T_0}$ - like for ideal engine

```
a0 = 295.2805
```

Flight speed [m/s]

$V_0 = M_0 * a_0$ - like for ideal engine

```
V0 = 265.7525
```

Section 2 Compressor inlet

Total temperature [K]

$T_{t2} = T_{t0}$ - like for ideal engine

```
Tt2 = 252.1540
```

Total pressure [Pa]

$P_{t2} = \sigma_{\mathrm{IN}} * P_{t0}$

```
Pt2 = 3.6092e+04
```

Section 3 - Compressor outlet / Burner inlet

Total temperature [K]

$$T_{t3} = T_{t2} * \left(1 + \frac{\mathrm{CPR}^{\frac{k-1}{k}} - 1}{\eta_C}\right)$$

```
Tt3 = 566.2641
```

Total pressure [Pa]

$P_{t3} = P_{t2} * \mathrm{CPR}$

```
Pt3 = 4.3311e+05
```

COMPRESSOR

Compressor work [J/kg]

$W_C = c_p * (T_{t3} - T_{t2})$

```
WC = 3.1568e+05
```

Compressor power [W]

$P_C = m_0 * W_C$

```
PC = 6.3136e+06
```

Section 4 Burner outlet / Turbine inlet

Total temperature [K]

T_{t4}

```
Tt4 = 1300
```

Total pressure [Pa]

$$P_{t4} = \sigma_B * P_{t3}$$

```
Pt4 = 4.2445e+05
```

BURNER

Fuel-air ratio

$$f_B = c_{pB} * \frac{T_{t4} - T_{t3}}{\mathrm{FHV} * \eta_B}$$

```
fB = 0.0209
```

Fuel mass flow [kg/s]

$$m_{fB} = m_0 * f_B$$

```
mfB = 0.4179
```

Section 5 Turbine outlet / Nozzle inlet

Total temperature [K]

$$T_{t5} = T_{t4} - \frac{W_C}{\eta_M * (1 + f_B) * c_{pt}}$$

```
Tt5 = 1.0330e+03
```

Total pressure [Pa]

$$P_{t5} = P_{t4}\left(\frac{\eta_T + \frac{T_{t5}}{T_{t4}} - 1}{\eta_T}\right)^{\frac{kt}{kt-1}}$$

```
Pt5 = 1.4945e+05
```

Section 9 Engine Nozzle outlet

Total temperature [K]

$$T_{t9} = T_{t5}$$

```
Tt9 = 1.0330e+03
```

Total pressure [Pa]

$$P_{t9} = P_{t5} * \sigma_N$$

```
Pt9 = 1.4347e+05
```

Static pressure [Pa]

$$P_9 = P_0$$

```
P9 = 22000
```

Static temperature [K]

$$T_9 = T_{t9} * \left(\frac{P_9}{P_{t9}}\right)^{\frac{kt-1}{kt}}$$

```
T9 = 648.7254
```

Jet stream Mach No

$$M_9 = \sqrt{\left(\frac{T_{t9}}{T_9} - 1\right) * \frac{2}{kt - 1}}$$

```
M9 = 1.8948
```

Speed of sound [m/s]

$$a_9 = \sqrt{kt * Rt * T_9}$$

```
a9 = 500.2133
```

Jet speed [m/s]

$$V_9 = M_9 * a_9$$

```
V9 = 947.8210
```

TURBOJET ENGINE PERFORMANCE CALCULATION

Thrust [N]

$$T = m_0 * (1 + f_B) * V_9 - m_0 * V_0$$

```
T = 1.4037e+04
```

Specific thrust [Ns/kg]

$$\mathrm{ST} = \frac{T}{m_0} = (1 + f_B) * V_9 - V_0$$

```
ST = 701.8725
```

Specific fuel consumption [kg/N/s]

$$\mathrm{SFC} = \frac{m_{fB}}{T}$$

```
SFC = 2.9769e-05
```

Specific fuel consumption [kg/N/h]

$$\mathrm{SFC} = \mathrm{SFC} * 3600$$

```
SFC = 0.1072
```

Thermal efficiency

$$\eta_{th} = \frac{(1 + f_B) * {V_9}^2 - {V_0}^2}{2 * f_B * \mathrm{FHV}}$$

```
etha_th = 0.4711
```

Propulsive efficiency

$$\eta_p = \frac{2 * V_0 * \mathrm{ST}}{(1 + f_B) * {V_9}^2 - {V_0}^2}$$

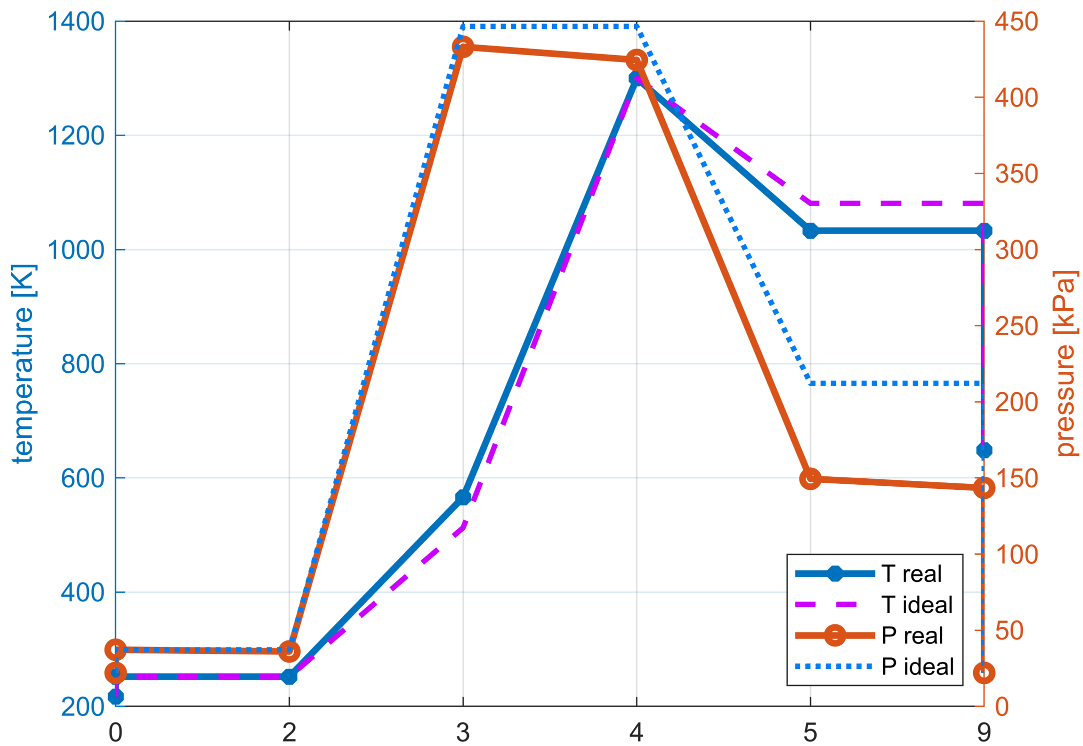
```
etha_p = 0.4407
```

Overall efficiency

$$\eta_o = \frac{V_0 * \mathrm{ST}}{f_B * \mathrm{FHV}} = \eta_{\mathrm{th}} * \eta_p$$

```
etha_o = 0.2076
```

Temperature, pressure vs engine sections plot




```
Tabela = 8×5 table
```

	Section	T real [K]	T ideal [K]	P real [kPa]	P ideal [kPa]
1	'0'	217	217	22	22
2	't0'	252.1540	252.1540	37.2087	37.2087
3	't2'	252.1540	252.1540	36.0924	37.2087
4	't3'	566.2641	512.8654	433.1089	446.5040
5	't4'	1300	1300	424.4467	446.5040
6	't5'	1.0330e+03	1.0809e+03	149.4512	212.1852
7	't9'	1.0330e+03	1.0809e+03	143.4731	212.1852

	Section	T real [K]	T ideal [K]	P real [kPa]	P ideal [kPa]
8	'9'	648.7254	615.9567	22	22

Performance comparison of real and ideal turbojet engine

```
Tabela = 8×4 table
```

	Parameter	Unit	Real turbojet	Ideal turbojet
1	'Thrust'	'kN'	14.0374	15.9926
2	'Specific Thrust'	'N*s/kg'	701.8725	799.6289
3	'Fuel consumption'	'kg/s'	0.4179	0.4393
4	'Specific fuel consump'	'kg/N/h'	0.1072	0.0989
5	'therm. efficiency'	'-'	0.4711	0.5505
6	'prop. efficiency'	'-'	0.4407	0.4087
7	'overall efficiency'	'-'	0.2076	0.2250
8	'V9'	'm/s'	947.8210	1.0425e+03

CONCLUSIONS

Real to ideal jet engine comparison shows

- Total pressure in engine sections is lower in real engine
- Total temperature after compressor is higher, but after turbine is lower in real engine
- Higher temperature after compressor causes lower fuel consumption of the real engine - TIT (turbine inlet temperature) is the same in both cases
- Lower total temperature in the nozzle inlet and higher static temperature in the nozzle outlet causes lower outlet flow velocity and by this way lower thrust and specific thrust of real jet engine
- Specific fuel consumption is higher due to lwer thrust and thermal and overall efficiences are lower in real engine

Temperature - entropy plot

Entropy growth is calculated from equations:

Inlet entropy grow [J/kg/K] :

$$\Delta s_{\mathrm{IN}} = -R * \ln(\sigma_{\mathrm{IN}})$$

```
dS_IN = 8.7418
```

Compressor entropy grow [J/kg/K] :

$$\Delta s_C = c_p * \ln\frac{T_{t3}}{T_{t2}} - R * \ln(\mathrm{CPR})$$

```
dS_C = 99.8974
```

Combustor entropy grow [J/kg/K] :

$$\Delta s_B = c_{pB} * \ln\frac{T_{t4}}{T_{t3}} - R_t * \ln(\sigma_B)$$

```
dS_B = 1.0031e+03
```

Turbine entropy grow   [J/kg/K] :

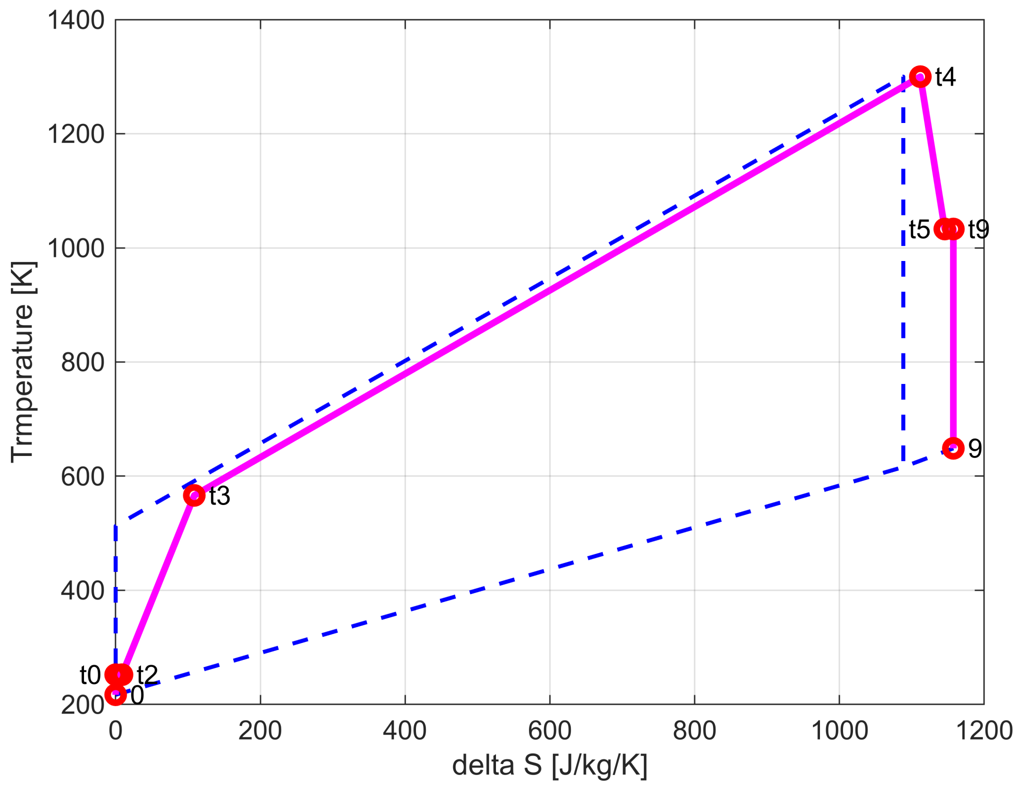
$$\Delta s_T = c_{pt} * \ln\frac{T_{t5}}{T_{t4}} - R_t * \ln\left(\frac{P_{t5}}{P_{t4}}\right)$$

```
dS_T = 33.7726
```

Nozzle entropy grow   [J/kg/K] :

$$\Delta s_N = -\mathrm{Rt} * \ln(\sigma_N)$$

```
dS_N = 11.8384
```



CONCLUSIONS

All processes genarte entropy increase.Significant entropy growth is in the burner, compressor and turbine.

TURBOJET ENGINE WITH AFTERBURNER (AB-ON)

Caclculation done for turbojet engine for afterburner OFF mode from section 0 to 5 is valid for the engine with the afterburner ON mode. Differences statr from AB (7) section.

Section AB - AFTERBURNER

Total temperature [K]

T_{tAB}

```
TtAB = 1750
```

Total pressure [Pa]

$P_{\mathrm{tAB}} = \sigma_{\mathrm{AB}} * P_{\mathrm{t5}}$

```
PtAB = 1.4945e+05
```

Fuel-air ratio

$$f_{\mathrm{AB}} = (1 + f_B) * c_{\mathrm{pAB}} * \frac{T_{\mathrm{tAB}} - T_{\mathrm{t5}}}{\mathrm{FHV} * \eta_{\mathrm{AB}}}$$

```
fAB = 0.0224
```

Afterburner fuel mass flow [kg/s]

$m_{\mathrm{fAB}} = m_0 * f_{\mathrm{AB}}$

```
mfAB = 0.4479
```

Section 9 AB ON

Total temperature [K]

$T_{\mathrm{t9AB}} = T_{\mathrm{tAB}}$

```
Tt9AB = 1750
```

Total pressure [Pa]

$P_{\mathrm{t9AB}} = \sigma_N * P_{\mathrm{tAB}}$

```
Pt9AB = 1.4347e+05
```

Staticl pressure [Pa]

$P_{\mathrm{9AB}} = P_0$

```
P9AB = 22000
```

Static temperature [K]

$$T_{\mathrm{9AB}} = T_{\mathrm{t9AB}} * \left(\frac{P_{\mathrm{9AB}}}{P_{\mathrm{t9AB}}}\right)^{\frac{\mathrm{kAB}-1}{\mathrm{kAB}}}$$

```
T9AB = 1.1353e+03
```

Jet stream Mach No

$$M_{\mathrm{9AB}} = \sqrt{\left(\frac{T_{\mathrm{t9AB}}}{T_{\mathrm{9AB}}} - 1\right) * \frac{2}{\mathrm{k_{AB}} - 1}}$$

```
M9AB = 1.8999
```

Speed of sound [m/s]

$$a_{9\mathrm{AB}} = \sqrt{\mathrm{k}_{\mathrm{AB}} * R_{\mathrm{AB}} * T_{9\mathrm{AB}}}$$

```
a9AB = 662.0727
```

Jet speed [m/s]

$$V_{9\mathrm{AB}} = M_{9\mathrm{AB}} * a_{9\mathrm{AB}}$$

```
V9AB = 1.2579e+03
```

TURBOJET ENGINE with AFTERBURNER - PERFORMANCE CALCULATION

Total fuel-air ratio

$$f = f_B + f_{\mathrm{AB}}$$

```
f_AB = 0.0433
```

Total fuel consumption [kg/s]

$$m_f = m_{\mathrm{fB}} + m_{\mathrm{fAB}}$$

```
mf_AB = 0.8658
```

Thrust [N]

$$T_{\mathrm{AB}} = m_0 * (1 + f) * V_{9\mathrm{AB}} - m_0 * V_0$$

```
T_AB = 2.0931e+04
```

Specific thrust [Ns/kg]

$$\mathrm{ST}_{\mathrm{AB}} = \frac{T_{\mathrm{AB}}}{m_0} = (1 + f) * V_{9\mathrm{AB}} - V_0$$

```
ST_AB = 1.0466e+03
```

Specific fuel consumption [kg/N/s]

$$\mathrm{SFC}_{\mathrm{AB}} = \frac{m_f}{T_{\mathrm{AB}}}$$

```
SFC_AB = 4.1365e-05
```

Specific fuel consumption [kg/N/h]

$$\mathrm{SFC}_{\mathrm{AB}} = \mathrm{SFC}_{\mathrm{AB}} * 3600$$

```
SFC_AB = 0.1489
```

Thermal efficiency

$$\eta_{\mathrm{thAB}} = \frac{(1 + f) * {V_{9\mathrm{AB}}}^2 - {V_0}^2}{2 * f * \mathrm{FHV}}$$

```
etha_th_AB = 0.4244
```

Propulsive efficiency

$$\eta_{\mathrm{pAB}} = \frac{2 * V_0 * \mathrm{ST}_{\mathrm{AB}}}{(1 + f) * {V_{9\mathrm{ANB}}}^2 - {V_0}^2}$$

```
etha_p_AB = 0.2361
```

Overall efficiency

$$\eta_{\mathrm{oAB}} = \frac{V_0 * \mathrm{ST}_{\mathrm{AB}}}{f * \mathrm{FHV}} = \eta_{\mathrm{thAB}} * \eta_{\mathrm{pAB}}$$

```
etha_o_AB = 0.1002
```

TURBOJET ENGINE AFTERBURNER OFF/ ON COMPARISON

```
Tabela = 8×5 table
```

	Parameter	Unit	AB OFF	AB ON	AB ON ideal
1	'Thrust'	'kN'	14.0374	20.9314	22.9077
2	'Specific Thrust'	'N*s/kg'	701.8725	1.0466e+03	1.1454e+03
3	'V9'	'm/s'	947.8210	1.2579e+03	1.3545e+03
4	'Fuel consumption'	'kg/s'	0.4179	0.8658	0.8369
5	'Specific fuel consump'	'kg/N/h'	0.1072	0.1489	0.1315
6	'therm. efficiency'	'-'	0.4711	0.4244	0.5115
7	'prop. efficiency'	'-'	0.4407	0.2361	0.2309
8	'overall efficiency'	'-'	0.2076	0.1002	0.1181

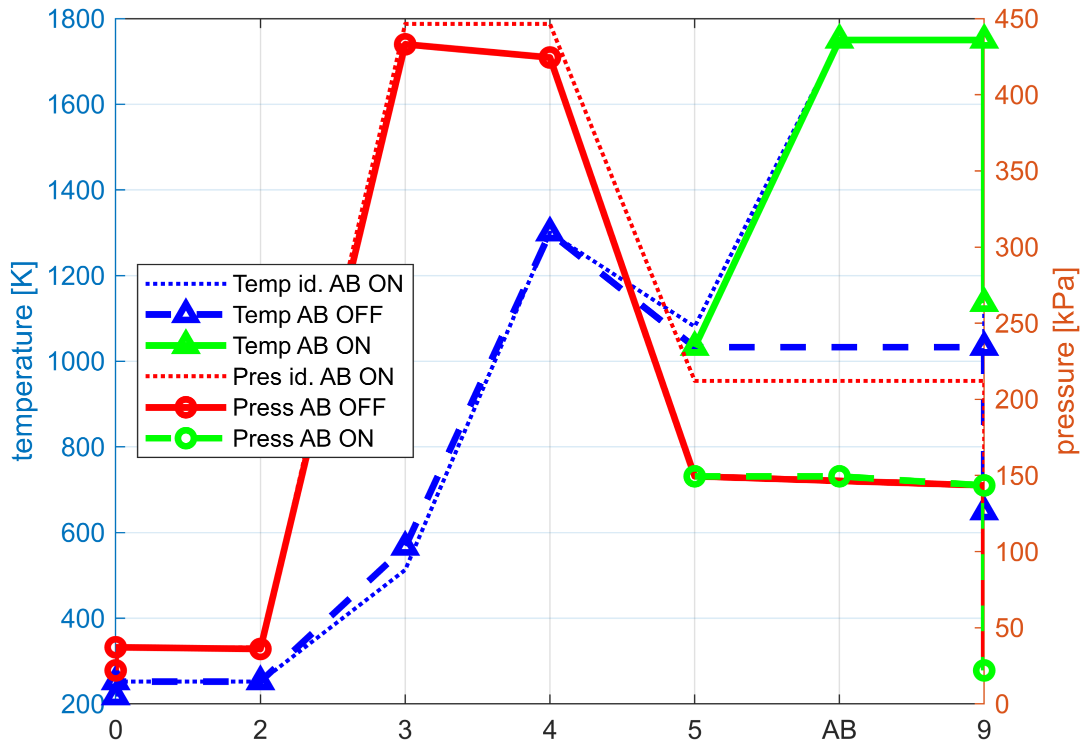
CONCLUSIONS:

Engine with AB ON has:

- higher thrust and specific thust due to higher jet speed (+)
- significantly higher fuel consumption and specific fuel consumption (-)
- lower thermal and overall efficiences (-)

Real engine has worse performance efficiency than ideal engine

Temperature, pressure plot vs engine sections for ideal and real turbojet with AB ON



CONCLUSIONS:

When AB is on than pressure profile in the engine is unchanged and temperature profile is hanged from AB section (temperature is significantly higher in afterburner and propelling nozzle)

Temperature - entropy plot

Afterburner entropy growth [J/kg/K]

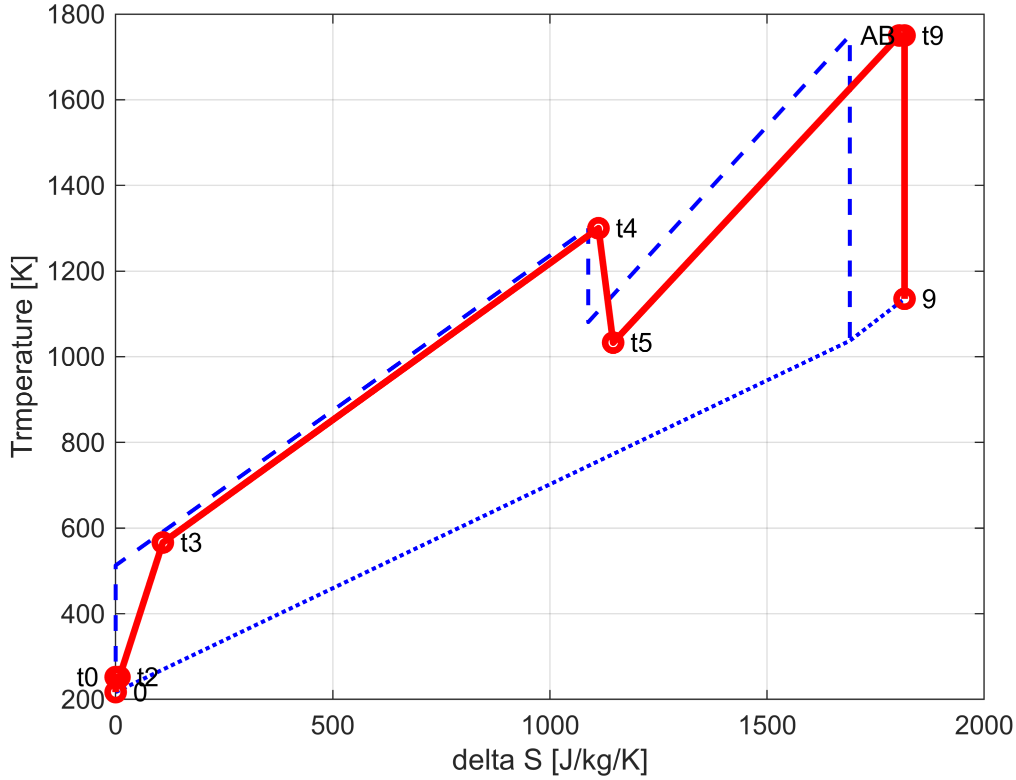
$$\Delta s_{\mathrm{AB}} = c_{\mathrm{pAB}} * \ln \frac{T_{\mathrm{tAB}}}{T_{\mathrm{t5}}} - R_{\mathrm{AB}} * \ln \frac{P_{\mathrm{tAB}}}{P_{\mathrm{t5}}}$$

```
dS_AB = 658.8873
```

Nozzle entropy growth [J/kg/K]

$$\Delta s_N = -R_{\mathrm{AB}} * \ln(\sigma_N)$$

```
dS_N = 12.1241
```



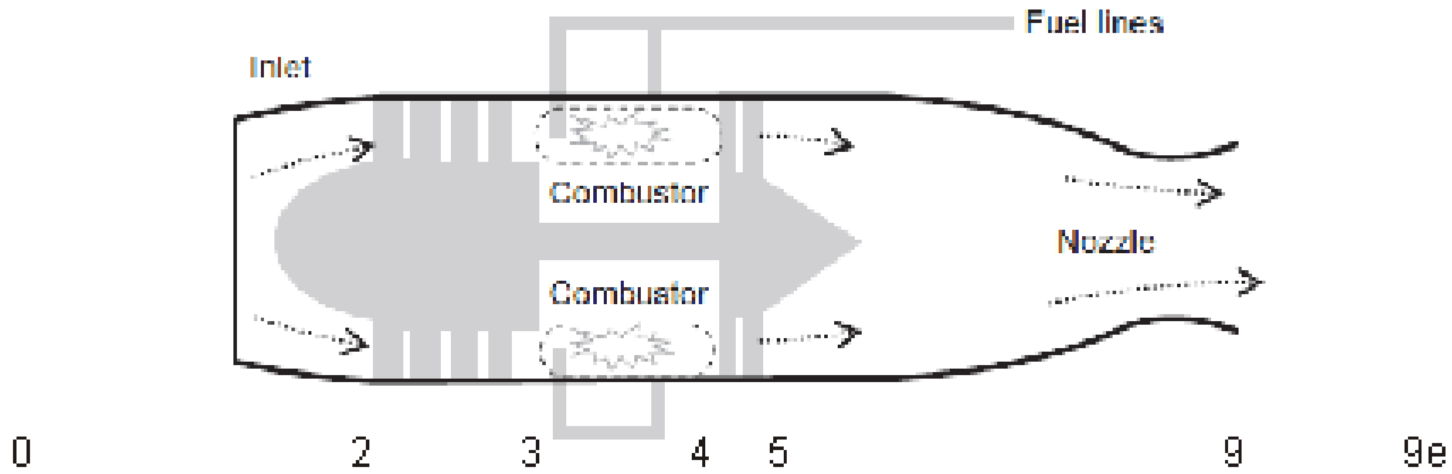
Conclusions:

In AB ON mode additional entropy increase is in the afterburner process

TURBOJET ENGINE - INCOMPLETE EXPANSION IN THE PROPELLING NOZZLE

Calculation example of a turbojet engine with incomplete expansion in the propelling nozzle is presented on the example done above for turbojet engin without afterburner. Calcilation conditions are the same like for turbojet engine without afterburner (first example). The difference is only that the static pressure in section 9 isn't equal ambient pressure, but is higher. In the example p9=Pt9/1.85, than expansion is conducted outside the nozzle in free stream process.

To present the calculation the additional section 9e is added - in this section jet stream acheieve ambient pressure.



All parameters from section 0 to 9t are the same as for previosly was calculated for the engine without afterburner. The difference is in section 9 static parameters calculation

Section 9 - incomplete decompression in the nozzle

Total pressure [Pa]

P_{t9}

```
Pt9 = 1.4347e+05
```

Total temperature [K]

T_{t9}

```
Tt9 = 1.0330e+03
```

Static pressure [Pa]

$$P_{9\,\mathrm{IE}} = \frac{P_{\mathrm{t9}}}{1.85}$$

```
P9_IE = 7.7553e+04
```

Static temperature   [K]

$$T_{9\,\mathrm{IE}} = T_{\mathrm{t9}} * \left(\frac{P_{9\,\mathrm{IE}}}{P_{\mathrm{t9}}}\right)^{\frac{\mathrm{kt}-1}{\mathrm{kt}}}$$

```
T9_IE = 886.8015
```

Mach No. in section 9

$$M_{9\,\mathrm{IE}} = \sqrt{\left(\frac{T_{\mathrm{t9}}}{T_{9\mathrm{IE}}} - 1\right) * \frac{2}{\mathrm{k}_t - 1}}$$

```
M9_IE = 0.9997
```

Speed of sound [m/s]

$$a_{9\,\mathrm{IE}} = \sqrt{\mathrm{k}_t * R_t * T_{9\,\mathrm{IE}}}$$

```
a9_IE = 584.8413
```

Jet speed in section 9 [m/s]

$$V_{9\,\mathrm{IE}} = M_{9\,\mathrm{IE}} * a_{9\,\mathrm{IE}}$$

```
V9_IE = 584.6740
```

Gas density in section 9 [kg/m^3]

$$\rho_{9\,\mathrm{IE}} = \frac{P_{9\,\mathrm{IE}}}{R_t * T_{9\,\mathrm{IE}}}$$

```
RO9_IE = 0.3016
```

Section 9e

Jet speed after decompression to ambient pressure [m/s]

$$V_{9\,e} = V_{9\,\mathrm{IE}} + \frac{P_{9\,\mathrm{IE}} - P_0}{\rho_{9\,\mathrm{IE}} * V_{9\,\mathrm{IE}}}$$

```
V9e = 899.7531
```

Static temperature [K]

$$T_{9\mathrm{e}} = T_{\mathrm{t}9} - \frac{{V_{9e}}^2}{2 * c_{\mathrm{pt}}}$$

```
T9e = 687.0761
```

Staticl pressure [Pa]

$$P_{9\,e} = P_0$$

```
P9e = 22000
```

PERFORMANCE OF TURBOJET ENGINE WITH INCOMPLTE EXPANSION IN THE NOZZLE

Thrust [N]

$$T_{\mathrm{IE}} = m_0 * (1 + f_B) * V_{9e} - m_0 * V_0$$

```
T_IE = 1.3056e+04
```

Specific thrust [Ns/kg]

$$\mathrm{ST}_{\mathrm{IE}} = \frac{T_{\mathrm{IE}}}{m_0} = (1 + f_B) * V_{9e} - V_0$$

```
ST_IE = 652.8003
```

Specific fuel consumption [kg/N/s]

$$\mathrm{SFC}_{\mathrm{IE}} = \frac{m_{\mathrm{fB}}}{T_{\mathrm{IE}}}$$

```
SFC_IE = 3.2007e-05
```

Specific fuel consumption   [kg/N/h]

$$\mathrm{SFC}_{\mathrm{IE}} = \mathrm{SFC}_{\mathrm{IE}} * 3600$$

```
SFC_IE = 0.1152
```

Thermal efficiency

$$\eta_{\mathrm{th\ IE}} = \frac{(1+f_B) * {V_{9}}_e^2 - V_0^2}{2 * f_B * \mathrm{FHV}}$$

```
etha_th_IE = 0.4206
```

Propulsive efficiency

$$\eta_{p\ \mathrm{IE}} = \frac{2 * V_0 * \mathrm{ST}_{\mathrm{IE}}}{(1+f_B) * {V_{9}}_e^2 - V_0^2}$$

```
etha_p_IE = 0.4590
```

Overall efficiency

$$\eta_{o\ \mathrm{IE}} = \frac{V_0 * \mathrm{ST}_{\mathrm{IE}}}{f_B * \mathrm{FHV}} = \eta_{\mathrm{th\ IE}} * \eta_{p\ \mathrm{IE}}$$

```
etha_o_IE = 0.1931
```

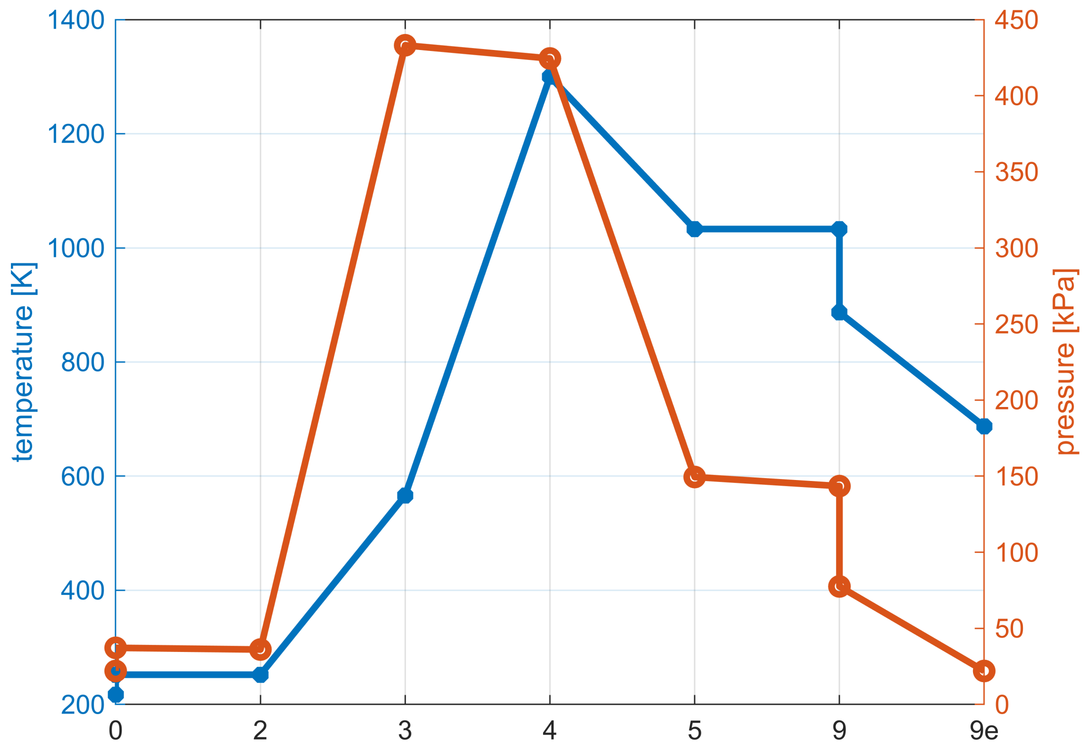
FULL EXPANSION IN PROPELING NOZZLE vs. ICOMPLETE EXPANSION IN PROPELING NOZZLE ENGINE PERFORMANCE COMPARISON

```
Tabela = 13×4 table
```

	Parameter	Unit	FULL EXP.	INCOMPLETE EXP.
1	'Tt9'	'K'	1.0330e+03	1.0330e+03
2	'Pt9'	'kPa'	143.4731	143.4731
3	'T9'	'K'	648.7254	886.8015
4	'V9'	'm/s'	947.8210	584.6740
5	'P9'	'kPa'	22	77.5530
6	'T9e'	'K'	648.7254	687.0761
7	'V9e'	'm/s'	947.8210	899.7531
8	'Thrust'	'kN'	14.0374	13.0560
9	'Specific Thrust'	'N*s/kg'	701.8725	652.8003
10	'Specific fuel consump'	'kg/N/h'	0.1072	0.1152
11	'therm. efficiency'	'-'	0.4711	0.4206
12	'prop. efficiency'	'-'	0.4407	0.4590

	Parameter	Unit	FULL EXP.	INCOMPLETE EXP.
13	'overall efficiency'	'-'	0.2076	0.1931

Temperature, pressure vs engine sections plot

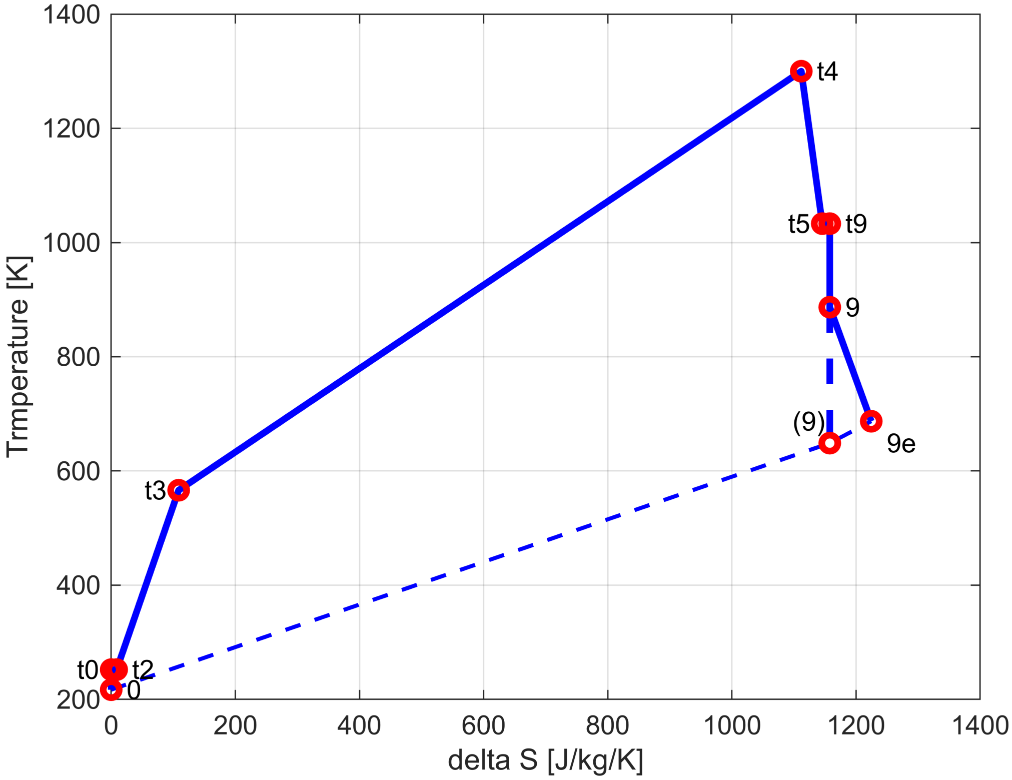


Temperature - entropy plot

Additional entropy growth 9-9e [J/kg/K]

$$\Delta s_{9-9e} = c_{\mathrm{pt}} * \ln \frac{T_{9e}}{T_{9\,\mathrm{IE}}} - R_t * \ln \frac{P_{9e}}{P_{9\,\mathrm{IE}}}$$

```
deltaS_9_9e = 66.8206
```



CONCLUSIONS:

Incomplete expansion in the engine propelling nozzle causes:

- Lower thrust and specific thrust than it is in full decompression mode
- Higher specific fuel consumption
- Additional entropy increas caused by jet decompression outside the nozzle

TURBOJET ENGINE WITH CONVERGENT NOZZLE and WITHOUT AFTERBURNER

Given

T0=288 K, P0=100 kPa, M0=0, compressor pressure ratio 4 and 8, Turbine inlet temperature Tt4=1100 K, mass flow m=20 kg/s.

inlet pressure losses coefficient σ_{IN} 0.97, burner pessure losses coefficient σ_B 0.98, nozzle pressure losses coefficient σ_N 0.96, compressor efficeincy η_C 0.8, turbine efficiency η_T 0.85, burner efficiency η_B 0.98, mechanical efficiency η_M= 0.99. Engine exit nozzle is convergent

Gas parameters:

Air: k=1.4; cp=1005 J/kg/K, R=287 J/kg/K,

Fumes in turbine and nozzle kt=1.33, cpt=1170 J/kg/K, Rt=290 J/kg/K,

For combustion in combustor cpB=1200 J/kg/K, Fuel heat value: FHV=43 MJ/kg

Flight Mach No

```
M0 = 0
```

Air Mass flow [kg/s]

```
m0 = 20
```

Turbine inlet temperature [K]

```
Tt4 = 1100
```

Compressor pressure ratio (first option)

```
CPR1 = 4
```

Compressor pressure ratio (second option)

```
CPR2 = 8
```

Ambient conditions

Static temperature [K]

```
T0 = 288
```

Static pressure   [Pa]

```
P0 = 100000
```

CALCULATION

Section 0

Total temperature   [K]

$T_{t0} = T_0\left(1 + \frac{k-1}{2}M_0^2\right)$ - like for ideal engine

```
Tt0 = 288
```

Total pressure   [Pa]

$P_{t0} = P_0\left(1 + \frac{k-1}{2}M_0^2\right)^{\frac{k}{k-1}}$ - like for ideal engine

```
Pt0 = 100000
```

Speed of sound   [m/s]

$a_0 = \sqrt{k * R * T_0}$ - like for ideal engine

```
a0 = 340.1741
```

Flight speed   [m/s]

$V_0 = M_0 * a_0$ - like for ideal engine

```
V0 = 0
```

Section 2 Compressor inlet

Total temperature   [K]

$T_{t2} = T_{t0}$ - like for ideal engine

```
Tt2 = 288
```

Total pressure   [Pa]

$P_{t2} = \sigma_{IN} * P_{t0}$

```
Pt2 = 97000
```

Section 3 - Compressor outlet / Burner inlet

Total temperature   [K]

$$T_{t3} = T_{t2} * \left(1 + \frac{\mathrm{CPR}^{\frac{k-1}{k}} - 1}{\eta_C}\right)$$

first option for CPR1

```
Tt3_1 = 462.9579
```

second option for CPR2

```
Tt3_2 = 580.1210
```

Total pressure   [Pa]

$P_{t3} = P_{t2} * \mathrm{CPR}$

first option for CPR1

```
Pt3_1 = 388000
```

second option for CPR2

```
Pt3_2 = 776000
```

COMPRESSOR

Compressor work   [J/kg]

$W_C = c_p * (T_{t3} - T_{t2})$

first option for CPR1

```
WC_1 = 1.7583e+05
```

second option for CPR2

```
WC_2 = 2.9358e+05
```

Section 4 Burner outlet / Turbine inlet

Total temperature   [K]

T_{t4}

```
Tt4 = 1100
```

Total pressure [Pa]

$P_{t4} = \sigma_B * P_{t3}$

first option for CPR1

```
Pt4_1 = 380240
```

second option for CPR2

```
Pt4_2 = 760480
```

BURNER

Fuel-air ratio

$$f_B = c_{pB} * \frac{T_{t4} - T_{t3}}{\mathrm{FHV} * \eta_B}$$

first option for CPR1

```
fB_1 = 0.0181
```

second option for CPR2

```
fB_2 = 0.0148
```

Fuel mass flow [kg/s]

$m_{fB} = m_0 * f_B$

first option for CPR1

```
mfB_1 = 0.3628
```

second option for CPR2

```
mfB_2 = 0.2961
```

Section 5 Turbine outlet / Nozzle inlet

Total temperature [K]

$$T_{t5} = T_{t4} - \frac{W_C}{\eta_M * (1 + f_B) * c_{pt}}$$

first option for CPR1

```
Tt5_1 = 950.9023
```

second option for CPR2

```
Tt5_2 = 850.2385
```

Total pressure [Pa]

$$P_{t5} = P_{t4}\left(\frac{\eta_T + \frac{T_{t5}}{T_{t4}} - 1}{\eta_T}\right)^{\frac{kt}{kt-1}}$$

first option for CPR1

```
Pt5_1 = 1.8880e+05
```

second option for CPR2

```
Pt5_2 = 2.1733e+05
```

Section 9 Engine Nozzle outlet

Total temperature   [K]

$$T_{t9} = T_{t5}$$

first option for CPR1

```
Tt9_1 = 950.9023
```

second option for CPR2

```
Tt9_2 = 850.2385
```

Total pressure [Pa]

$$P_{t9} = P_{t5} * \sigma_N$$

first option for CPR1

```
Pt9_1 = 1.8125e+05
```

second option for CPR2

```
Pt9_2 = 2.0864e+05
```

Critical pressure parameter

$$\beta_{cr} = \left(\frac{1 + k_t}{2}\right)^{\frac{kt}{kt-1}}$$

```
beta_cr = 1.8506
```

Checking of outlet nozzle pressure ratio vs critical pressure parameter

first option for CPR1

$P_{t9}/P_0 =$

```
ans = 1.8125
```

second option for CPR2

$P_{t9}/P_0 =$f

```
ans = 2.0864
```

In the first option Pt9/P0<β_{cr} therefore the full expansion is in the nozzle.

In the second option Pt9/P0>β_{cr}therefore gas expansion in the nozzle is to critical parameters. Expansion process is continued outside the nozzle.

Calculation of full expansion in the nozzle - first option

Static pressure [Pa]

$$P_9 = P_0$$

```
P9_1 = 100000
```

Static temperature   [K]

$$T_9 = T_{t9} * \left(\frac{P_9}{P_{t9}}\right)^{\frac{\mathrm{kt}-1}{\mathrm{kt}}}$$

```
T9_1 = 820.4530
```

Jet stream Mach No

$$M_9 = \sqrt{\left(\frac{T_{t9}}{T_9} - 1\right) * \frac{2}{\mathrm{kt} - 1}}$$

```
M9_1 = 0.9816
```

Speed of sound   [m/s]

$$a_9 = \sqrt{\mathrm{kt} * \mathrm{Rt} * T_9}$$

```
a9_1 = 562.5377
```

Jet speed   [m/s]

$$V_9 = M_9 * a_9$$

```
V9_1 = 552.2094
```

PERFORMANCE CALCULATION

Thrust   [N]

$$T = m_0 * (1 + f_B) * V_9 - m_0 * V_0$$

```
T_1 = 1.1245e+04
```

Specific thrust   [Ns/kg]

$$\mathrm{ST} = \frac{T}{m_0} = (1 + f_B) * V_9 - V_0$$

```
ST_1 = 562.2269
```

Specific fuel consumption   [kg/N/s]

$$\mathrm{SFC} = \frac{m_{\mathrm{fB}}}{T}$$

```
SFC_1 = 3.2266e-05
```

Specific fuel consumption   [kg/N/h]

$$\mathrm{SFC} = \mathrm{SFC} * 3600$$

```
SFC_1 = 0.1162
```

Thermal efficiency

$$\eta_{\mathrm{th}} = \frac{(1 + f_B) * {V_9}^2 - {V_0}^2}{2 * f_B * \mathrm{FHV}}$$

```
etha_th_1 = 0.1990
```

Propulsive efficiency

$$\eta_p = \frac{2 * V_0 * \mathrm{ST}}{(1 + f_B) * {V_9}^2 - {V_0}^2}$$

```
etha_p_1 = 0
```

Overall efficiency

$$\eta_o = \frac{V_0 * \mathrm{ST}}{f_B * \mathrm{FHV}} = \eta_{\mathrm{th}} * \eta_p$$

```
etha_o_1 = 0
```

Calculation of critical expansion in the nozzle - second option

Static pressure [Pa]

$$P_9 = \frac{P_{\mathrm{t9}}}{\beta_{\mathrm{cr}}}$$

```
P9_2 = 1.1274e+05
```

Static temperature   [K]

$$T_9 = T_{\mathrm{t9}} * \left(\frac{P_9}{P_{\mathrm{t9}}}\right)^{\frac{\mathrm{kt}-1}{\mathrm{kt}}}$$

```
T9_2 = 729.8184
```

Gas velocity = Speed of sound   [m/s]

$$V_9 = a_9 = \sqrt{\mathrm{k}_t * R_t * T_{9\,\mathrm{IE}}}$$

```
V9_2 = 530.5572
```

Gas density in section 9 [kg/m^3]

$$\rho_9 = \frac{P_9}{R_t * T_9}$$

```
RO9_2 = 0.5327
```

Section 9e

Jet speed after decompression to ambient pressure [m/s]

$$V_{9\,e} = V_9 + \frac{P_9 - P_0}{\rho_9 * V_9}$$

```
V9e = 575.6343
```

Static temperature   [K]

$$T_{9\mathrm{e}} = T_{\mathrm{t}9} - \frac{{V_{9_e}}^2}{2 * c_{\mathrm{pt}}}$$

```
T9e = 708.6338
```

Staticl pressure   [Pa]

$$P_{9\,e} = P_0$$

```
P9e = 100000
```

PERFORMANCE OF TURBOJET ENGINE WITH INCOMPLTE EXPANSION IN THE NOZZLE

Thrust   [N]

$$T = m_0 * (1 + f_B) * V_{9e} - m_0 * V_0$$

```
T_2 = 1.1683e+04
```

Specific thrust   [Ns/kg]

$$\mathrm{ST} = \frac{T}{m_0} = (1 + f_B) * V_{9e} - V_0$$

```
ST_2 = 584.1562
```

Specific fuel consumption   [kg/N/s]

$$\mathrm{SFC} = \frac{m_{\mathrm{fB}}}{T_{\mathrm{IE}}}$$

```
SFC_2 = 2.5343e-05
```

Specific fuel consumption   [kg/N/h]

$$\mathrm{SFC} = \mathrm{SFC} * 3600$$

```
SFC_2 = 0.0912
```

Thermal efficiency

$$\eta_{\mathrm{th}} = \frac{(1 + f_B) * {V_{9\,e}}^2 - {V_0}^2}{2 * f_B * \mathrm{FHV}}$$

```
etha_th_2 = 0.2641
```

Propulsive efficiency

$$\eta_p = \frac{2 * V_0 * \mathrm{ST}_{\mathrm{IE}}}{(1 + f_B) * {V_9}_e^2 - V_0^2}$$

```
etha_p_2 = 0
```

Overall efficiency

$$\eta_o = \frac{V_0 * \mathrm{ST}}{f_B * \mathrm{FHV}} = \eta_{\mathrm{th}} * \eta_p$$

```
etha_o_2 = 0
```

Option 1 vs Option 2 comparison

```
Tabela = 14×4 table
```

	Parameter	Unit	FULL EXP. (first option)	CRITICAL EXP. (second option)
1	'CPR'	'-'	4	8
2	'Tt9'	'K'	950.9023	850.2385
3	'Pt9'	'kPa'	181.2470	208.6362
4	'T9'	'K'	820.4530	729.8184
5	'V9'	'm/s'	552.2094	530.5572
6	'P9'	'kPa'	100	112.7395
7	'T9e'	'K'	820.4530	708.6338
8	'V9e'	'm/s'	552.2094	575.6343
9	'Thrust'	'kN'	11.2445	11.6831
10	'Specific Thrust'	'N*s/kg'	562.2269	584.1562
11	'Specific fuel c...	'kg/N/h'	0.1162	0.0912
12	'therm. efficiency'	'-'	0.1990	0.2641
13	'prop. efficiency'	'-'	0	0
14	'overall efficiency'	'-'	0	0